Published in "Journal of risk and insurance", Volume 89, Issue 1, 2022, pp. 5-34, which should be cited to refer to this work.
DOI: 10.1111/jori.12330

Incentive and welfare effects of correlated returns

Christophe Courbage[*], Richard Peter[†], Béatrice Rey[‡]

Abstract

We provide a microeconomic analysis of the incentive and welfare effects of correlated returns. While most of the existing literature has focused on risky returns as an aggregate shock, we introduce correlation between returns and the individual's non-financial endowment. Using a simple consumption-saving model with two periods, time-separable utility, and two states allows us to rewrite the correlated return in terms of a transfer rate that measures the spread between the return in the good and the bad state. We find that a critical level of the transfer rate separates savers from borrowers. We also identify restrictions on the individual's risk preferences for a larger transfer rate to raise optimal saving. We analyze the welfare effects of correlated returns by characterizing the transfer rate that maximizes intertemporal expected utility. The welfare benefits of correlated returns derive from their insurance effects.

Keywords: correlation attitude · risk preferences · saving · correlated returns

JEL-Classification: D14 · D15 · D60 · D81 · E21 · E43

[*]Geneva School of Business Administration (HES-SO), E-Mail: christophe.courbage@hesge.ch.

[†]University of Iowa, Department of Finance, E-Mail: richard-peter@uiowa.edu.

[‡]Universite de Lyon and CNRS, GATE Lyon Saint-Etienne, E-Mail: rey-fournier@gate.cnrs.fr.

1 Introduction

A rich literature has explored the effects of risk on risk-taking behavior (see Eeckhoudt and Gollier, 2013, for a review). While one strand has focused on independent financial background risk (Kihlstrom et al., 1981; Nachman, 1982; Gollier and Pratt, 1996), the importance of dependent background risk has long been recognized (Tsetlin and Winkler, 2005; Li, 2011), especially in the context of insurance (Doherty and Schlesinger, 1983; Rey, 2003). In this paper, we contribute to this literature and study the incentive and welfare effects of correlated return risk. In particular, we analyze consumption-saving decisions when the return on investment correlates with a non-financial variable.

The recent experience with Covid-19 and pandemic events more generally illustrate the relevance of shocks that affect people's health and investment returns simultaneously. Even though capital markets rebound relatively fast during Covid-19 (Richter and Wilson, 2020), the long-term effects are yet unclear and different industries are affected differently (Sinagl, 2020). Some argue that todays markets are more susceptible to unforeseen events like a global pandemic due to social-media driven sentiment, the interconnectedness of global supply chains and a pricey stock market.[1]

Several financial tools have correlated return features. Insurance contracts may serve as an example because they only provide an indemnity payment if the policyholder suffers a loss. If no loss happens, the "return" on insurance is zero. Another example are health savings accounts (HSAs).[2] Savings via an HSA are tax-exempt if utilized for medical expenditures, implying a higher realized return on each before-tax dollar in case of sickness. If savings are withdrawn for non-medical consumption, individuals have to repay the tax advantage and face an additional penalty, which lowers the return on saving. So the uncertainty over returns correlates with the individual's health. Retirement accounts and annuities may serve as another example. For retirement accounts, individuals can only access their savings if they reach retirement age. In case of premature death, the accumulated savings can be subject to taxation, reducing their effective return, or might be forfeited altogether in the absence of heirs. At the time contributions are made, the effective return is uncertain and correlated with the individual's life expectancy. Similarly, if individuals annuitize their retirement savings, the net present value of payouts is higher the longer they life. At the time people decide about annuitization, the return is uncertain and correlates with life expectancy.[3]

---

[1] See `https://www.marketwatch.com/` for an inverview with Seema Shah, chief strategist at Principal Global Investors.

[2] Specific forms of HSAs can be found in the U.S., South Africa, Canada, Singapore and China (see Peter et al., 2016). In the U.S., HSAs were introduced in 2004 and have been attracting a growing number of individuals since then covering 21.8 million Americans as of January 2016 (see the AHIP 2017 HSA census available at `https://www.ahip.org/`).

[3] This has given rise to enhanced annuities with higher payouts in case of impaired health (Steinorth, 2012).

We also contribute to the literature on risk-induced saving by identifying restrictions on preferences for clear-cut comparative statics. Leland (1968) and Sandmo (1970) were the first to point out that income risk can create a precautionary demand for saving. Kimball (1990) called such behavior "prudent" and showed that it is equivalent to a positive third derivative of the utility function in the time-separable expected utility model. These works paved the way to study the effect of risks other than income risk on saving behavior. Non-financial risks and, in particular health risks have been shown to affect optimal saving (see, e.g., Jappelli et al., 2007; Nocetti and Smith, 2010, 2011; Denuit et al., 2011; Liu and Menegatti, 2019a). Others have studied interest rate risk and saving behavior. Eeckhoudt and Schlesinger (2008) and Chiu et al. (2012) provide sufficient conditions for an increase in interest rate risk to raise savings. Jouini et al. (2013) obtain necessary and sufficient conditions for Nth-degree risk changes in the interest rate to increase saving.[4] More recently, Liu and Menegatti (2019b) derive the conditions for an interest rate risk and an independent health risk to increase saving. In those papers, return risk represents an aggregate shock to the individual, which is uncorrelated with his endowment.

Only a few papers have looked at dependence in the context of consumption-saving behavior. Courbage and Rey (2007) identify the role of cross-prudence for precautionary saving in response to income risk and a correlated non-financial background risk (see also Menegatti, 2009b). Menegatti (2009a) obtains additional results in the same setting by restricting both risks to be small. Li (2012), Baiardi et al. (2015, 2014) and Magnani and Menegatti (2015) study precautionary saving for labor income risk and interest rate risk and allow for correlation between the two.[5] The only paper that considers return risk and a correlated non-financial background risk is Baiardi et al. (2014) but they restrict the analysis to small risks and work with Taylor approximations. All of these papers focus exclusively on the level of saving. We also investigate whether individuals are actually savers or would rather prefer to borrow and examine welfare. We make no assumption on the size of the risks involved.

We use a simple model for our analysis. We use Kimball's (1990) two-period consumption-saving model with time-separable utility and consider an individual who faces a binary non-financial risk (e.g., health status, environmental quality, life expectancy, etc.). We then introduce dependence between this risk and the rate of return on the individual's deposits. To focus on riskiness, we set the expected return equal to the market rate of return to obtain a mean-preserving spread à la Rothschild and Stiglitz (1970). Under this comparability assumption, we can rewrite the correlated return with the help of a transfer rate that measures the spread between the return in the good and the bad state. Correlated returns have insurance characteristics because they redistribute income across states.

[4] Atalay et al. (2014) and Jindapon et al. (2019) provide evidence that return risk can incentivize savings via prize-linked savings accounts.

[5] The first paper uses positive quadrant dependence, the second one covariance, and the last two restrict the size of both risks to be small and also use covariance as the measure of dependence.

We abstract from institutional features and analyze how correlated returns and preferences jointly determine behavior. When looking at the individual's decision whether to save or to borrow, we find a critical level of the transfer rate. It interacts with the individual's correlation attitude (Epstein and Tanny, 1980) and separates savers from borrowers. When looking at the optimal level of saving (or borrowing), we identify restrictions on partial prudence in wealth and partial cross-prudence in the non-financial attribute so that a larger transfer rate raises optimal saving (or lowers optimal borrowing). We also uncover the welfare benefits of correlated returns. They arise from their insurance characteristics. To bring out the insurance effects more explicitly, we discuss a model of insurance and saving with a correlated return and discuss briefly a possible generalization to multiple states.

The paper is organized as follows. We introduce our model of correlated returns in Section 2. Section 3 analyzes the extensive margin, so whether individuals save or borrow. In Section 4, we examine the optimal level of saving (or borrowing), that is, the intensive margin. In Section 5, we discuss the welfare effects of correlated returns, study the optimal transfer rate and discuss its relationship with insurance. Section 6 presents an extension to multiple states and a final section concludes.

2 The benchmark model

We consider a simple two-period model and an individual whose preferences are characterized by bivariate vNM utility functions $u(w, H)$ and $v(w, H)$ in period one and two, where w denotes wealth or consumption and H a non-financial variable (e.g., health status, life expectancy, *et cetera*). Throughout the analysis, the level of the non-financial variable is certain in the first period, and we suppress it to simplify the notation. We denote by $v^{(i,j)}$ the (i, j)th cross derivative of v with respect to its first and second argument,

$$v^{(i,j)}(w, H) = \frac{\partial^{i+j} v(w, H)}{\partial w^i \partial H^j}, \ i, j \geq 0. \tag{1}$$

For $j = 0$ or $i = 0$, we obtain unidirectional derivatives with respect to only w or only H. We make standard assumptions on individual preferences: u is strictly increasing and concave, $u' > 0$ and $u'' < 0$, and v is strictly increasing and concave in each argument, $v^{(1,0)} > 0$, $v^{(0,1)} > 0, v^{(2,0)} < 0$ and $v^{(0,2)} < 0$. The individual's preferences are non-satiated and risk-averse with respect to each argument.[6]

For now, we impose no restriction on the cross derivative $v^{(1,1)}$. In the terminology of Epstein and Tanny (1980), the individual is said to be correlation loving (neutral, averse) if $v^{(1,1)} > 0$ $(= 0, < 0)$. For such an individual, the marginal utility of consumption increases (remains constant, decreases) in the non-financial variable. If the non-financial variable is

[6] Our main simplifying assumption is intertemporal separability as in Kimball's (1990) model. Some of our results continue to hold if we relax this assumption, which we point out explicitly.

health, existing results suggest that marginal utility of consumption is increasing or constant in health status for severe injuries and decreasing in health status for minor injuries (see Viscusi and Evans, 1990; Evans and Viscusi, 1991; Sloan et al., 1998; Carthy et al., 1998). Finkelstein et al. (2013) estimate that a one-standard deviation increase in the number of chronic diseases leads to a 10-25% decrease in marginal utility of consumption, consistent with correlation loving. Ebert and van de Kuilen (2017) instead find experimental evidence in favor of correlation aversion for the economic domains of time preferences, social preferences and waiting time. The non-financial variable can be a complement or a substitute for consumption in the Edgeworth sense depending on whether preferences are correlation loving or averse. The notion of correlation attitude has recently regained attention in the risk literature (e.g., Eeckhoudt et al., 2007; Denuit et al., 2010; Crainich et al., 2016).

The individual has a certain income of w_1 in the first period. In the second period, he has a certain income of w_2 and faces a binary non-financial risk taking value H_g with probability $(1-p)$ and H_b with probability p such that $H_g > H_b$. Subscripts g and b are shorthand for the good and the bad state. Our specification with a non-financial risk of loss corresponds to Cook and Graham's (1977) setting who study insurance demand for irreplaceable commodities. We will point out some commonalities in Section 5.2. Expected utility in the second period is discounted by the utility discount factor $\beta \leq 1$ to allow for impatience. The individual decides how much to save in the first period. We formulate all our results in terms of a saving decision but we may think of investment or risk-taking more generally. As a benchmark, we first assume a certain rate of return, denoted by r. The individual maximizes expected lifetime utility according to the following objective:

$$\max_s \Big\{ u(w_1 - s) + \beta \left[pv(w_2 + (1+r)s, H_b) + (1-p)v(w_2 + (1+r)s, H_g) \right] \Big\}. \tag{2}$$

A purely monetary loss is a special case of a bivariate utility function with $v(w, H)$ being additive, $v(w, H) = v(w + H)$. In this case, risk aversion over wealth implies that marginal utility of consumption is decreasing in H. So a purely monetary loss is an application of the bivariate model with $v^{(1,1)} < 0$. The first-order condition for the optimal level of saving, denoted by s_0, is given by

$$\begin{aligned} -u'(w_1 - s_0) + \beta(1+r) \Big[& pv^{(1,0)}(w_2 + (1+r)s_0, H_b) \\ & + (1-p)v^{(1,0)}(w_2 + (1+r)s_0, H_g) \Big] \; = \; 0. \end{aligned} \tag{3}$$

The second-order condition holds because the objective function is globally concave in s due to the concavity of u and v in their first argument.

Consider now that the return is uncertain at the time the individual makes his decision. Existing research has mainly focused on cases where return risk represents an aggregate shock (e.g., Eeckhoudt and Schlesinger, 2008; Chiu et al., 2012; Jouini et al., 2013). We in turn analyze a *correlated return* because the level of the return correlates with the individual's endowment. In our model, we assume a return of r_b in the bad state ($H = H_b$) and a return of r_g in the good state ($H = H_g$). We use the term *transfer rate* for the spread between the two levels of the return, $\Delta = r_g - r_b$. For reasons of comparability, we set the expected value of the correlated return equal to the prevailing market rate of return, $pr_b + (1-p)r_g = r$. The correlated return then represents a mean-preserving spread of the prevailing market rate of return (Rothschild and Stiglitz, 1970).

This allows us to rewrite the two return levels as $r_b = r - (1-p)\Delta$ in the bad state and $r_g = r + p\Delta$ in the good state. If we require $r_b \geq -1$ and $r_g \geq -1$, the transfer rate is in the compact interval $[-(1+r)/p, (1+r)/(1-p)]$. For $\Delta = 0$, the return is the same in both states ($r_b = r_g = r$), and the individual's decision problem is the one in Eq. (2). If $\Delta > 0$ (< 0), then $r_b < r < r_g$ ($r_b > r > r_g$), and there is perfect positive (negative) correlation between the return and the individual's non-financial endowment. The return is higher (lower) in the good state than the bad state.[7] An increase in the absolute value of Δ represents a mean-preserving spread of the correlated return, see Ebert (2015). We provide an extension to multiple states in Section 6.

The intuition behind correlated returns is that shocks may affect the individual's non-financial endowment and returns on investments at the same time. Pandemic events like Covid-19 illustrate how people's health and financial markets can be both subject to the same source of uncertainty. Certain financial instruments use correlated returns to provide incentives. For HSAs the return on saving depends on the individual's health state. If the individual stays healthy, savings can be forfeited corresponding to $r_g = -1$ in our notation.[8] We can then infer the value of Δ that equates the expected return to the market rate of return. It is given by $\Delta = -(1+r)/p < 0$, and the return when sick is then $r_b = (r + (1-p))/p > r$. The rationale behind HSAs is that individuals benefit from a higher return on savings when they become sick. Another example is insurance. Consider for simplicity the case of a purely monetary loss with $H_b = -L$ and $H_g = 0$, and set $v(w, H) = v(w + H)$. Then, consumption in the bad state is given by $w_2 + s(1+r) - s(1-p)\Delta - L$ and consumption in the good state by $w_2 + s(1+r) + sp\Delta$. If we set $s = pL/(1+r)$ and $\Delta = -(1+r)/p$, consumption in both states becomes w_2 and the individual is fully insured. This is achieved by payment of an upfront premium in the first period corresponding to the discounted actuarially fair value of $pL/(1+r)$. In this framework, smaller values of s represent partial insurance.

As evident from these examples, for $\Delta = 0$ saving smooths consumption by redistributing income across time. The case of conventional insurance is the polar case with $\Delta = -(1+r)/p$, and saving then redistributes income from the first period into the bad state in the second

[7] The return and the non-financial variable are positive (negative) quadrant dependent if $\Delta > 0$ (< 0), see Lehmann (1966). This is a stronger notion of dependence than correlation, see Li (2011).

[8] This is the case, for example, for flexible spending accounts where savings do not roll over and are therefore lost if they cannot be spent on medical consumption. In other cases, individuals pay a penalty if savings are withdrawn for non-medical consumption. Then, $r_g \in (-1, r)$, and its exact value depends on the penalty.

period. Values of Δ between $-(1+r)/p$ and 0 represent a mix between insurance and saving. In the opposite case with $\Delta = (1+r)/(1-p)$, the individual reallocates income from the first period into the good state in the second period (i.e., "anti-insurance"), and values of Δ between 0 and $(1+r)/(1-p)$ represent a mix between anti-insurance and saving. So values of the transfer rate further away from zero indicate stronger insurance characteristics because returns are more state-contingent while values of the transfer rate closer to zero imply stronger consumption-smoothing because returns are less sensitive to the state.

With a correlated return, the individual's optimal level of saving solves the following maximization problem:

$$\max_s \left\{ u(w_1 - s) + \beta \left[pv(w_2 + (1+r_b)s, H_b) + (1-p)v(w_2 + (1+r_g)s, H_g) \right] \right\}. \tag{4}$$

$U(s;\Delta)$ denotes the expected intertemporal consumption utility as a function of savings and the transfer rate. We use subscripts for derivatives with respect to parameters. The optimal level of saving, denoted s^*, is determined implicitly by the following first-order condition:

$$\begin{aligned} -u'(w_1 - s^*) + \beta(1+r) \left[pv^{(1,0)}(B^*) + (1-p)v^{(1,0)}(G^*) \right] & \\ +\beta p(1-p)\Delta \left[v^{(1,0)}(G^*) - v^{(1,0)}(B^*) \right] &= 0. \end{aligned} \tag{5}$$

B^* and G^* abbreviate consumption at the optimal level of saving s^* and the non-financial variable in the bad and the good state, $B^* = (w_2 + (1+r_b)s^*, H_b)$ and $G^* = (w_2 + (1+r_g)s^*, H_g)$. Our assumptions on u and v ensure that the individual's intertemporal objective function is globally concave in s for any transfer rate.

In the remainder of this paper, we will analyze the incentive effects of correlated returns at the extensive margin and the intensive margin and their effects on individual welfare.

3 Incentive effects at the extensive margin

We first investigate incentive effects at the extensive margin. This refers to the individual's decision whether to save or to borrow. The individual finds it optimal to save ($s^* > 0$) if and only if $U_s(0;\Delta) > 0$, that is,

$$\begin{aligned} -u'(w_1) + \beta(1+r) \left[pv^{(1,0)}(B_0) + (1-p)v^{(1,0)}(G_0) \right] & \\ +\beta p(1-p)\Delta \left[v^{(1,0)}(G_0) - v^{(1,0)}(B_0) \right] &> 0. \end{aligned} \tag{6}$$

$B_0 = (w_2, H_b)$ and $G_0 = (w_2, H_g)$ are shorthand for the outcomes if $s = 0$. The equivalence between criterion (6) and the optimality of a positive amount of saving follows from the concavity of the objective function. If the individual finds it optimal to save, any amount less than s^* would be too low and *a fortiori* no savings at all so that (6) is satisfied. Likewise, if (6) holds, the optimal level of saving must be to the right of $s = 0$ per concavity of the objective function. With this simple argument, we can use criterion (6) as a litmus test for whether an individual is a saver or a borrower.

Condition (6) also shows that the transfer rate interacts with the individual's correlation attitude. The last term in (6) is positive if $\text{sgn}(\Delta) = \text{sgn}\left(v^{(1,1)}\right)$ and negative if $\text{sgn}(\Delta) = -\text{sgn}\left(v^{(1,1)}\right)$. So whenever the individual is correlation neutral ($v^{(1,1)} = 0$), the transfer rate is irrelevant at the extensive margin. In all other cases, we can rearrange (6) to find the value of the transfer rate that separates savers from borrowers. This critical level is given by

$$\Delta_{crit} = -\frac{-u'(w_1) + \beta(1+r)\left[pv^{(1,0)}(B_0) + (1-p)v^{(1,0)}(G_0)\right]}{\beta p(1-p)\left[v^{(1,0)}(G_0) - v^{(1,0)}(B_0),\right]}. \tag{7}$$

The numerator is positive if the individual saves under the market interest rate (i.e., if $s_0 > 0$) and negative if he borrows under the market interest rate (i.e., if $s_0 < 0$). The sign of the denominator depends on the individual's correlation attitude. A simple rearrangement of condition (6) establishes the following result.

Proposition 1. *Consider a correlated return with transfer rate* Δ.

(i) A correlation lover saves (borrows) if $\Delta > (<)\ \Delta_{crit}$.

(ii) A correlation averter saves (borrows) if $\Delta < (>)\ \Delta_{crit}$.

We point out that Proposition 1 continues to hold if we relax the assumption of intertemporally separable preferences. Δ_{crit} compares the individual's saving decision under the market rate of return to the effect of the transfer rate on saving incentives. Under correlation loving, the marginal utility of consumption is higher in the good state than in the bad state, while the reverse is true under correlation aversion. So the denominator of Δ_{crit} is positive under correlation loving and negative under correlation aversion. For a correlation lover who borrows under the market rate (i.e., $s_0 < 0$), Δ_{crit} is positive and the correlated return needs to provide a sufficiently high transfer rate to give strong enough incentives for the individual to start saving. If a correlation lover already saves under the market interest rate (i.e., $s_0 > 0$), then Δ_{crit} is negative and the individual will still find it optimal to save under a correlated return with a positive transfer rate. Similar reasoning applies to correlation averters.

This has implications for the incentive effects of correlated returns in an economy with heterogeneous individuals. Assume that individuals differ in their financial and non-financial endowments, risk and time preferences, etc., but that everybody is correlation loving. This heterogeneity will generate a distribution over Δ_{crit} with one value for each individual. Those who already save under the market rate will have a negative Δ_{crit}, those who borrow under the market rate will have a positive Δ_{crit}. If we now introduce a saving instrument with a correlated return and raise Δ gradually, Proposition 1 predicts that individuals with a negative Δ_{crit} will still save and individuals with a positive Δ_{crit} will start to save as soon as Δ is large enough. The reverse argument applies to an economy of correlation averters. We conclude that saving instruments with a correlated return and a positive (negative) transfer rate lead to more saving behavior in an economy of correlation lovers (averters).

We now provide comparative statics of the critical transfer rate. Moving the transfer rate further away from zero represents a mean-preserving spread of the correlated return. Take the case of a correlation lover who borrows under the market interest rate so that $\Delta_{crit} > 0$. The higher Δ_{crit}, the more the market rate needs to be distorted to turn this individual into a saver. So comparative statics of Δ_{crit} identify factors that indicate a stronger need to provide saving incentives. An individual who does not save under the market rate of return satisfies $U_s(0;0) \leq 0$. His marginal rate of substitution of first-period income for second-period income, $\mu_{1,2}$, does not exceed $1/(1+r)$.[9] For this individual, a marginal reduction of first-period income, $\mathrm{d}w_1 < 0$, needs to be compensated by an increase in second-period income of $\mathrm{d}w_2 = (-\mathrm{d}w_1)/\mu_{1,2}$ to keep expected lifetime utility constant. If $\mu_{1,2}$ is bounded by $1/(1+r)$, such a change can only be effectuated if $-\mathrm{d}w_2/\mathrm{d}w_1 \geq (1+r)$. So the available return on the market is not high enough for this individual to engage in saving.

To keep the presentation tractable, we only cover the case of a correlation lover and relegate the proof to Appendix A.1. When the non-financial attribute is health, many empirical studies suggest correlation loving as the predominant case (e.g., Finkelstein et al., 2013).[10]

Proposition 2. *Assume a correlation lover who does not save under the market rate of return ($s_0 \leq 0$). The critical transfer rate to induce saving is:*

a) decreasing in the utility discount factor, the market rate of return, and first-period income,

b) increasing in second-period income if $v^{(2,1)} \leq 0$, or if $v^{(2,1)} > 0$ and $\mu^g_{1,2} \geq 1/(1+r)$,

c) decreasing in the high non-financial endowment,

d) increasing in the low non-financial endowment if and only if $\mu^g_{1,2} < 1/(1+r)$,

e) increasing in the probability of loss if $p \geq 1/2$, or if $p < 1/2$ and $\mu^g_{1,2} \geq 1/(1+r)$.

We provide the economic intuition for these effects. A higher utility discount factor, a higher market rate of return, and higher first-period income all have the effect to reinforce the incentive to save under the market rate of return and/or to strengthen the incentive effect

[9] The formal definition is

$$\mu_{1,2} = U_{w_2}(0;0)/U_{w_1}(0;0) = \beta \left[p v^{(1,0)}(w_2, H_b) + (1-p) v^{(1,0)}(w_2, H_g) \right] / u'(w_1). \tag{8}$$

$\mu_{1,2} \leq 1/(1+r)$ if and only if $U_s(0;0) \leq 0$. We can similarly define $\mu^g_{1,2}$ and $\mu^b_{1,2}$ as the individual's marginal rate of substitution if the the non-financial variable is H_g or H_b with certainty, i.e.,

$$\mu^g_{1,2} = \beta v^{(1,0)}(w_2, H_g)/u'(w_1) \quad \text{and} \quad \mu^b_{1,2} = \beta v^{(1,0)}(w_2, H_b)/u'(w_1). \tag{9}$$

[10] The results for a correlation averse individual are quite similar. In most cases, the effects are simply flipped.

of the transfer rate. Therefore, the critical transfer rate decreases because it takes less to "persuade" the individual to engage in saving, thus explaining *a)*. Result *c)* has a similar intuition. For *b)*, notice that higher second-period income lowers the incentive to save under the market rate of return. If the individual is not cross-prudent in the non-financial variable ($v^{(2,1)} \leq 0$), higher second-period income also lowers the incentive effect of the transfer rate because the difference between marginal utility of consumption is decreasing in second-period income. Under cross-prudence in the non-financial variable ($v^{(2,1)} > 0$), higher second-period income now increases the incentive effect of the transfer rate, which is conflicting with its effect to lower the incentive to save under the market rate of return. This is why an additional restriction is required. Result *d)* is obtained because the low level of the non-financial variable entails a trade-off; it increases the incentive to save under the market rate but decreases the incentive effect of the transfer rate. The second effect dominates if $\mu^g_{1,2} < 1/(1+r)$, and the critical transfer rate must increase. Statement *e)* follows because a higher loss probability makes the state more likely where marginal utility of consumption is low. This reduces the incentive to save under the market rate. The incentive effect of the transfer rate depends on the probability of loss via the riskiness of the correlated return with variance $p(1-p)\Delta^2$. The incentive effect increases for $p < 1/2$, peaks at $p = 1/2$, and decreases for $p > 1/2$. This explains the additional condition for $p < 1/2$ and the unambiguous effect for $p \geq 1/2$. For correlation lovers, we have $\mu_{1,2} < \mu^g_{1,2}$; therefore, $U_s(0;0) \geq 0$ or, equivalently, $\mu_{1,2} \leq 1/(1+r)$ does *not* restrict the size of $\mu^g_{1,2}$ relative to $1/(1+r)$. In summary, the level of the critical transfer rate is jointly determined by individual risk and time preferences, financial and non-financial endowments and market conditions.

4 Incentive effects at the intensive margin

In the previous section we investigated how correlated returns affect the optimality of saving versus borrowing. A critical transfer rate separates savers from borrowers, and correlated returns can provide stronger incentives to save than the market rate of return. We now turn to the optimal level of saving (or borrowing).

We differentiate the first-order condition (5) with respect to the transfer rate:

$$\begin{aligned} U_{s\Delta}(s^*;\Delta) \;&=\; \beta p(1-p)\Big\{-v^{(1,0)}(B^*) + v^{(1,0)}(G^*) \\ &\quad -(1+r_b)s^* v^{(2,0)}(B^*) + (1+r_g)s^* v^{(2,0)}(G^*)\Big\}. \end{aligned} \tag{10}$$

The four terms in the curly bracket denominate different effects that individuals trade off as the transfer rate increases. The first term is negative because a higher transfer rate reduces the return on saving in the bad state (substitution effect). The second term is positive because a higher transfer rate increases the return on saving in the good state (substitution effect). The third term is positive for $s^* > 0$ because a higher transfer rate reduces the individual's wealth in the bad state, which increases his marginal utility of consumption (wealth effect).

The fourth term is negative for $s^* > 0$ because a higher transfer rate increases the individual's wealth in the good state, which reduces his marginal utility of consumption (wealth effect). There is a substitution and a wealth effect in each state, which differ in sign. The two substitution effects and the two wealth effects also differ in sign across states. A change in the transfer rate introduces complex effects into the consumption-saving trade-off.

The risk literature has identified partial risk aversion (Menezes and Hanson, 1970, []) as a determinant of the comparative statics of optimal saving when the (certain) interest rate changes (Chiu et al., 2012). It is defined as follows.

Definition 1. The individual's *partial risk aversion in wealth* is $\mathcal{R}(x+y,H) = -y\frac{v^{(2,0)}(x+y,H)}{v^{(1,0)}(x+y,H)}$.

We can then rewrite the curly bracket in Eq. (10) as follows:

$$v^{(1,0)}(B^*)\big[\mathcal{R}(w_2+(1+r_b)s^*,H_b)-1\big]-v^{(1,0)}(G^*)\big[\mathcal{R}(w_2+(1+r_g)s^*,H_g)-1\big]. \qquad (11)$$

We obtain a clear effect if the two square brackets differ in sign. If partial risk aversion is less than unity in the bad state and larger than unity in the good state, the optimal level of saving increases as the transfer rate increases. The approach based on partial risk aversion is unsatisfactory in case of a correlated return because a common assumption on partial risk aversion is that it is either uniformly above or below unity (e.g., Chiu et al., 2012). We cannot sign (11) in this case. We introduce two other intensity measures of the individual's risk preferences to obtain definitive comparative statics.

Definition 2. For the individual's second-period utility function v, we define:

a) *Partial prudence in wealth:* $\mathcal{P}(x+y,H) = -y\frac{v^{(3,0)}(x+y,H)}{v^{(2,0)}(x+y,H)}$.

b) *Partial cross-prudence in the non-financial variable:* $\mathcal{C}(x+y,H) = -y\frac{v^{(2,1)}(x+y,H)}{v^{(1,1)}(x+y,H)}$.

We decompose possible values of the transfer rate into intermediate values between 0 and Δ_{crit}, that is, $I = (\min\{0,\Delta_{crit}\},\max\{0,\Delta_{crit}\})$, and values not between 0 and Δ_{crit}, that is, $J = [-(1+r)/p,\min\{0,\Delta_{crit}\})\cup(\max\{0,\Delta_{crit}\},(1+r)/(1-p)]$.[11] We formulate the following proposition.

Proposition 3. *Consider a correlated return with transfer rate Δ and assume that the individual's partial cross-prudence in the non-financial variable is bounded by unity. If $\Delta \in I$ and the individual's partial prudence in wealth is bounded by 2, or if $\Delta \in J$ and the individual's partial prudence in wealth exceeds 2, then:*

(i) Correlation lovers will save more after a marginal increase of the transfer rate.

(ii) Correlation averters will save more after a marginal decrease of the transfer rate.

[11] If $\Delta_{crit} < -(1+r)/p$ or $\Delta_{crit} > (1+r)/(1-p)$, then J will be a single interval and the upper or lower bound of I needs to be adjusted accordingly. In all other cases, J is the union of two intervals.

The proof is given in Appendix A.2. Table 1 dissects these conditions. Consider a correlation lover; for combinations on the diagonal he either saves due to a sufficiently high positive transfer rate (upper left) or he borrows due to a sufficiently low negative transfer rate (lower right). Both cases are summarized as $\Delta \in J$ in Proposition 3, and partial prudence in wealth above 2 ensures more saving or less borrowing as the transfer rate increases. On the off-diagonal are situations where the individual saves despite a negative transfer rate (lower left) or where he borrows despite a positive transfer rate (upper right). They are summarized as $\Delta \in I$ in Proposition 3, and partial prudence in wealth below 2 ensures more saving or less borrowing as the transfer rate increases. In either case, the additional restriction on partial cross-prudence in the non-financial variable is needed to sign the effect.

$\mathcal{C} < 1$	$\Delta > \Delta_{crit}$	$\Delta < \Delta_{crit}$
$\Delta > 0$	$\mathcal{P} > 2$	$\mathcal{P} < 2$
$\Delta < 0$	$\mathcal{P} < 2$	$\mathcal{P} > 2$

Table 1: Sufficient conditions for $\mathrm{d}s^*/\mathrm{d}\Delta$ and $v^{(1,1)}$ to have the same sign

Two knife-edge cases are not covered in Proposition 3 (i.e., $\Delta \in \{0, \Delta_{crit}\}$). We formulate them as separate corollaries because they admit simpler conditions. This helps develop intuition and makes the role of our assumptions transparent.

Corollary 1 ($\Delta = 0$)**.** *Starting at the market rate of return, if $\Delta_{crit} < 0$ and the individual is cross-prudent in the non-financial variable, or if $\Delta_{crit} > 0$ and the individual is cross-imprudent in the non-financial variable, then:*

(i) A correlation lover will save more after a marginal increase of the transfer rate.

(ii) A correlation averter will save more after a marginal decrease of the transfer rate.

Corollary 2 ($\Delta = \Delta_{crit}$)**.** *Starting at the critical transfer rate, a marginal increase (decrease) of the transfer rate increases saving if the individual is correlation loving (averse).*

A proof is given in Appendix A.3. Corollary 2 is a special case of Proposition 1. If the transfer rate is at the critical level, the individual neither saves nor borrows because his endowed intertemporal consumption stream is already optimal. But if $s^* = 0$, the two wealth effects in Eq. (10) disappear and the change in the transfer rate reduces to a comparison between both substitution effects. For $s^* = 0$, wealth levels in the second period do not depend on the state, and the comparison between the two substitution effects depends entirely on the individual's correlation attitude. As we know from Proposition 1, the statement in Corollary 2 does not only hold at the margin but globally for any change of the transfer rate relative to Δ_{crit}.

When starting at the market rate of return (i.e., $\Delta = 0$), the return on saving is certain and consumption levels do not depend on the state in the second period. Then, correlation attitude ranks the two substitution effects and cross-prudence in the non-financial variable ranks the two wealth effects in Eq. (10). Assumptions on the individual's risk attitudes suffice and we do not have to restrict their intensities.

In the general case of Proposition 3, we aggregate the substitution and the wealth effect in each state. If this incentive effect is larger in the good state than the bad state, a marginal increase of the transfer rate increases savings. Each incentive effect depends on the consumption level and the value of the non-financial variable in that state. For correlation lovers, the restriction on partial prudence in wealth ensures that the incentive effect increases when replacing wealth in the bad state with wealth in the good state. Likewise, for correlation lovers the restriction on partial cross-prudence in the non-financial variable ensures that the incentive effect increases when going from the low level of the non-financial variable to its high level. For correlation averters instead, the preference conditions induce the opposite ranking of the incentive effect in the good state versus the bad state. This intuition also reveals why the prudence measure needs to be bounded from below in some cases and from above in other cases whereas the cross-prudence measure is always bounded from above. The wealth level in the good state may or may not exceed that in the bad state depending on whether the individual saves or borrows and on the sign of the transfer rate (see Proposition 1); however, it is always the value of the non-financial variable in the good state which is larger.

The conditions in Proposition 3 may appear complex at first sight, but they are well known in the literature. Adopting Chiu et al.'s (2012) argument from a univariate context, partial prudence in wealth exceeds 2 for all $y > 0$ and $x \geq 0$ if and only if $-yv^{(3,0)}(y, H)/v^{(2,0)}(y, H) > 2$ for all $y > 0$. This measure is *relative* prudence in wealth. The comparison of relative prudence with 2 often appears in the literature; examples include the comparative statics of the demand for a risky asset with respect to changes in the return distribution (Hadar and Seo, 1990; Choi et al., 2001), to sign the effect of uncertainty on bargaining outcomes (White, 2008), the effect of changes in interest rate risk on optimal saving (Eeckhoudt and Schlesinger, 2008), and in more general saving, portfolio choice and output choice problems under uncertainty (Chiu et al., 2012; Jouini et al., 2013; Menegatti and Peter, 2020). Recently, Liu and Menegatti (2019b) show that the comparison of partial prudence in wealth with 2 coupled with assumptions on cross-prudence in health determine precautionary saving behavior in the presence of endogenous health investment, see their Propositions 4 and 5.

To the best of our knowledge, the expression $-yv^{(2,1)}(x + y, H)/v^{(1,1)}(x + y, H)$ has not been introduced into the literature yet. We refer to this coefficient as an intensity measure of partial cross-prudence in the non-financial variable because $v^{(2,1)}$ is the individual's cross-prudence in the non-financial variable (Eeckhoudt et al., 2007).[12] As in Chiu et al. (2012), this

[12] We justify this terminology in Appendix A.4 with the help of the partial prudence premium. See Trautmann and van de Kuilen (2018) for a recent survey of the broader evidence on higher order risk preferences.

measure exceeds 1 for all $y > 0$ and $x \geq 0$ if and only if $-yv^{(2,1)}(y,H)/v^{(1,1)}(y,H) > 1$ for all $y > 0$, where the latter is an intensity measure of *relative* cross-prudence in the non-financial variable. The comparison of a prudence index to unity is less prevalent in the literature. Still, two recent papers demonstrate the usefulness of this threshold to determine the impact of inequality and economic convergence on the efficient discount rate (Gollier, 2015) and to explain a individual's attitude to an increase in initial wealth when facing two interdependent multiplicative risks (Denuit and Rey, 2014).

There is little - if not to say no - empirical guidance to judge how restrictive the conditions in Proposition 3 are. Rey and Rochet (2004) discuss several specifications for bivariate preferences that can help shed some light on the issue.[13] The conditions are sufficient but not necessary. Among those individuals who do not satisfy them, some will increase saving (or decrease borrowing) and some will react in the opposite way as the transfer rate changes.

We derive some other comparative statics of saving behavior at the intensive margin. Many of them are straightforward.

Remark 1. *The optimal level of saving under correlated return risk is:*

a) increasing in the utility discount factor and first-period income,

b) decreasing in second-period income,

c) increasing in the market rate of return for borrowers; for savers it is increasing in the market rate of return if partial risk aversion in wealth is less than unity,

d) increasing (decreasing) in the high and the low non-financial endowment for correlation lovers (averters).

The proof is given in Appendix A.5. These results follow from the effects on the marginal benefit and the marginal cost of saving. Result *c)* contains the usual trade-off that an increase in the rate of return has two conflicting effects when individuals save, a positive substitution effect because a higher return makes saving more attractive, and a negative wealth effect because the individual's wealth in the second period increases. Partial risk aversion is uniformly below unity if and only if relative risk aversion is (see Chiu et al., 2012). This sufficient condi-

[13] Under additive separability ($v^{(1,1)} = 0$), a correlated return does not affect saving behavior at the extensive margin (see Proposition 1) but it still has an effect at the intensive margin. The restriction on $\mathcal{C}$ can be dropped in this case, and only the condition on the prudence measure is relevant. In Noussair et al. (2013), 62% of their demographically representative sample exhibit relative prudence above 2. For multiplicative separability, the utility of the non-financial variable cancels out of the preferences coefficients in Definition 2. The restriction on $\mathcal{C}$ then becomes a restriction on partial risk aversion in wealth. For savers, many utility functions satisfy $\mathcal{C} < 1$ and $\mathcal{P} < 2$, for example, iso-elastic utility with relative risk aversion of unity or higher. A necessary condition to obtain $\mathcal{C} < 1$ and $\mathcal{P} > 2$ is that absolute risk aversion increases in wealth. For borrowers, $\mathcal{C} < 1$ is always satisfied under multiplicative separability and $\mathcal{P} < 2$ holds under prudence while $\mathcal{P} > 2$ requires a sufficient degree of imprudence.

tion is well-known in the consumption-saving literature.[14] The comparative statics properties in Remark 1 also hold when interest rate risk represents an aggregate shock.

5 Individual welfare

Correlated returns affect saving decisions at the extensive and the intensive margin. We will now investigate their effects on individual welfare. We can then answer the question to what extent individuals benefit from correlated returns. The welfare benefits derive from the insurance characteristics of correlated returns. Therefore, we first analyze the individually-optimal transfer rate and then provide a joint assessment of correlated returns and insurance.

5.1 The individually-optimal transfer rate

For a transfer rate of Δ, the individual's indirect utility function is given by $U(s^*; \Delta)$ with s^* defined in Eq. (5). It measures the individual's welfare at his optimal level of saving for a given level of the transfer rate. The envelope theorem yields

$$\frac{\mathrm{d}U}{\mathrm{d}\Delta} = \frac{\partial U}{\partial \Delta} + \frac{\partial U}{\partial s}\frac{\mathrm{d}s}{\mathrm{d}\Delta} = \frac{\partial U}{\partial \Delta} = \beta p(1-p)s^* \left[v^{(1,0)}(G^*) - v^{(1,0)}(B^*)\right], \tag{12}$$

because $\partial U/\partial s = 0$ from the optimality of s^*. Eq. (12) informs us how changes in the transfer rate affect the individual's welfare. We can identify some values of the transfer rate as suboptimal and rule them out as potential maximizers of $U(s^*; \Delta)$. We summarize our results in the following proposition and provide a proof in Appendix A.6.

Proposition 4. *If an individual is not correlation neutral and $\Delta \in I$, a marginal increase (decrease) of the transfer rate raises his welfare for $\Delta_{crit} < (>)\ 0$.*

Transfer rates between 0 and Δ_{crit} are not optimal because the individual can be made better off. We point out two knife-edge cases as separate corollaries and will use them to develop some intuition. Proofs are given in Appendix A.7.

Corollary 3 ($\Delta = 0$)**.** *Starting at the market rate of return, a marginal increase (decrease) of the transfer rate raises the individual's welfare if $\Delta_{crit} < (>)\ 0$.*

Corollary 4 ($\Delta = \Delta_{crit}$)**.** *The individual's welfare has a local minimum at Δ_{crit}.*

Proposition 4 and Corollaries 3 and 4 continue to hold if we relax intertemporal separability of preferences. Corollary 4 rules out the critical transfer rate as a maximizer of the individual's welfare. The individual neither saves nor borrows when $\Delta = \Delta_{crit}$, see Proposition 1. For

[14] See, for example, Eeckhoudt and Schlesinger (2008) and Chiu et al. (2012) who prove the sufficiency of this condition for first-order stochastic changes in the interest rate, and Courbage and Rey (2007) and Menegatti (2009a) for optimal saving in the presence of a non-financial risk. Jouini et al. (2013) provide conditions under which the restriction on relative risk aversion is also necessary for definitive comparative statics.

both correlation lovers and correlation averters, marginal utility of consumption differs across states; hence, any small deviation from the critical transfer rate raises welfare because it allows the individual to transfer wealth from the low to the high marginal utility state. A similar rationale holds for Corollary 3. When saving under the market rate of return, wealth in the second period does not depend on the state. A correlation attitude other than neutral drives a wedge between marginal utility of consumption across states. A small deviation from the market rate of return improves the individual's consumption stream and increases intertemporal welfare. Proposition 4 extends this argument to any transfer rate between 0 and Δ_{crit}. The welfare benefits of a correlated return arise from its insurance effects because it facilitates redistribution from the low to the high marginal utility state.

Corollary 3 also tells us who would opt into a savings plan with a correlated return. Assume a small positive transfer rate. According to Corollary 3, individuals with $\Delta_{crit} < 0$ benefit from such a plan. Proposition 1 identifies these individuals as correlation lovers who save and correlation averters who borrow under the market rate of return. Similarly, a correlated return with a small negative transfer rate attracts correlation loving borrowers and correlation averse savers. So correlation attitude interacts with saving behavior under the market rate of return to determine the preference of a correlated return over the status-quo.

We may then wonder about the change in saving behavior of those who opt in. Consider a savings plan with correlated return and a small positive transfer rate. It attracts correlation loving savers and correlation averse borrowers, see Corollary 3. Due to Corollary 1, correlation loving savers who are cross-prudent in the non-financial variable will save more under the savings plan with a correlated return. But correlation averse borrowers who are cross-imprudent in the non-financial variable *borrow* more under this plan. Both groups benefit from the correlated return but only one group saves more, whereas the other group saves less and borrows more compared to their behavior under the market rate of return. The change in saving behavior does not inform us about the change in welfare and vice versa.

In the specific example of HSAs, the return on saving is higher in the bad state than the good state, corresponding to $\Delta < 0$. When the non-financial variable is health, many papers suggest correlation loving, see Finkelstein et al. (2013). This appears to be in contrast to Proposition 4 and Corollary 3 because correlation loving savers benefit from a marginally positive, not negative, transfer rate. We suggest a possible reconciliation based on Liu's (2004) approach of endogenous health care spending. If individuals who experience a negative health shock can spend money to (partially) restore their health, a negative health shock becomes a negative wealth shock. As explained in Section 2, a monetary loss is a special case of a bivariate utility function with $v^{(1,1)} < 0$. But then savers exhibit $\Delta_{crit} > 0$ and benefit from a marginally negative transfer rate. This is in line with the rationale of HSAs, which allow individuals to accumulate tax-favored savings to cover health care expenditures.[15]

[15] Steinorth (2012) and Peter et al. (2016) analyze saving behavior under HSAs and model some of their institutional features directly.

Starting from Eq. (12), we will now analyze the transfer rate that maximizes the individual's welfare. We require $r_b \geq -1$ and $r_g \geq -1$ so that the admissible values for the transfer rate lie in $[-(1+r)/p, (1+r)/(1-p)]$, a compact interval of $\mathbb{R}$. We know from the extreme value theorem that $U(s^*; \Delta)$ attains a maximum because it is a continuous function in Δ. Proposition 4 and Corollaries 3 and 4 show that any maximizer must lie in J. In the sequel, we focus on interior solutions. A prerequisite for an interior solution is that the effect of the non-financial risk on marginal utility of consumption can be offset monetarily. We can then find wealth levels w_{2g} and w_{2b} in the good and the bad state such that $v^{(1,0)}(w_{2g}, H_g) = v^{(1,0)}(w_{2b}, H_b)$. We provide some comparative statics of the welfare-maximizing transfer rate in the next proposition and state a proof in Appendix A.8.

Proposition 5. *Let Δ^* be an interior maximizer of the individual's welfare as a function of the transfer rate, $U(s^*; \Delta)$. The individually-optimal transfer rate is:*

a) increasing in the high and decreasing in the low non-financial variable for correlation loving savers and correlation averse borrowers,

b) decreasing in the high and increasing in the low non-financial variable for correlation loving borrowers and correlation averse savers,

c) increasing in the utility discount factor and first-period income if and only if partial risk aversion is higher in the bad state than the good state.

To develop intuition, we explain how a parameter change affects the trade-off that an individually-optimal transfer rate solves. At $\Delta = \Delta^*$, marginal utility of consumption is equal across states. If the high non-financial variable increases, this raises the marginal utility of consumption in the good state for a correlation lover. To counterbalance this effect, the transfer rate needs to be adjusted to increase wealth and lower marginal utility. If the individual saves, this is achieved by a higher transfer rate, while the reverse is true if he borrows. A similar reasoning applies to a correlation averter and the low non-financial variable. This direct channel dominates because the indirect effect on saving behavior cannot upset it.

The utility discount factor and first-period income do not directly affect marginal utility of consumption in the second period but only indirectly via their effect on saving. According to Remark 1, individuals save more when the utility discount factor or first-period income increase. This results in two wealth effects, one in each state, and both of them lower the marginal utility of consumption. If the effect is equally strong in both states, the optimality condition for Δ^* remains unaffected and no adjustment is required. But if the two effects differ in size, the individually-optimal transfer rate changes. For example, if the effect is stronger in the bad state than the good state, marginal utility in the bad state drops by more than in the good state, and the individually-optimal transfer rate increases to redistribute wealth from the low to the high marginal utility state.

The comparative statics reveal that the individually-optimal transfer rate is jointly determined by a variety of factors, including the individual's preferences and endowments. In an economy of heterogeneous individuals, policymakers have to trade off these various determinants when setting a transfer rate that applies uniformly across individuals. Proposition 5 informs about some of the complexities associated with this task.

5.2 Insurance effects

To bring out the insurance effects of correlated returns more explicitly, we present a modified model that includes both saving and insurance. We extend the model pioneered by Dionne and Eeckhoudt (1984) and studied more recently by Hofmann and Peter (2016), who focus on a purely monetary risk. The non-financial variable is not insurable, so we let the non-financial risk be flanked by a monetary risk. A good example is the case of a health risk where a loss in health is accompanied by treatment expenses, or the risk of disability which results in reduced productivity on the labor market. Health insurance can reimburse treatment expenses and long-term disability insurance can replace a portion of the individual's income.

In our model, we assume a financial loss of T associated with the low outcome of the non-financial variable H_b in the second period. Insurance reimburses a fraction α of the financial loss against payment of an upfront premium π. The price of insurance is proportional to its discounted actuarial value, $\pi = m\alpha pT/(1+r)$, where m is a loading factor. The insurance contract is called actuarially favorable (fair, unfair) if $m < (=, >)\ 1$. The individual's objective function is then given by

$$U(s,\alpha;\Delta) = u(w_1 - s - \pi) + \beta\left[pv(w_2 + (1+r_b)s - (1-\alpha)T, H_b) + (1-p)v(w_2 + (1+r_g)s, H_g)\right]. \tag{13}$$

He chooses saving and a level of insurance coverage to maximize intertemporal expected utility. The objective function is globally concave in (s, α) if both u and v are concave in wealth, see Appendix A.9, regardless of the individual's correlation attitude. We now present the welfare effects of correlated returns in our modified model.

Proposition 6. *Assume that U admits an interior solution and consider the individual's welfare as a function of the transfer rate at the optimal levels of saving and insurance.*

(i) If insurance is actuarially unfair, a marginal decrease (increase) of the transfer rate raises the welfare of savers (borrowers).

(ii) If insurance is actuarially fair, the transfer rate does not affect the individual's welfare.

(iii) If insurance is actuarially favorable, a marginal increase (decrease) of the transfer rate raises the welfare of savers (borrowers).

A proof is given in Appendix A.10. We emphasize that it does not rely on intertemporal separability. Proposition 1 generalizes to the combined saving-insurance problem in the sense that a critical threshold on the transfer rate, which only depends on exogenous parameters and preferences, determines whether individuals are classified as savers or borrowers.[16] Proposition 6 states that a correlated return raises welfare if and only if there are cost differences between the saving and the insurance mechanism. Our comparability assumption $pr_b + (1-p)r_g = r$ is the analog to the assumption of actuarial fairness in insurance. It states that saving under the correlated return is ex-ante budget neutral. If actuarially fair insurance is available (i.e., $m = 1$), saving under the market rate of return combined with an optimal level of insurance can perfectly smooth differences in marginal utility across states and time. Then there is no reason to deviate from the status quo. In all other cases (i.e., $m \neq 1$), the individual's optimal behavior under the market rate of return leaves some difference between marginal utility in the bad versus the good state, and correlated returns can add value.

Correlated returns can also be valuable due to institutional constraints on the insurance market. Proposition 6 applies for an interior solution and does not restrict the level of insurance coverage *a priori*. For a purely monetary risk, full coverage is optimal if insurance is actuarially fair and partial insurance is optimal if insurance is actuarially unfair (Mossin, 1968). Dionne and Eeckhoudt (1984) extended this result to a two-period model with endogenous saving but retain the assumption of a purely monetary risk. Cook and Graham (1977) show in a single-period model that Mossin's result no longer holds when a non-financial risk is present (see also Rey, 2003). We extend this result to our two-period model with endogenous saving. Starting at the market rate of return, individuals would like to overinsure (i.e., $\alpha^* > 1$) if and only if the loading factor is below a threshold value. This threshold is equal to 1 for correlation neutral individuals, consistent with Dionne and Eeckhoudt (1984), below 1 for correlation lovers and above 1 for correlation averters. A correlation averter then prefers a correlated return with a small positive or negative transfer rate, depending on whether he is a saver or borrower, over the market rate of return even if actuarially fair insurance is available because insurance contracts typically do not reimburse more than the actual loss amount.

6 An extension to multiple states

Our analysis of correlated returns is based on a binary risk for the non-financial variable. We will now discuss a possible extension to multiple states. The binary-risk assumption is primarily for convenience and ease of exposition. Let the non-financial endowment be given by random variable $\widetilde{H}$ with values in $[\underline{H}, \overline{H}]$. We denote the correlated return by $r + \delta\Delta(H)$. The transfer function Δ measures how the return depends on the realization of the non-financial variable and parameter δ is the weight on the correlated component relative to the market rate of return. We assume r to be certain. We could allow for a risky market return as long as it is independent from $\widetilde{H}$. The model with two states permits only perfect positive or

[16] Using the techniques in Courbage et al. (2017), the individual saves if and only if $U_s(0, \alpha_0; \Delta) > 0$ for $\alpha_0 = \arg\max_{\alpha \in [0,1]} U(0, \alpha; \Delta)$. This provides the value of Δ_{crit} in the modified model. We formulate Proposition 6 in terms of savers and borrowers for compactness, but could rewrite it with exogenous parameters.

perfect negative correlation between the return and the non-financial variable. Our extended framework can represent any correlation between -1 and 1. The comparability assumption to focus on risk effects is now $\mathbb{E}\Delta(\widetilde{H}) = 0$. Under this assumption, $r + \delta'\Delta(\widetilde{H})$ is riskier than $r + \delta\Delta(\widetilde{H})$ in the sense of Rothschild and Stiglitz if either $\delta' > \delta \geq 0$ or $\delta' < \delta \leq 0$.

The individual solves the following objective:

$$\max_s \left\{ u(w_1 - s) + \beta \mathbb{E} v(w_2 + (1 + r + \delta\Delta(\widetilde{H}))s, \widetilde{H}) \right\}. \tag{14}$$

Familiar arguments establish a critical level on δ,

$$\delta_{crit} = -\frac{-u'(w_1) + \beta(1+r)\mathbb{E}v^{(1,0)}(w_2, \widetilde{H})}{\beta \mathrm{Cov}(\Delta(\widetilde{H}), v^{(1,0)}(w_2, \widetilde{H}))}. \tag{15}$$

For δ_{crit} to be well-defined, the correlation between $\Delta(\widetilde{H})$ and $v^{(1,0)}(w_2, \widetilde{H})$ needs to be different from zero. This can only hold if the individual is not correlation neutral ($v^{(1,1)} \neq 0$). We obtain the following characterization of saving behavior at the extensive margin.

Proposition 7. *Consider a correlated return with increasing transfer function* Δ*.*

(i) A correlation lover saves (borrows) if $\delta > (<)\ \delta_{crit}$*.*

(ii) A correlation averter saves (borrows) if $\delta < (>)\ \delta_{crit}$*.*

For a correlation lover $v^{(1,0)}(w_2, H)$ is increasing in H. If Δ is also increasing in H, the covariance in Eq. (15) is positive. For a correlation averter, the covariance is negative and the inequalities flip. Clearly, Proposition 7 generalizes Proposition 1 and has a similar intuition.

For the intensive margin, let $U(s; \delta)$ denote the individual's objective function and s^* his optimal level of saving for a given δ. Let $\widetilde{S}^* = (1 + r + \delta\Delta(\widetilde{H}))s^*$ be shorthand for the endogenous component of consumption. The cross-derivative of the objective function is

$$\begin{aligned} U_{s\delta}(s^*; \delta) &= \beta \mathbb{E}\Delta(\widetilde{H}) v^{(1,0)}(w_2 + \widetilde{S}^*, \widetilde{H}) + \beta \mathbb{E}\widetilde{S}^* \Delta(\widetilde{H}) v^{(2,0)}(w_2 + \widetilde{S}^*, \widetilde{H}) \\ &= \beta \mathrm{Cov}\left(\Delta(\widetilde{H}), v^{(1,0)}(w_2 + \widetilde{S}^*, \widetilde{H}) + \widetilde{S}^* v^{(2,0)}(w_2 + \widetilde{S}^*, \widetilde{H}) \right). \end{aligned} \tag{16}$$

Consider an increasing transfer function Δ. The monotonicity of the second argument of the covariance in H determines the sign of $U_{s\delta}(s^*; \delta)$. Define $I = [\min\{0, \delta_{crit}\}, \max\{0, \delta_{crit}\}]$. We obtain the following result, which generalizes Proposition 3.

Proposition 8. *Consider a correlated return with increasing transfer function* Δ *and assume partial cross-prudence in the non-financial variable below unity. For* $\delta \in I$*, let the individual's partial prudence in wealth be bounded by* 2*; for* $\delta \notin I$*, let the individual's partial prudence in wealth exceed* 2*. It holds that:*

(i) Correlation lovers will save more after a marginal increase of δ*.*

(ii) Correlation averters will save more after a marginal decrease of δ*.*

The proof is similar to the proof of Proposition 3 in Appendix A.2 and is omitted. The assumptions on partial prudence in wealth and partial cross-prudence in the non-financial variable establish the monotonicity of the second argument of the covariance as a function of H. If $\delta = 0$, consumption is risk-free and assumptions on correlation attitude and cross-prudence suffice to sign $U_{s\delta}(s^*;\delta)$. For $\delta = \delta_{crit}$, saving is zero and the monotonicity depends solely on correlation attitude. This extends Corollaries 1 and 2 in Section 4.

For the welfare effect, the envelope theorem yields

$$U_\delta(s^*;\delta) = \beta s^* \mathbb{E}\Delta(\widetilde{H}) v^{(1,0)}(w_2 + \widetilde{S}^*, \widetilde{H}) = \beta s^* \mathrm{Cov}\left(\Delta(\widetilde{H}), v^{(1,0)}(w_2 + \widetilde{S}^*, \widetilde{H})\right). \tag{17}$$

The following result extends Proposition 4.

Proposition 9. *Consider a correlated return with increasing transfer function Δ; if the individual is not correlation neutral and $\delta \in I$, the individual's welfare can be increased.*

Take $\delta \in (\delta_{crit}, 0)$ and consider a correlation lover. Proposition 7 implies $s^* > 0$ so that $(1 + r + \delta\Delta(H))s^*$ is decreasing in H due to $\delta < 0$. The second argument of the covariance in Eq. (17) is then increasing in H because of $v^{(2,0)} < 0$ and $v^{(1,1)} > 0$. The covariance is positive, the level of saving is positive and therefore $U_\delta(s^*;\delta) > 0$. A marginal increase of δ raises the individual's welfare. The other combinations follow a similar argument. We can also extend Corollaries 3 and 4 from Section 5 for the knife-edge cases $\delta = 0$ and $\delta = \delta_{crit}$.

Under the binary-risk assumption, actuarially fair insurance is a perfect substitute for correlated returns, see Proposition 6(ii). This is no longer the case with multiple states. Let the financial loss be generated by the non-financial variable, say $T(\widetilde{H})$ with T decreasing. In the health example, this means that better health outcomes require less treatment. The individual's objective function is now

$$U(s, \alpha; \delta) = u(w_1 - s - \pi) + \beta \mathbb{E} v(w_2 + (1 + r + \delta\Delta(\widetilde{H}))s - (1 - \alpha)T(\widetilde{H}), \widetilde{H}) \tag{18}$$

with insurance premium $\pi = m\alpha\mathbb{E}T(\widetilde{H})/(1 + r)$. Consider an interior maximizer (s^*, α^*) and let S^* denote the endogenous component of the individual's consumption in the second period. A marginal change in δ affects the individual's welfare if and only if

$$U_\delta(s^*, \alpha^*; \delta) = \beta s^* \mathrm{Cov}\left(\Delta(\widetilde{H}), v^{(1,0)}(w_2 + \widetilde{S}^* - (1 - \alpha^*)T(\widetilde{H}), \widetilde{H})\right) \neq 0$$

according to the envelope theorem. We state our final result.

Proposition 10. *In the model with more than two states, a correlated return can increase the individual's welfare even if actuarially fair insurance is available.*

We provide a short numerical illustration in Appendix A.11 to show an explicit example for such a welfare increase. For intuition, take the case of $m = 1$ and $\delta = 0$. The first-order conditions for optimal saving and insurance imply

$$\mathrm{Cov}\left(T(\widetilde{H}), v^{(1,0)}(w_2 + \widetilde{S}^* - (1-\alpha^*)T(\widetilde{H}), \widetilde{H})\right) = 0. \tag{19}$$

Under the binary-risk assumption, marginal utility of consumption in the second period must then be constant in H. We can see this directly in the proof of Proposition 6 in Appendix A.10 for $m = 1$. With more than two states, the covariance in Eq. (17) can be zero even if the second argument is not constant in H. This is the case in the numerical example. With only two periods and two states, saving under the market rate of return and actuarially fair insurance perfectly equalize marginal utility across states and time. The resulting allocation cannot be further improved. With multiple states, correlated returns can add value beyond what can be achieved by consumption smoothing and actuarially fair insurance.

7 Conclusion

In this paper, we studied the incentive and welfare effects of correlated returns in a simple two-period consumption-saving model à la Kimball (1990). One motivation are shocks that affect non-financial outcomes and investment returns at the same time such as pandemics like Covid-19. Another motivation derives from financial instruments with correlated return features such as insurance. We found a critical transfer rate that interacts with the individual's correlation attitude to separate savers from borrowers. Measures of partial prudence in wealth and partial cross-prudence in the non-financial attribute determine whether larger transfer rates increase optimal saving. Finally, individuals who are not correlation neutral prefer a positive level of exposure to correlated return risk due to its insurance effects.

Potential extensions include multiple periods, economies with heterogeneous individuals or overlapping generations. An extension to multiple periods could be developed along the lines of Edwards (2010), who studies optimal portfolio choice dynamics in the presence of health shocks. Our two-period model is deliberately abstract to identify the preferences conditions for definitive effects without imposing functional form assumptions on utility.

Correlated returns have the potential to motivate individuals to accumulate more savings and, when appropriately designed, these incentives can be provided in a welfare-enhancing way. We argue that correlated returns can be a useful tool to counteract low household saving rates around the globe.[17] Our results can inform the study of correlated returns in applied settings that take specific institutional details into account. Our findings may also contribute to the development and implementation of new financial tools with correlated returns.

[17] According to the OECD, household saving rates are 6.0% in the U.S., 4.1% in the European Union and only 0.7% in Japan (see `https://data.oecd.org/`). They are substantially lower today compared to the time period from the 1960s to the 1990s (see Maddison, 1992). In Lusardi et al. (2011), over half of the surveyed households could not come up with $2,000 in case of emergency. Low levels of financial liquidity put households at risk and have the potential to create negative externalities in the economy.

References

Atalay, K., Bakhtiar, F., Cheung, S., and Slonim, R. (2014). Savings and prize-linked savings accounts. *Journal of Economic Behavior & Organization*, 107(Part A): 86–106.

Baiardi, D., De Donno, M., Magnani, M., and Menegatti, M. (2015). New results on precautionary saving under two risks. *Economics Letters*, 130: 17–20.

Baiardi, D., Magnani, M., and Menegatti, M. (2014). Precautionary saving under many risks. *Journal of Economics*, 113(3): 211–228.

Carthy, T., Chilton, S., Covey, J., Hopkins, L., Jones-Lee, M., Loomes, G., Pidgeon, N., and Spencer, A. (1998). On the contingent valuation of safety and the safety of contingent valuation: Part 2 - The CV/SG "chained" approach. *Journal of Risk and Uncertainty*, 17(3): 187–214.

Chiu, H., Eeckhoudt, L., and Rey, B. (2012). On relative and partial risk attitudes: Theory and implications. *Economic Theory*, 50(1): 151–167.

Choi, G., Kim, I., and Snow, A. (2001). Comparative statics predictions for changes in uncertainty in the portfolio and savings problems. *Bulletin of Economic Research*, 53(1): 61–72.

Cook, P. and Graham, D. (1977). The demand for insurance and protection: The case of irreplaceable commodities. *Quarterly Journal of Economics*, 91(1): 143–1564.

Courbage, C., Loubergé, H., and Peter, R. (2017). Optimal prevention for multiple risks. *Journal of Risk and Insurance*, 84(3): 899–922.

Courbage, C. and Rey, B. (2007). Precautionary saving in the presence of other risks. *Economic Theory*, 32(2): 417–424.

Crainich, D., Eeckhoudt, L., and Le Courtois, O. (2016). An index of (absolute) correlation aversion: Theory and some implications. *IESEG School of Management (Working Paper)*.

Denuit, M., Eeckhoudt, L., and Menegatti, M. (2011). Correlated risks, bivariate utility and optimal choices. *Economic Theory*, 46(1): 39–54.

Denuit, M., Eeckhoudt, L., and Rey, B. (2010). Some consequences of correlation aversion in decision science. *Annals of Operations Research*, 176(1): 259–269.

Denuit, M. and Rey, B. (2014). Benchmark values for higher order coefficients of relative risk aversion. *Theory and Decision*, 76(1): 81–94.

Dionne, G. and Eeckhoudt, L. (1984). Insurance and saving: some further results. *Insurance: Mathematics and Economics*, 3(2): 101–110.

Doherty, N. A. and Schlesinger, H. (1983). Optimal insurance in incomplete markets. *Journal of Political Economy*, 91(6): 1045–1054.

Ebert, S. (2015). On skewed risks in economic models and experiments. *Journal of Economic Behavior & Organization*, 112: 85–97.

Ebert, S. and van de Kuilen, G. (2017). Measuring multivariate risk preferences. *Tilburg University (Working Paper)*.

Edwards, R. D. (2010). Optimal portfolio choice when utility depends on health. *International Journal of Economic Theory*, 6(2): 205–225.

Eeckhoudt, L. and Gollier, C. (2013). The effects of changes in risk on risk taking: A survey. In Dionne, G., editor, *Handbook of Insurance*, chapter 5, pages 123–134. Springer, 2 edition.

Eeckhoudt, L., Rey, B., and Schlesinger, H. (2007). A good sign for multivariate risk taking. *Management Science*, 53(1): 117–124.

Eeckhoudt, L. and Schlesinger, H. (2008). Changes in risk and the demand for saving. *Journal of Monetary Economics*, 55(7): 1329–1336.

Epstein, L. and Tanny, S. (1980). Increasing generalized correlation: A definition and some economic consequences. *Canadian Journal of Economics*, 13(1): 16–34.

Evans, W. and Viscusi, K. (1991). Estimation of state-dependent utility functions using survey data. *The Review of Economics and Statistics*, 73(1): 94–104.

Finkelstein, A., Luttmer, E., and Notowidigdo, M. (2013). What good is wealth without health? The effect of health on the marginal utility of consumption. *Journal of the European Economic Association*, 11(suppl_1): 221–258.

Gollier, C. (2015). Discounting, inequality and economic convergence. *Journal of Environmental Economics and Management*, 69: 53–61.

Gollier, C. and Pratt, J. W. (1996). Risk vulnerability and the tempering effect of background risk. *Econometrica*, 64(5): 1109–1123.

Hadar, J. and Seo, T. (1990). The effects of shifts in a return distribution on optimal portfolios. *International Economic Review*, 31(3): 721–736.

Hofmann, A. and Peter, R. (2016). Self-insurance, self-protection, and saving: On consumption smoothing and risk management. *Journal of Risk and Insurance*, 83(3): 719–734.

Jappelli, T., Pistaferri, L., and Weber, G. (2007). Health care quality, economic inequality, and precautionary saving. *Health Economics*, 16(4): 327–346.

Jindapon, P., Sujarittanonta, P., and Viriyavipart, A. (2019). Prize-linked savings games: Theory and experiment. *Working Paper (University of Alabama)*.

Jouini, E., Napp, C., and Nocetti, D. (2013). Economic consequences of nth-degree risk increases and nth-degree risk attitudes. *Journal of Risk and Uncertainty*, 47(2): 199–224.

Kihlstrom, R. E., Romer, D., and Williams, S. (1981). Risk aversion with random initial wealth. *Econometrica*, 49(4): 911–920.

Kimball, M. (1990). Precautionary saving in the small and in the large. *Econometrica*, 58(1): 53–73.

Lehmann, E. L. (1966). Some concepts of dependence. *The Annals of Mathematical Statistics*, 37(5): 1137–1153.

Leland, H. (1968). Saving and uncertainty: The precautionary demand for saving. *The Quarterly Journal of Economics*, 82(3): 465–473.

Li, J. (2011). The demand for a risky asset in the presence of a background risk. *Journal of Economic Theory*, 146(1): 372–391.

Li, J. (2012). Precautionary saving in the presence of labor income and interest rate risks. *Journal of Economics*, 106(3): 251–266.

Liu, D. and Menegatti, M. (2019a). Optimal saving and health prevention. *Journal of Economics*, 128(2): 177–191.

Liu, D. and Menegatti, M. (2019b). Precautionary investment in wealth and health. *Journal of Risk and Insurance*, 86(1): 237–255.

Liu, L. (2004). Comorbidities and the willingness to pay for reducing the risk of a targeted disease: introducing endogenous effort for risk reduction. *Health Economics*, 13(5): 493–498.

Lusardi, A., Schneider, D., and Tufano, P. (2011). Financially fragile households: Evidence and implications. Technical report, National Bureau of Economic Research.

Maddison, A. (1992). A long-run perspective on saving. *The Scandinavian Journal of Economics*, 94(2):181–196.

Magnani, M. and Menegatti, M. (2015). Precautionary saving and changes in risk correlation. *Economics Bulletin*, 35(1): 561–570.

Menegatti, M. (2009a). Optimal saving in the presence of two risks. *Journal of Economics*, 96(3):277–288.

Menegatti, M. (2009b). Precautionary saving in the presence of other risks: a comment. *Economic Theory*, 39(3): 473–476.

Menegatti, M. and Peter, R. (2020). Risk effects on optimal decision making–a question of the right order. *Available at SSRN 3585380*.

Menezes, C. and Hanson, D. (1970). On the theory of risk aversion. *International Economic Review*, 11(3): 481–487.

Mossin, J. (1968). Aspects of rational insurance purchasing. *Journal of Political Economy*, 46(4): 553–568.

Nachman, D. C. (1982). Preservation of "more risk averse" under expectations. *Journal of Economic Theory*, 28(2): 361–368.

Nocetti, D. and Smith, W. T. (2010). Uncertainty, the demand for health care, and precautionary saving. *The BE Journal of Economic Analysis & Policy*, 10(1): 1–29.

Nocetti, D. and Smith, W. T. (2011). Precautionary saving and endogenous labor supply with and without intertemporal expected utility. *Journal of Money, Credit and Banking*, 43(7): 1475–1504.

Noussair, C., Trautmann, S., and van de Kuilen, G. (2013). Higher order risk attitudes, demographics, and financial decisions. *Review of Economic Studies*, 81(1): 325–355.

Peter, R., Soika, S., and Steinorth, P. (2016). Health insurance, health savings accounts and healthcare utilization. *Health Economics*, 25(3): 357–371.

Rey, B. (2003). A note on optimal insurance in the presence of a nonpecuniary background risk. *Theory and Decision*, 54(1): 73–83.

Rey, B. and Rochet, J.-C. (2004). Health and wealth: How do they affect individual preferences? *The Geneva Papers on Risk and Insurance Theory*, 29(1): 43–54.

Richter, A. and Wilson, T. (2020). Covid-19: Implications for insurer risk management and the insurability of pandemic risk. *Working Paper (LMU Munich).*

Rothschild, M. and Stiglitz, J. (1970). Increasing risk: I. A definition. *Journal of Economic Theory*, 2(3): 225–243.

Sandmo, A. (1970). The effect of uncertainty on saving decisions. *The Review of Economic Studies*, 37(3): 353–360.

Sinagl, P. (2020). When does cash-flow risk matter to investors? Evidence from the covid-19 pandemic. *Working Paper (University of Iowa).*

Sloan, F., Viscusi, K., Chesson, H., Conover, C., and Whetten-Goldstein, K. (1998). Alternative approaches to valuing intangible health losses: The evidence for multiple sclerosis. *Journal of Health Economics*, 17(4): 475–497.

Steinorth, P. (2012). The demand for enhanced annuities. *Journal of Public Economics*, 96(11): 973–980.

Trautmann, S. and van de Kuilen, G. (2018). Higher order risk attitudes: A review of experimental evidence. *European Economic Review*, 103: 108–124.

Tsetlin, I. and Winkler, R. L. (2005). Risky choices and correlated background risk. *Management Science*, 51(9): 1336–1345.

Viscusi, K. and Evans, W. (1990). Utility functions that depend on health status: Estimates and economic implications. *The American Economic Review*, 80(3): 353–374.

White, L. (2008). Prudence in bargaining: The effect of uncertainty on bargaining outcomes. *Games and Economic Behavior*, 62(1): 211–231.

A Mathematical proofs

A.1 Proof of Proposition 2

We first repeat the definition of the critical transfer rate:

$$\Delta_{crit} = -\frac{-u'(w_1) + \beta(1+r)\left[pv^{(1,0)}(B_0) + (1-p)v^{(1,0)}(G_0)\right]}{\beta p(1-p)\left[v^{(1,0)}(G_0) - v^{(1,0)}(B_0)\right]}. \tag{20}$$

The numerator $N = -u'(w_1) + \beta(1+r)\left[pv^{(1,0)}(B_0) + (1-p)v^{(1,0)}(G_0)\right]$ is non-positive because the individual does not save under the market interest rate (i.e., $U_s(0;0) \leq 0$). The denominator $D = \beta p(1-p)\left[v^{(1,0)}(G_0) - v^{(1,0)}(B_0)\right]$ is positive due to $v^{(1,1)} > 0$.

The derivative of Δ_{crit} with respect to the utility discount factor β is

$$\frac{\mathrm{d}\Delta_{crit}}{\mathrm{d}\beta} = -\frac{u'(w_1)}{\beta D} < 0. \tag{21}$$

The derivative of Δ_{crit} with respect to the market rate of return r is

$$\frac{\mathrm{d}\Delta_{crit}}{\mathrm{d}r} = -\frac{\beta\left[pv^{(1,0)}(B_0) + (1-p)v^{(1,0)}(G_0)\right]}{D} < 0. \tag{22}$$

If we differentiate Δ_{crit} with respect to first-period income w_1, we obtain

$$\frac{\mathrm{d}\Delta_{crit}}{\mathrm{d}w_1} = \frac{u''(w_1)}{D} < 0. \tag{23}$$

This proves *a)*.

To show *b)* to *e)*, we first state the corresponding four derivatives and provide sufficient conditions to sign them in the following paragraphs. Differentiating Δ_{crit} with respect to second-period income w_2 yields

$$\begin{aligned}\frac{\mathrm{d}\Delta_{crit}}{\mathrm{d}w_2} &= \frac{\beta p(1-p)}{D^2}\Big\{-\beta(1+r)\left[v^{(1,0)}(G_0) - v^{(1,0)}(B_0)\right]\left[pv^{(2,0)}(B_0) + (1-p)v^{(2,0)}(B_0)\right] \\ &\quad + N\left[v^{(2,0)}(G_0) - v^{(2,0)}(B_0)\right]\Big\},\end{aligned} \tag{24}$$

which we rewrite as follows:

$$\begin{aligned}\frac{\mathrm{d}\Delta_{crit}}{\mathrm{d}w_2} &= \frac{\beta p(1-p)}{D^2}\Big\{-\beta(1+r)v^{(2,0)}(G_0)\left[v^{(1,0)}(G_0) - v^{(1,0)}(B_0)\right] \\ &\quad + \left[v^{(2,0)}(G_0) - v^{(2,0)}(B_0)\right]\left[\beta(1+r)v^{(1,0)}(G_0) - u'(w_1)\right]\Big\}.\end{aligned} \tag{25}$$

The derivative of Δ_{crit} with respect to the high value of the non-financial variable H_g is

$$\frac{\mathrm{d}\Delta_{crit}}{\mathrm{d}H_g} = \frac{\beta p(1-p)}{D^2}\Big\{\beta(1+r)v^{(1,0)}(B_0) - u'(w_1)\Big\}v^{(1,1)}(G_0). \tag{26}$$

The derivative of Δ_{crit} with respect to the low value of the non-financial variable H_b is

$$\frac{\mathrm{d}\Delta_{crit}}{\mathrm{d}H_b} = -\frac{\beta p(1-p)}{D^2}\left\{\beta(1+r)v^{(1,0)}(G_0) - u'(w_1)\right\} v^{(1,1)}(B_0). \tag{27}$$

We also compute the derivative of Δ_{crit} with respect to the probability p:

$$\frac{\mathrm{d}\Delta_{crit}}{\mathrm{d}p} = \frac{1}{p(1-p)D}\left\{\beta(1+r)p(1-p)\left[v^{(1,0)}(G_0) - v^{(1,0)}(B_0)\right] + N(1-2p)\right\}, \tag{28}$$

which we rearrange to

$$\begin{aligned}\frac{\mathrm{d}\Delta_{crit}}{\mathrm{d}p} &= \frac{1}{p(1-p)D}\left\{\beta(1+r)p^2\left[v^{(1,0)}(G_0) - v^{(1,0)}(B_0)\right]\right.\\ &\quad \left. +(1-2p)\left[\beta(1+r)v^{(1,0)}(G_0) - u'(w_1)\right]\right\}.\end{aligned} \tag{29}$$

Each of these four derivatives contains either the term $\left[\beta(1+r)v^{(1,0)}(G_0) - u'(w_1)\right]$ or the term $\left[\beta(1+r)v^{(1,0)}(B_0) - u'(w_1)\right]$. To sign these terms, we compare $\mu^g_{1,2}$ and $\mu^b_{1,2}$ as defined in Fn. 9 with $1/(1+r)$. When $v^{(1,1)} > 0$, we obtain

$$\mu^b_{1,2} = \frac{\beta v^{(1,0)}(w_2, H_b)}{u'(w_1)} < \frac{\beta v^{(1,0)}(w_2, H_g)}{u'(w_1)} = \mu^g_{1,2}, \tag{30}$$

so that $\mu^b_{1,2} < \mu_{1,2} < \mu^g_{1,2}$. The individual does not save under the market rate of return; therefore, $U_s(0;0) \leq 0$, which is equivalent to $\mu_{1,2} \leq 1/(1+r)$. As a result, $\mu^b_{1,2} < 1/(1+r)$ while $\mu^g_{1,2}$ may or may not be below $1/(1+r)$.

The first term in the curly bracket of Eq. (24) is always positive whereas the second term is non-negative if $v^{(2,1)} \leq 0$; then, $\mathrm{d}\Delta_{crit}/\mathrm{d}w_2 > 0$. If $v^{(2,1)} > 0$, then $\mu^g_{1,2} \geq 1/(1+r)$ is sufficient for $\mathrm{d}\Delta_{crit}/\mathrm{d}w_2 > 0$ from Eq. (25). Now $\mu^b_{1,2}$ is below $1/(1+r)$ so the curly bracket in Eq. (26) is negative, implying $\mathrm{d}\Delta_{crit}/\mathrm{d}H_g < 0$. The sign of $\mathrm{d}\Delta_{crit}/\mathrm{d}H_b$ coincides with the sign of the curly bracket in Eq. (27); therefore, $\mu^g_{1,2} < 1/(1+r)$ is equivalent to $\mathrm{d}\Delta_{crit}/\mathrm{d}H_b > 0$. Eq. (28) shows that $\mathrm{d}\Delta_{crit}/\mathrm{d}p > 0$ for $p \geq 1/2$. If $p < 1/2$, it can be seen from Eq. (29) that $\mu^g_{1,2} \geq 1/(1+r)$ is sufficient for $\mathrm{d}\Delta_{crit}/\mathrm{d}p > 0$.

A.2 Proof of Proposition 3

We first define $F(x+y, H) = v^{(1,0)}(x+y, H) + yv^{(2,0)}(x+y, H)$ for any x, y, H. The terms $F(w_2+(1+r_b)s^*, H_b)$ and $F(w_2+(1+r_g)s^*, H_g)$ denote the net effect between the substitution and the wealth effect in the bad and the good state. We can then rewrite the curly bracket in Eq. (10) as $F(w_2+(1+r_g)s^*, H_g) - F(w_2+(1+r_b)s^*, H_b)$. The incentive effect in the good state dominates the incentive effect in the bad state if and only if a marginal increase in the transfer rate increases saving.

We first analyze correlation lovers ($v^{(1,1)} > 0$). We have $\min\{0, \Delta_{crit}\} < \Delta < \max\{0, \Delta_{crit}\}$ for $\Delta \in I$. Then, either $0 < \Delta < \Delta_{crit}$ so that $r_g > r_b$ and $s^* < 0$ per Proposition 1(i), or $\Delta_{crit} < \Delta < 0$ so that $r_b > r_g$ and $s^* > 0$ per Proposition 1(i). In both cases we obtain $(1+r_b)s^* > (1+r_g)s^*$ so that $F(w_2 + (1+r_b)s^*, H) < F(w_2 + (1+r_g)s^*, H)$ if $F(x+y, H)$ is decreasing in y (i.e., if $\mathrm{d}F(x+y,H)/\mathrm{d}y < 0$). The last condition is equivalent to $-yv^{(3,0)}(x+y,H)/v^{(2,0)}(x+y,H) < 2$. Now if $F(x+y,H)$ is increasing in H (i.e., if $\mathrm{d}F(x+y,H)/\mathrm{d}H > 0$), which is equivalent to $-yv^{(2,1)}(x+y,H)/v^{(1,1)}(x+y,H) < 1$ due to $v^{(1,1)} > 0$, the following chain of inequalities holds:

$$F(w_2 + (1+r_b)s^*, H_b) < F(w_2 + (1+r_g)s^*, H_b) < F(w_2 + (1+r_g)s^*, H_g). \tag{31}$$

If $\Delta \in J$, then $(1+r_b)s^* < (1+r_g)s^*$ so $F(x+y,H)$ being decreasing in y ensures that the incentive effect increases when replacing wealth in the bad state by wealth in the good state.

The case of a correlation averter ($v^{(1,1)} < 0$) is similar. For $\Delta \in I$, we have $(1+r_b)s^* < (1+r_g)s^*$ and for $\Delta \in J$ we have $(1+r_b)s^* > (1+r_g)s^*$ due to Proposition 1(ii). The restriction on partial prudence in wealth ensures that the incentive effect decreases when replacing wealth in the bad state by wealth in the good state. Also, $-yv^{(2,1)}(x+y,H)/v^{(1,1)}(x+y,H) < 1$ is now equivalent to $F(x+y,H)$ being decreasing in H (i.e., $\mathrm{d}F(x+y,H)/\mathrm{d}H < 0$) due to $v^{(1,1)} < 0$. This yields the following chain of inequalities:

$$F(w_2 + (1+r_b)s^*, H_b) > F(w_2 + (1+r_g)s^*, H_b) > F(w_2 + (1+r_g)s^*, H_g). \tag{32}$$

Hence, saving increases following a marginal decrease of the transfer rate.

A.3 Proof of Corollaries 1 and 2

For $\Delta = 0$, we obtain $r_b = r_g = r$ so that G^* and B^* only differ in the non-financial variable. The curly bracket in Eq. (10) simplifies to

$$\left[v^{(1,0)}(G^*) - v^{(1,0)}(B^*)\right] + (1+r)s^* \left[v^{(2,0)}(G^*) - v^{(2,0)}(B^*)\right]. \tag{33}$$

The first square bracket has the same sign as $v^{(1,1)}$, the second square bracket has the same sign as $v^{(2,1)}$.

For $\Delta = \Delta_{crit}$, the optimal level of saving is zero ($s^* = 0$) and the curly bracket in Eq. (10) simplifies to $v^{(1,0)}(G_0) - v^{(1,0)}(B_0)$. It has the same sign as $v^{(1,1)}$.

A.4 The intensity of partial cross-prudence in the non-financial variable

Let $\widetilde{\varepsilon}$ denote a small zero-mean risk on wealth, $\mathbb{E}\widetilde{\varepsilon} = 0$. If the individual is not cross-prudence neutral ($v^{(2,1)} \neq 0$), the introduction of this wealth risk affects the marginal utility of the non-financial variable. If the individual is cross-prudent in the non-financial variable, it increases expected marginal utility of the non-financial variable. If the financial outcome $x+y$ and we replace y by $y(1+\widetilde{\varepsilon})$, we define the proportional reduction ϕ of y that has the same effect on the marginal utility of the non-financial variable:

$$\mathbb{E}v^{(0,1)}(x+y(1+\widetilde{\varepsilon}),H)=v^{(0,1)}(x+y(1-\phi),H). \tag{34}$$

We call ϕ a *partial cross-prudence premium.* It measures by how much the $\widetilde{\varepsilon}$ risk affects the marginal utility of the non-financial variable in units of the financial variable. Analogous to Kimball (1990), we apply Taylor approximations to Eq. (34) and obtain

$$\phi\simeq-\frac{1}{2}\cdot\mathrm{Var}(\widetilde{\varepsilon})\cdot\frac{yv^{(2,1)}(x+y,H)}{v^{(1,1)}(x+y,H)}. \tag{35}$$

$\mathrm{Var}(\widetilde{\varepsilon})$ denotes the variance of the $\widetilde{\varepsilon}$-risk. This justifies $-yv^{(2,1)}(x+y,H)/v^{(1,1)}(x+y,H)$ as an intensity measure of partial cross-prudence in the non-financial variable. For small risks it is proportional to the size of the partial cross-prudence premium.

A.5 Proof of Remark 1

The proof follows by taking the derivative of the first-order expression (5) with respect to the exogenous parameters. For the utility discount factor β, this yields

$$U_{s\beta}=(1+r_b)pv^{(1,0)}(B^*)+(1+r_g)(1-p)v^{(1,0)}(G^*)>0. \tag{36}$$

For first-period income w_1, we obtain

$$U_{sw_1}=-u''(w_1-s^*)>0. \tag{37}$$

For second-period income w_2, we derive

$$U_{sw_2}=\beta(1+r_b)pv^{(2,0)}(B^*)+\beta(1+r_g)(1-p)v^{(2,0)}(G^*)<0. \tag{38}$$

For the market rate of return r, we find

$$\begin{aligned}U_{sr} &= \beta pv^{(1,0)}(B^*)+\beta(1-p)v^{(1,0)}(G^*)\\ &\quad+\beta(1+r_b)s^*pv^{(2,0)}(B^*)+\beta(1+r_g)s^*(1-p)v^{(2,0)}(G^*).\end{aligned} \tag{39}$$

For individuals who borrow,[18] this is always positive indicating that a higher market rate of return leads to less borrowing (i.e., more saving). For individuals who save, we can rearrange U_{sr} as follows:

$$\begin{aligned}U_{sr} &= \beta pv^{(1,0)}(B^*)\big[1-\mathcal{R}(w_2+(1+r_b)s^*,H_b)\big]\\ &\quad+\beta(1-p)v^{(1,0)}(G^*)\big[1-\mathcal{R}(w_2+(1+r_g)s^*,H_g)\big].\end{aligned} \tag{40}$$

[18] That is, if $\Delta<\Delta_{crit}$ for correlation lovers or $\Delta>\Delta_{crit}$ for correlation averters, see Proposition 1.

If partial risk aversion in wealth is less than unity, both square brackets are positive indicating that an increase in the market rate of return raises saving. Finally, we obtain

$$U_{sH_b} = \beta(1+r_b)(1-p)v^{(1,1)}(B^*) \quad \text{and} \quad U_{sH_g} = \beta(1+r_g)pv^{(1,1)}(G^*) \tag{41}$$

for the non-financial variables. Both terms have the same sign as $v^{(1,1)}$.

A.6 Proof of Proposition 4

For $\Delta \in (\Delta_{crit}, 0)$, we have $r_b > r_g$, $s^* > 0$ for correlation lovers and $s^* < 0$ for correlation averters (see Proposition 1). So $w_2 + (1+r_b)s^* > w_2 + (1+r_g)s^*$ for correlation lovers, which renders the square bracket in (12) positive, and $w_2+(1+r_b)s^* < w_2+(1+r_g)s^*$ for correlation averters, which renders the square bracket in (12) negative. We always obtain $\mathrm{d}U/\mathrm{d}\Delta > 0$ because the square bracket in (12) and the individual's saving choice have the same sign.

Similarly, if $\Delta \in (0, \Delta_{crit})$, we have $r_g > r_b$, $s^* < 0$ for correlation lovers and $s^* > 0$ for correlation averters (see Proposition 1). Then, $w_2+(1+r_b)s^* > w_2+(1+r_g)s^*$ for correlation lovers, which renders the square bracket in (12) positive, and $w_2+(1+r_b)s^* < w_2+(1+r_g)s^*$ for correlation averters, which makes the square bracket in (12) negative. Hence, $\mathrm{d}U/\mathrm{d}\Delta < 0$ because the square bracket in (12) and the individual's saving choice have opposite signs.

A.7 Proof of Corollaries 3 and 4

For $\Delta = 0$, we have $r_b = r_g = r$, and the square bracket in Eq. (12) is positive (negative) for correlation lovers (averters). If $\Delta_{crit} < 0$, correlation lovers save while correlation averters borrow under the market rate of return. In either case, $\mathrm{d}U/\mathrm{d}\Delta > 0$. If $\Delta_{crit} > 0$, correlation lovers borrow while correlation averters save under the market rate, and $\mathrm{d}U/\mathrm{d}\Delta < 0$.

At $\Delta = \Delta_{crit}$, behavior switches from borrowing to saving for correlation lovers and from saving to borrowing for correlation averters. In a neighborhood of Δ_{crit}, the square bracket in Eq. (12) is strictly positive (negative) for correlation lovers (averters). In either case, $U(s^*;\Delta)$ switches from strictly decreasing to strictly increasing at Δ_{crit}.

A.8 Proof of Proposition 5

If Δ^* is an interior maximizer of $U(s^*;\Delta)$, then $\mathrm{d}U(s^*;\Delta^*)/\mathrm{d}\Delta = 0$ and $\mathrm{d}^2U(s^*;\Delta^*)/\mathrm{d}\Delta^2 < 0$. Taking into account that s^* depends on Δ, we obtain

$$\frac{\mathrm{d}^2U}{\mathrm{d}\Delta^2} = U_{\Delta\Delta} + U_{s\Delta}\frac{\mathrm{d}s^*}{\mathrm{d}\Delta}, \tag{42}$$

which is negative for $\Delta = \Delta^*$ due to the second-order condition.

Let k an exogenous parameter of our model, $k \in \{H_g, H_b, \beta, w_1\}$. To examine the effect of k on Δ^*, we need to sign $\mathrm{d}\Delta^*/\mathrm{d}k$. We write $\Delta^*(k)$ because Δ^* depends on k; parameter k affects the optimal level of saving in two ways, directly via first-order condition (5), and indirectly through its effect on Δ^*; we write $s^*(\Delta^*(k), k)$. The effect of k on the first-order optimality condition for Δ^* is threefold; there is a direct effect if k appears directly in the first-order condition $U_\Delta = 0$, there is an indirect effect via the optimal transfer rate and another indirect effect via the optimal level of saving. To capture all these effects, write

$$U_\Delta(s^*(\Delta^*(k), k), \Delta^*(k), k) = 0. \tag{43}$$

The net effect of a marginal variation in k must be such that the first-order optimality condition remains satisfied, i.e.,

$$U_{\Delta s}\left(\frac{\mathrm{d}s^*}{\mathrm{d}\Delta} \cdot \frac{\mathrm{d}\Delta^*}{\mathrm{d}k} + \frac{\mathrm{d}s^*}{\mathrm{d}k}\right) + U_{\Delta\Delta} \cdot \frac{\mathrm{d}\Delta^*}{\mathrm{d}k} + U_{\Delta k} = 0. \tag{44}$$

Solving for $\mathrm{d}\Delta^*/\mathrm{d}k$ renders

$$\frac{\mathrm{d}\Delta^*}{\mathrm{d}k} = -\frac{U_{\Delta k} + U_{\Delta s} \cdot \frac{\mathrm{d}s^*}{\mathrm{d}k}}{U_{\Delta\Delta} + U_{\Delta s} \cdot \frac{\mathrm{d}s^*}{\mathrm{d}\Delta}}. \tag{45}$$

The denominator is negative due to the second-order condition for Δ^*. Therefore, the sign of $\mathrm{d}\Delta^*/\mathrm{d}k$ coincides with the sign of its numerator. We apply the implicit function rule and rearrange to obtain:

$$U_{\Delta k} + U_{\Delta s} \cdot \frac{\mathrm{d}s^*}{\mathrm{d}k} = U_{\Delta k} - U_{\Delta s}\frac{U_{sk}}{U_{ss}} = -\frac{1}{U_{ss}} \cdot \Big[U_{\Delta s}U_{sk} - U_{\Delta k}U_{ss}\Big]. \tag{46}$$

So the sign of $\mathrm{d}\Delta^*/\mathrm{d}k$ coincides with the sign of the square bracket in Eq. (46). In the sequel, we will determine this sign for $k \in \{H_g, H_b, \beta, w_1\}$, taking into account that both $U_s(s^*; \Delta^*) = 0$ and $U_\Delta(s^*; \Delta^*) = 0$ hold at an interior maximizer of $U(s^*; \Delta)$.

For $k = H_g$, we obtain

$$U_{\Delta s}U_{sH_g} - U_{\Delta H_g}U_{ss} = \beta p(1-p)v^{(1,1)}(G^*)s^* \underbrace{\Big[-u''(w_1 - s^*) - \beta(1+r_b)(1+r)v^{(2,0)}(B^*)\Big]}_{>0}, \tag{47}$$

and the sign is jointly determined by the individual's correlation attitude and saving behavior. For $k = H_b$, we find

$$U_{\Delta s}U_{sH_b} - U_{\Delta H_b}U_{ss} = \beta p(1-p)v^{(1,1)}(B^*)s^* \underbrace{\Big[u''(w_1 - s^*) + \beta(1+r_g)(1+r)v^{(2,0)}(G^*)\Big]}_{<0}, \tag{48}$$

and the sign is also jointly determined by the individual's correlation attitude and saving behavior. This proves *a)* and *b)*. Notice that $\operatorname{sgn}(\mathrm{d}\Delta^*/\mathrm{d}H_g) = -\operatorname{sgn}(\mathrm{d}\Delta^*/\mathrm{d}H_b)$ irrespective of the individual's correlation attitude and saving behavior. The comparative statics of the two non-financial variables always go in opposite directions.

To show *c)*, we set $k = \beta$ and calculate

$$\begin{aligned} U_{\Delta s}U_{s\beta} - U_{\Delta\beta}U_{ss} &= \beta(1+r)p(1-p)v^{(1,0)}(G^*)v^{(1,0)}(B^*) \\ &\quad \cdot \left[\mathcal{R}(w_2+(1+r_b)s^*, H_b) - \mathcal{R}(w_2+(1+r_g)s^*, H_g)\right], \end{aligned} \tag{49}$$

which is positive if and only if partial risk aversion is higher in the bad state than the good state. Similarly, for $k = w_1$ we find

$$\begin{aligned} U_{\Delta s}U_{sw_1} - U_{\Delta w_1}U_{ss} &= -\beta p(1-p)u''(w_1-s^*)v^{(1,0)}(G^*) \\ &\quad \cdot \left[\mathcal{R}(w_2+(1+r_b)s^*, H_b) - \mathcal{R}(w_2+(1+r_g)s^*, H_g)\right] \end{aligned} \tag{50}$$

and obtain the same equivalent condition for a positive sign.

A.9 Global concavity of objective function (13)

We suppress the argument of utility in the first period and use B and G to abbreviate the pairs of consumption and the non-financial variable in the bad and the good state at a given level of saving and insurance coverage. We obtain the following derivatives:

$$\begin{aligned} U_{ss} &= u'' + \beta(1+r_b)^2 p v^{(2,0)}(B) + \beta(1+r_g)^2(1-p)v^{(2,0)}(G) < 0, \\ U_{\alpha\alpha} &= \left(\frac{m}{1+r}pT\right)^2 u'' + \beta p T^2 v^{(2,0)}(B) < 0, \\ U_{s\alpha} &= \frac{m}{1+r}pTu'' + \beta pT(1+r_b)v^{(2,0)}(B) < 0. \end{aligned} \tag{51}$$

After some algebra, we calculate the determinant of the Hessian of U as follows:

$$\begin{aligned} U_{ss}U_{\alpha\alpha} - U_{s\alpha}^2 &= \beta p T^2 \left\{ \left(\frac{mp(1+r_g)}{1+r} - 1\right)^2 u''v^{(2,0)}(B) \right. \\ &\quad \left. + (1-p)(1+r_g)^2 v^{(2,0)}(G)\left(p\left(\frac{m}{1+r}\right)^2 u'' + v^{(2,0)}(B)\right)\right\} > 0. \end{aligned} \tag{52}$$

U is globally concave in (s, α) for any transfer rate as long as u and v are concave in wealth.

A.10 Proof of Proposition 6

An interior solution (s^*, α^*) is characterized by the following pair of first-order conditions,

$$\begin{aligned} U_s &= -u'(w_1^*) + \beta(1+r)\left[pv^{(1,0)}(B^*) + (1-p)v^{(1,0)}(G^*)\right] \\ &\quad + \beta p(1-p)\Delta\left[v^{(1,0)}(G^*) - v^{(1,0)}(B^*)\right] = 0, \end{aligned} \tag{53}$$

$$U_\alpha = -\frac{m}{1+r}pTu'(w_1^*) + \beta pTv^{(1,0)}(B^*) = 0. \tag{54}$$

w_1^*, B^* and G^* are shorthand for the arguments in the first and second period when evaluated at (s^*, α^*). Solve $U_\alpha = 0$ for $u'(w_1^*)$, substitute it into in $U_s = 0$, and rearrange to obtain

$$(1-p)(1+r_g)\left[v^{(1,0)}(G^*) - v^{(1,0)}(B^*)\right] + \frac{m-1}{m}(1+r)v^{(1,0)}(B^*) = 0. \tag{55}$$

The individual's indirect utility function is given by $U(s^*, \alpha^*; \Delta)$, and an application of the envelope theorem yields

$$\frac{\mathrm{d}U}{\mathrm{d}\Delta} = \beta p(1-p)s^*\left[v^{(1,0)}(G^*) - v^{(1,0)}(B^*)\right] = -\frac{\beta(1+r)ps^*}{(1+r_g)}\frac{m-1}{m}v^{(1,0)}(B^*). \tag{56}$$

The last equality is obtained by substituting from Eq. (55). If insurance is actuarially fair (i.e., $m = 1$), then $\mathrm{d}U/\mathrm{d}\Delta$ is zero, which proves result (*ii*). If insurance is actuarially unfair (i.e., $m > 1$), then $\mathrm{d}U/\mathrm{d}\Delta$ is negative for savers and positive for borrowers. This demonstrates result (*i*). If insurance is actuarially favorable (i.e., $m < 1$), then $\mathrm{d}U/\mathrm{d}\Delta$ is positive for savers and negative for borrowers, thus proving (*iii*).

A.11 Numerical example for Proposition 10

Let $w_1 = 1,200$ and $w_2 = 1000$ be the income levels in the first and second period. Consider a health risk with three equiprobable states, $H_g = 1$, $H_m = 0.8$ and $H_g = 0.6$. Assume a market interest rate of $r = 1\%$ and a utility discount factor of $\beta = 0.98$. The utility functions in the first and second period are $u(w) = \log(w)$ and $v(w, H) = \log(w) \cdot \sqrt{H}$ for $w > 1$. The individual is a correlation lover because $v^{(1,1)} > 0$. Let the financial risk take values $T_g = 0$, $T_m = 200$ and $T_b = 400$ for the three values of health. Lower health outcomes require more costly treatment. Assume further that actuarially fair insurance is available, $m = 1$.

The model is solved numerically. We obtain optimal saving of $s^* = 56$ and optimal insurance of $\alpha^* = 0.40$. $T(\widetilde{H})$ is uncorrelated with $v^{(1,0)}(w_2 + \widetilde{S}^* - (1-\alpha^*)T(\widetilde{H}), \widetilde{H})$ as required by condition (19). However, marginal utility of consumption in the second period is not constant across states. This allows us to increase welfare with the help of a correlated return. Take $\Delta(H_g) = 0.01$, $\Delta(H_m) = 0.02$ and $\Delta(H_b) = -0.03$; then $\mathbb{E}\Delta(\widetilde{H}) = 0$ and the correlation between $\widetilde{H}$ and $\Delta(\widetilde{H})$ is 75%. The correlation between $\Delta(\widetilde{H})$ and marginal utility of consumption in the second period is 65%, and we confirm numerically that welfare increases following a marginal increase of δ. In fact, the welfare-maximizing exposure to correlated return risk is achieved for $\delta^* = 1.26$ in the example.