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Friction Modeling of a High-Precision Positioning System

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***Abstract*— Friction modeling of a high-precision positioning system using linear permanent magnet synchronous motors is investigated. The friction force is measured precisely by some specific experiments to eliminate the parasitic ripple force. The experimental data show that the relation between the lubrication force and the velocity is not linear as it is assumed in the conventional friction models used in automatic control. A new model for the lubrication force based on tribological observations is proposed and introduced to the LuGre friction model. The new model can characterize the lubrication force saturation which is encountered in the acquired data. The parameters of the new friction model are identified and compared with the standard ones.**

I. INTRODUCTION

Friction, which occurs at the interface between two solids in contact, often influences the performance of mechanical systems. Mechanical positioning systems suffer from friction effects especially when precisions in the nanometer scale are required. Friction causes static error, limit cycle and many other performance degradations when a trajectory has to be followed by a positioning system. Control systems can compensate for the friction effects if an accurate friction model is available. The problem of friction modeling has already been studied by several research groups and different models have been proposed in the literature. A good survey of these models and the methods to compensate it can be found in [1].

The well-known LuGre friction model, proposed by C. Canudas de Wit *et al* in [2], is one of the most complete friction models used in automatic control. This model is based on the introduction of an internal state variable z to describe the deflection of bristles. The lubrication force is assumed proportional to the velocity according to Newton's law. However, it turns out that, depending on the lubricant used, the lubrication force can be a nonlinear function of the velocity. Experiments show that the assumption of Newton's law is, in fact, a coarse approximation of the reality.

This problem has already been encountered in diverse applications. To solve this problem, Nakanishi in [3] proposed a Coulomb-stiction model with a cubic viscosity force to fit to a brachiating robot system. Bona has encountered the same problem for a direct-drive manipulator in [4]. He proposed a modified LuGre friction model with a quadratic influence of viscosity to characterize the steady-state drive response. In theses two cases, the authors proposed new models for their particular problems which cannot be used for all lubricated systems. In a pseudo-plastic behavior, for example, where the friction force is saturated at high velocities, e. g. if it increases as the square root of the velocity, these models cannot characterize reality.

The force saturation problem has been studied in [5]. The proposed method leads to a Stribeck's curve with the Coulomb friction level above the adherence force threshold (also called stiction in literature). This proposition permits an inflection point on the Stribeck's curve which models a force saturation at large velocities. However, physically, this proposition seems to be unrealistic: adherence force level must be above the Coulomb force level.

In this paper, the friction modeling of a linear permanent magnet synchronous motor is investigated. A new model for the lubricated friction is proposed and introduced to the LuGre model. The new model can be used for systems with a pseudo-plastic behavior (force saturation with velocity) as well as where friction force increases faster than linearly with velocity. The parameters of the modified LuGre friction model for the positioning system is identified by several real-time experiments. A solution to the problem of friction force measurement in the presence of ripple force is proposed. The proposed model is compared with the classical friction models.

The paper is organized as follows. The modified LuGre model is described in Section II. The experimental setup, experiments and results are presented in Section III. Finally, section IV concludes the paper.

II. FRICTION MODELING

Although friction modeling has been largely investigated in tribology and several friction models have been developed, simple models which combine the basic dry friction model of Coulomb with the viscous lubrication model of Newton and Reynolds are used by the control community. In the viscous model, it is assumed that the viscous force is proportional to the velocity which is not a realistic assumption. Depending on the contact geometry, the surface rugosity and the lubricant type, the lubrification force becomes a nonlinear function of the velocity. In a polymeric lubricant, for example, the viscous force is a square root function of velocity. In others cases, a cubic root function can be observed. In view of these observations, a better model of the

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viscous lubrication force has been recently introduced in the basic tribological model [6], [7]. In this model, the viscous lubrication force is a power law of the sliding velocity as follows:

$$F_l = \sigma_2^{\sqrt{n}} |\dot{x}|^{n-1} \dot{x} \tag{1}$$

where F_l is the lubrication force, σ_2 a viscous coefficient, $\dot{x}$ the relative contact surface velocity and n an appropriate positive exponent which depends on the surface and lubricant type. The common linear viscous contribution in friction modeling, $\sigma_2 \dot{x}$, can be recovered by the new model with $n = 1$. The pseudo-plastic behavior or lubrication force saturation can be modeled by $0 < n < 1$ and $n > 1$ represents a lubrication force which increases faster than linearly.

Although $\sigma_2^{\sqrt{n}}$ still produces a constant, the coefficient $\sqrt{n}$ is added for normalization purposes. It should also be noted that, when $\dot{x} = 0$ and $n < 1$, the model can have numerical problems. To avoid these problems, the model can be implemented as follows:

$$F_l = \sigma_2^{\sqrt{n}} |\dot{x}|^{n} \operatorname{sgn}(\dot{x}) \tag{2}$$

which is strictly equal to (1).

This power model can be added on any friction model. Herein it is applied to the LuGre model [2] which seems to be the most complete model currently used in automatic control. The modified LuGre model is described by the following equations:

$$\begin{cases} F_f = \sigma_0 z + \sigma_1 \dot{z} + \sigma_2^{\sqrt{n}} |\dot{x}|^{n-1} \dot{x} \\ \dot{z} = \dot{x} - \frac{\sigma_0}{g(\dot{x})} z |\dot{x}| \\ g(\dot{x}) = F_c + (F_s - F_c) e^{-(\dot{x}/v_s)^2} \end{cases} \tag{3}$$

where z is the deflection of the bristles, σ_0 and σ_1 two dynamic parameters used to describe the stiffness and damping coefficients of the bristles. Dry friction parameters (also called static parameters in literature) are represented by the Coulomb friction threshold F_c, the adherence force level F_s and the Stribeck velocity v_s. Finally, F_f is the resulting friction force of this model.

III. Application to a linear synchronous motor

The modified LuGre model is used to model the friction in a linear, permanent magnet, synchronous motor (LPMSM). Such a system is represented in Fig. 1, where the experiments are carried on the upper axis. The friction junctions are composed of linear motion rolling guides with cylindrical roller bearings. LPMSMs have high acceleration and deceleration capabilities, high mechanical stiffness and no mechanical transmission components. They, therefore, do not suffer from backlash and thus allow very high positioning accuracy to be achieved. In this perspective, it is important to use a realistic and accurate friction model in order to compensate for the friction force accurately.

One of the main problems for identifying the parameters of the friction model for a permanent magnet motor is the presence of the ripple force. Ripple force is the combination of two forces: the cogging force and the reluctance force. The cogging force is due to the magnetic field discontinuities caused by translation from a magnet to another so it is a position dependent force. The reluctance force is due to magnetic fluctuations caused by the translator displacement and also by the current variation into the coil and therfore it is a position and current dependent force. Hence, as the velocity depends on the current, the ripple force is a position and velocity dependant force. Thus, Newton's equation of dynamics for the upper axis of the system becomes:

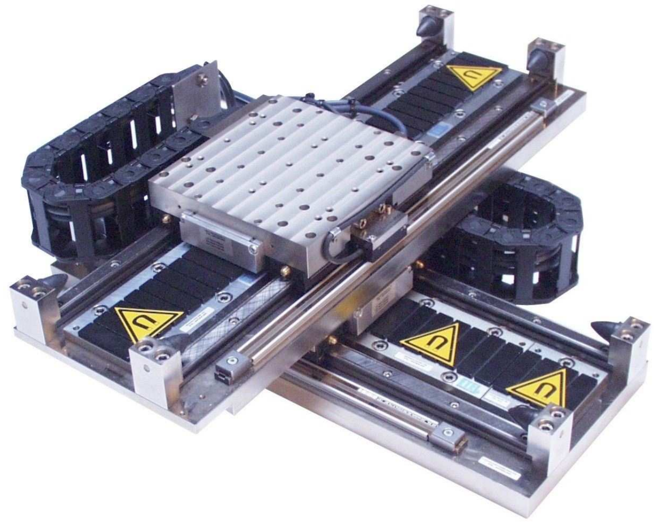

Fig. 1. Double-axis linear permanent magnet synchronous motor (with courtesy of ETEL).

$$m\ddot{x} = F_p - F_r - F_f \tag{4}$$

where m is the moving mass, $\ddot{x}$ its acceleration, F_p the pushing force delivered by the current, F_r the ripple force and F_f the friction force.

In order to identify the friction force parameters, the ripple force needs to be compensated for. As the experiment used to identify the dry friction parameters is run at constant velocities [8], the ripple force must be identified for each particular constant velocity. The experiment to identify the ripple force measures the force F_p needed during a motor stroke of 256 mm, and for several constant velocities. As $\ddot{x}$, the acceleration, is zero at constant velocity, (4) becomes :

$$F_p = F_r + F_f. \tag{5}$$

As a constant velocity induces a constant friction force, it is possible to remove its mean value $\overline{F_f}$ from the pushing force F_p. Thus, an approximation of the ripple force for a particular constant velocity is :

$$\hat{F}_r = F_p - \overline{F_f} \tag{6}$$

This experiment is run three times for each particular constant velocity in order to obtain an average value. The assumption of constant velocity is verified, because the maximal standard deviation of the velocity is 2%.

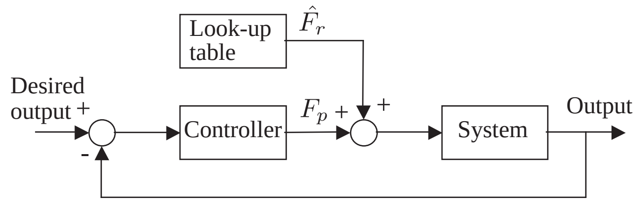

Fig. 2. Ripple force compensation schema.

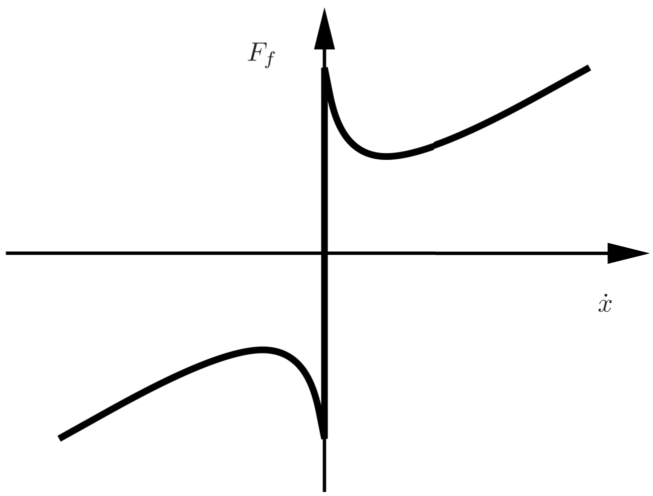

Fig. 3. Static standard characteristics (Stribeck curve).

To compensate for the ripple force, the pre-identified ripple force for each position and velocity of the motion is saved in a look-up table and is directly injected to the control system in a feedforward manner. Fig. 2 shows the ripple force compensation schema, where $\hat{F}_r$, the identified ripple force, is added to F_p, the pushing force.

This operation allows one to get rid of the parasitic ripple force and to get an accurate measurement of the friction force. This way, identification of the dry friction parameters of the LuGre model is possible. This identification is based on steady-state measurements. According to Newton's standard equation, for compensated steady-state velocities, where acceleration is zero, (4) becomes:

$$0 = F_p - F_f \tag{7}$$

The friction force is thus equal to the pushing force and can be measured. Six experiments are run for each particular velocity on a stroke of 256 mm. These experiments are done in a random order to get rid of the effect of the temperature that modifies the characteristics of the motor between the first and the last experiment.

In case of a viscous term proportional to the velocity, the expected response for this experiment would be a Stribeck curve as shown in Fig. 3. The friction force mean value, measured for different velocities, is plotted in a force-velocity graph. The acquired results are shown in Fig. 4. Each cross represents a mean force value measurement along all the displacement for a constant velocity. Confronted to this result, the Stribeck tribological law seems to be inadapted. This is the main motivation for the use of a new model.

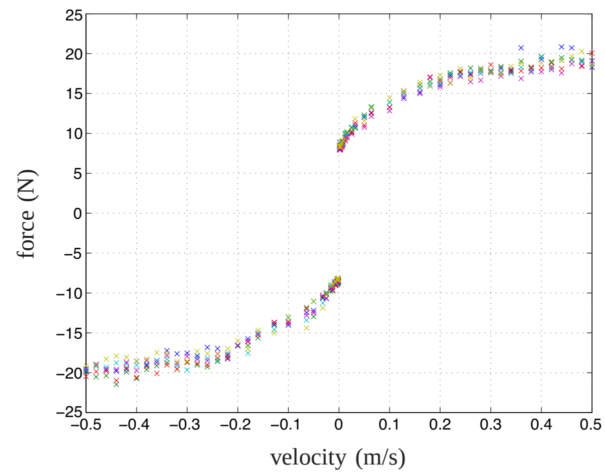

Fig. 4. Steady-state characteristics of the friction force (measured data).

The standard Stribeck tribological law is:

$$F_f = F_c + (F_s - F_c)e^{-(\dot{x}/v_s)^2} + \sigma_2\dot{x} \tag{8}$$

The proposed modification for the viscous lubrication part changes the standard Stribeck tribological law (8) into:

$$F_f = F_c + (F_s - F_c)e^{-(\dot{x}/v_s)^2} + \sigma_2^{\sqrt{n}}|\dot{x}|^{n-1}\dot{x} \tag{9}$$

Dry friction parameters can be identified from the proposed experiments and this equation.

The `fmincon` function available in MATLAB™ allows the optimization of a nonlinear function with constraints to be performed. This function is used to fit the experimental data with the proposed model. The sum of square errors between the measured friction force and the computed one is minimized under the constraints that all parameters have to be positive and that the adherence force peak, F_s, has to be above the Coulomb force threshold, F_c.

To initialize the optimization algorithm, a common method is to pre-identify the Coulomb force and the viscous part of the model with data acquired at a velocity greater than the Stribeck velocity. This way, the Stribeck velocity and the adherence force can be neglected and then the Stribeck tribological law equation reduces to :

$$F_f = F_c + \sigma_2^{\sqrt{n}}|\dot{x}|^{n-1}\dot{x} \tag{10}$$

The results obtained allow a closer initialisation of the parameters to their final values to be obtained which consequently reduces the probability to obtain a local minimum when using the function with all velocities.

Table I shows the dry friction parameters obtained for the whole range of velocities using the `fmincon` function.

Fig. 5 shows the friction force measurements and the force computed by the identified parameters. The points are the mean values of the measured data with the error-bars being the standard deviations, and the curve presents the force

TABLE I

IDENTIFIED DRY FRICTION PARAMETERS.

F_c [N]:	5.5993
σ_2 [N s/m]:	139.6884
n:	0.3494
F_s [N]:	6.8554
v_s [m/s]:	0.0025

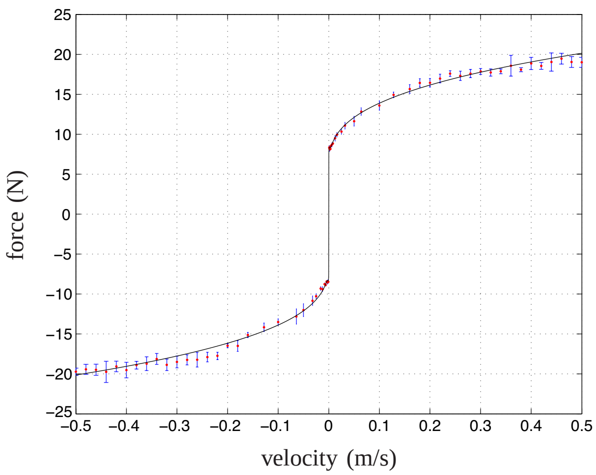

Fig. 5. The mean values and standard deviations of the measurements together with the computed friction force by the proposed model.

computed from the new static model. It can be observed that the physical phenomenon is realistically represented by this modification and the model accurately characterizes the influence of viscous lubrication.

Fig. 6 is a zoom on the Stribeck region. It shows that the modified LuGre model is very well identified and fits to the reality at low velocities as well as at high velocities.

In order to show the superiority of the proposed method with respect to other conventional approaches, a comparison with the standard LuGre model and Bona's modified model is carried out. The results are depicted in Fig. 7. The points and the error-bars represent the collected data, the dashed-dotted line the standard LuGre model, the dashed line the Bona's modified model (using a quadratic viscosity) and the solid line the proposed model. It is clear that even at low velocities the linear and quadratic functions do not give a good approximation of the measured data. The error will be much larger at high velocities.

IV. CONCLUSION

Control engineers often use a very simple model of friction for controller design. The most complete friction model used in automatic control considers a linear relation between the lubrication force and the velocity. It is shown in this paper that this relation is nonlinear and cannot be precisely modeled by the conventional models. Based on tribological observations, a power viscous model which characterizes the nonlinear behavior of the lubrication force is proposed and introduced to the LuGre model. The new viscous model can be applied to other friction models.

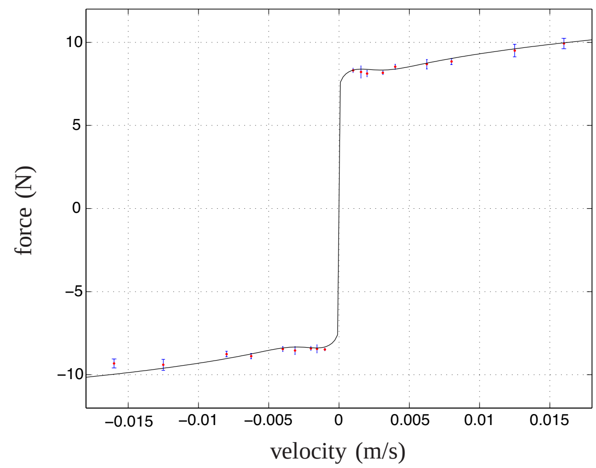

Fig. 6. Measured friction force and computed ones at low velocities: zoom on Stribeck region.

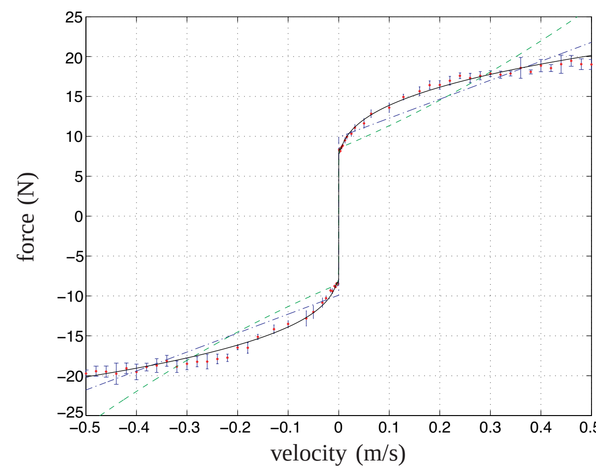

Fig. 7. Comparison between steady-state measurements and models characteristics (dashed-dotted: standard LuGre model, dashed: Bona's modified model, solid: proposed model).

The proposed method is applied to a linear, permanent magnet, synchronous motor for which the lubrication force is saturated at high velocities. The friction force is measured with some specific experiments in which the parasitic ripple force is eliminated. The final results show that the new viscous model gives much better fitting to the experimental data than the other methods.

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REFERENCES

[1] B. Armstrong-Hélouvry, P. Dupont, and C. Canudas de Wit, "A survey of models, analysis tools and compensation methods for the control of machines with friction," *Automatica*, vol. 30, no. 7, pp. 1083–1138, July 1994.

[2] C. Canudas de Wit, H. Olsson, K. J. Åström, and P. Lischinsky, "A new model for control of systems with friction," *IEEE Transactions on Automatic Control*, vol. 40, no. 3, pp. 419–425, March 1995.

[3] J. Nakanishi, T. Fukuda, and D. Koditschek, "A brachiating robot controller," *IEEE Transactions on Robotics and Automation*, vol. 16, no. 22, pp. 109–123, April 2000.

[4] B. Bona, M. Indri, and N. Smaldone, "Nonlinear friction estimation for digital control of direct-drive manipulators," European Control Conference, September 2003.

[5] P. Myszkorowski, F. Altpeter, and R. Longchamp, "Position control of drives with friction," IEEE Conference on Decision and Control, December 1999, pp. 5263–5267.

[6] A. Curnier, "Discrete contact mechanics," EPFL Press, CH-1015 Lausanne, Lecture notes, 2001.

[7] ——, *Mécaniques des Solides Déformables*. Presses Polytechniques et Universitaires Romandes, Oxford, 2005, vol. I Cinématique, dynamique, énergétique, ch. IV.

[8] F. Altpeter, "Friction modeling, identification and compensation," Ph.D. dissertation, N^o 1988 , École Polytechnique Fédérale de Lausanne-Switzerland, CH-1015 Lausanne, 1999. [Online]. Available: http://library.epfl.ch/theses/?nr=1988