

# Analytical versus end-to-end numerical modeling of adaptive optics systems: comparison between the code PAOLA and the Software Package CAOS

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**Abstract.** We compare in this paper the analytical approach and the Monte-Carlo (MC) end-to-end approach in the context of astronomical adaptive optics modeling. The two tools used for this purpose are the analytical code PAOLA and the MC Software Package CAOS. This is done to inter-validate the two codes, but also to help finding trade-offs between the MC approach, which can be very time-consuming but is expected to give more certainty on the obtained results, and the analytic approach, which is straightforward but based on a number of simplifying assumptions. We first test the fundamental fitting and anisoplanatic errors (the latter being equivalent in our test to the servo-lag error), and find a very satisfactory agreement. We make then a first attempt of a comparison including all error terms by simulating a complete 8-m telescope AO system, varying the wavefront sensing noise. Differences are found and thought to come essentially from an unmatched definition in the two codes of the deformable mirror modes.

## 1 Introduction

We compare in this paper the analytical approach and the Monte-Carlo (MC) end-to-end approach in the framework of numerical modeling of astronomical adaptive optics (AO) systems. The two tools used for this purpose are well-known and widely used within the astronomical AO community: PAOLA (Performance of Adaptive Optics for Large (or Little) Apertures [1,2]) on one hand, and the Software Package CAOS [3] on the other hand. This comparison is done to inter-validate the two codes, but also in order to help finding trade-offs permitting exploratory researches or large instrumental project performance evaluations while combining as far as possible the computing efficiency of the analytical approach and the robustness of the end-to-end MC approach.

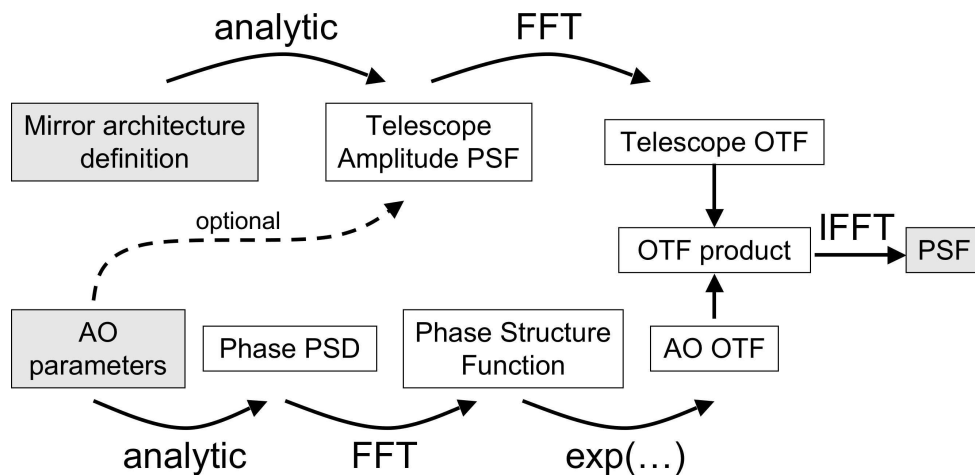
We briefly recall the main characteristics of both the modeling tools in Section 2. Then, as preliminaries to the full comparison, we test independently the modeling of the two most fundamental errors for an AO system: the fitting error and the anisoplanatic error (the latter being equivalent in our test to the servo-lag error), in Section 3. We find a very satisfactory agreement in both cases, and we discuss the choice of the DM basis which can have a strong impact on the PSF structure modeling, and hence has to be considered when using together the analytical approach and the MC approach for the study of a given system. We then make a first attempt of a full comparison (Section 4) by simulating a complete 8-m telescope AO system, increasing the wavefront sensor (WFS) noise error by increasing the guide star magnitude. Results and plans for foreseen tests are discussed in Section 5.

Some level of differences are expected between the AO performance prediction resulting from the analytical approach and the MC approach. Exploring these differences, and understanding where these are coming from, is the very objective of the on-going study reported in this paper.

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**Fig. 1.** The analytical code PAOLA flow chart. Top line, left to right: mirror architecture definition (input), computation of the amplitude PSF, computation of the telescope OTF. Bottom line, left to right: definition (input) of the AO parameters, then calculation of the residual phase PSD, the residual phase structure function, and finally the long-exposure AO OTF. Middle line: the telescope OTF is filtered by the AO OTF in order to obtain the global OTF, from which the final PSF is deduced.

## 2 Adaptive optics modeling codes used in this study

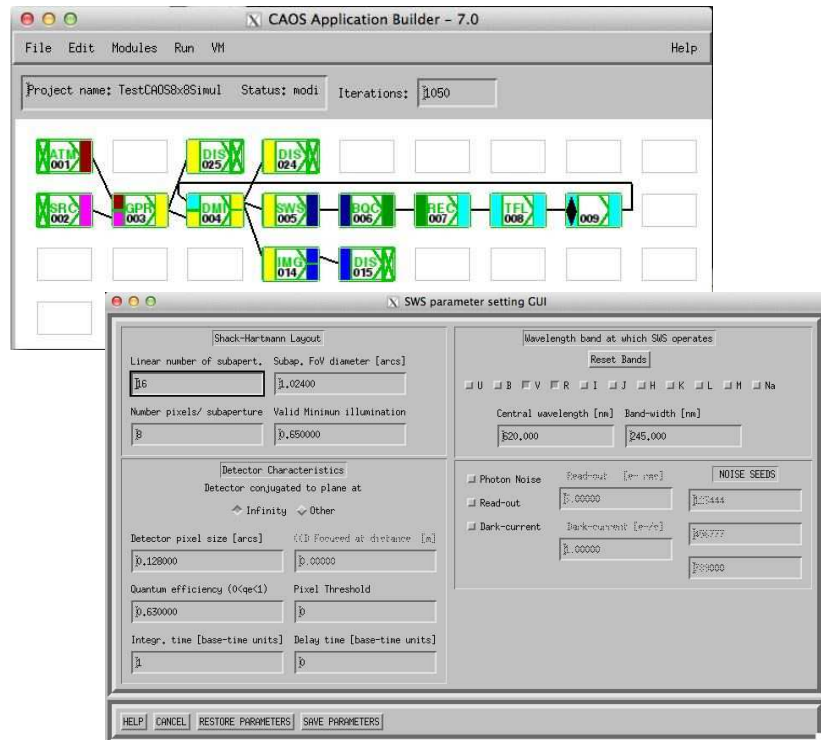
### 2.1 The synthetic code PAOLA

The code PAOLA (Performance of Adaptive Optics for Large (or Little) Apertures) [1,2] is a toolbox written in the IDL language for modeling the performances of an astronomical AO system<sup>1</sup>. Unlike most AO simulation packages, PAOLA is not a MC-based one. Instead of coding the individual behavior of each components of the AO loop, and then linking each boxes, as it is done in an MC code as CAOS (detailed in next subsection), a *synthetic* approach is adopted. Here the general behavior of the whole system at once is modeled. The core of the synthetic method is based on an analytic expression for the residual phase average (or long exposure) spatial power spectrum (or power spectral density, PSD), and its relationship with the long-exposure AO optical transfer function (OTF).

The PAOLA flow-chart is shown in Figure 1, where we see that, first of all, the mirror architecture is defined, then the amplitude PSF is computed, and the telescope OTF is deduced. In parallel, the AO parameters are defined, then the residual phase PSD is computed, hence the residual phase structure function is deduced, and eventually the long-exposure AO OTF. The telescope and AO OTF product is then computed and the final PSF obtained by an inverse FFT.

The main advantage over the MC approach is the gain in computation time, which can typically be of the order of  $10^3$  to  $10^4$ : a long exposure PSF can be calculated in a couple of seconds, instead of hours, or days, permitting a thorough exploration of the AO system parameter space. This has some costs: simplifying assumptions are made, and the main one is that the AO system is seen as a spatial filter applied on the turbulent phase — which would be exact if the AO correction was stationary within the pupil but we know that it is not 100% true (although close — see Keck AO system measurements in [4], this conference). Besides, not all sources of second order AO errors can be modeled via such a spatial filter approach, in particular non stationary pupil errors like WFS to DM mis-registrations. These are the reasons why we expect some level of differences between the AO performance prediction resulting from the two approaches, which makes this study necessary.

<sup>1</sup> PAOLA can be obtained from a direct request to [laurent.jolissaint@aquilaoptics.com](mailto:laurent.jolissaint@aquilaoptics.com)



**Fig. 2.** The CAOS Application Builder (background) and the GUI of module SWS. While the CAOS Application Builder shows the data flow built for the typical simulation of a natural-guide-star-based AO system, the GUI of module SWS shows the physical parameters chosen for the  $16 \times 16$ -subapertures SH sensor used throughout the simulations presented in this paper.

## 2.2 The Software Package CAOS

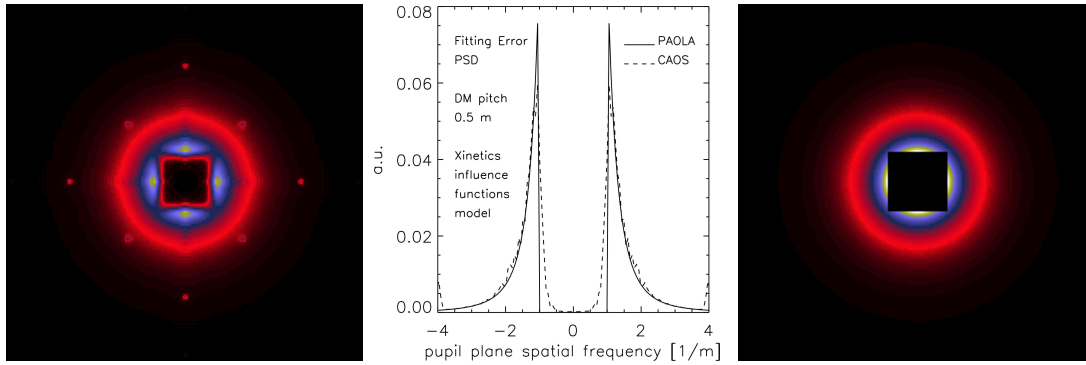
The Software Package CAOS [3] permits end-to-end MC numerical modeling of AO systems — CAOS stands for “Code for Adaptive Optics Systems”... It is also written in the IDL language and developed within the (homonymic) CAOS problem-solving environment (PSE, or “system”)[5,6], which allows to clearly separate in its own bosom the scientific part of the original Software Package from the global interface (the so-called CAOS Application Builder) and global structure of the tool (permitting also by the way to complete the whole suite with a number of other Software Packages).

For short, the Software Package CAOS is a software ensemble of modules designed for end-to-end simulations of generic astronomical AO systems, including a complete atmosphere turbulence modeling, sodium laser-guide star upward and downward propagation, observed object definition, Shack-Hartmann (SH) and pyramid WFSs detailed modeling, wavefront reconstruction and subsequent time-filtering tools, and wavefront correction via different kind of correctors; but also image formation, Fizeau interferometry, coronagraphy, etc. It is clearly a tool dedicated to optical astronomy detailed studies, being based on a wide range of somehow low-level physical modeling.

The background of Fig. 2 shows the example of the simulation of a  $16 \times 16$  SH-based AO system for an 8-m class telescope aiming at correcting the turbulent atmosphere at near-infrared wavelengths, while its foreground shows, as an example again, the graphical user interface (GUI) of module SWS which models the SH WFS (SWS standing for Shack-Hartmann Wavefront Sensor), showing the physical parameters chosen for it.

The Software Package CAOS is freely distributed from <http://lagrange.oca.eu/caos>, and subscription to its mailing-list is recommended in order to receive upgrades and new versions of either the Software Package CAOS or any other relevant part of the CAOS PSE.

## AO for ELT II



**Fig. 3.** Fitting error PSD. Left: CAOS PSD bidimensional representation. Right: PAOLA PSD bidimensional representation. Middle: comparative plot of both.

### 3 Preliminary comparisons

#### 3.1 Fitting Error Modeling

Within the CAOS-based model, in this fitting error (FE) analysis, the residual wavefront is simply modeled as the difference between the incoming turbulent atmosphere wavefront and its projection onto a deformable mirror (DM) influence function (IF) basis. A statistically-averaged PSF is then deduced by running a large number of independent realizations of the turbulent atmosphere. Instead, the PAOLA model considers the Kolmogorov phase spatial PSD, set to zero the AO-corrected spatial frequency domain ( $|f_x| < 1/(2\Lambda)$  and  $|f_y| < 1/(2\Lambda)$ , where  $\Lambda$  is the lenslet and DM pitch, and  $f_x$  and  $f_y$  are the components of the pupil plane spatial frequency along the  $x$  and  $y$  axes), for computing the FE structure function, from which the AO OTF is deduced, and so on up to the overall PSF.

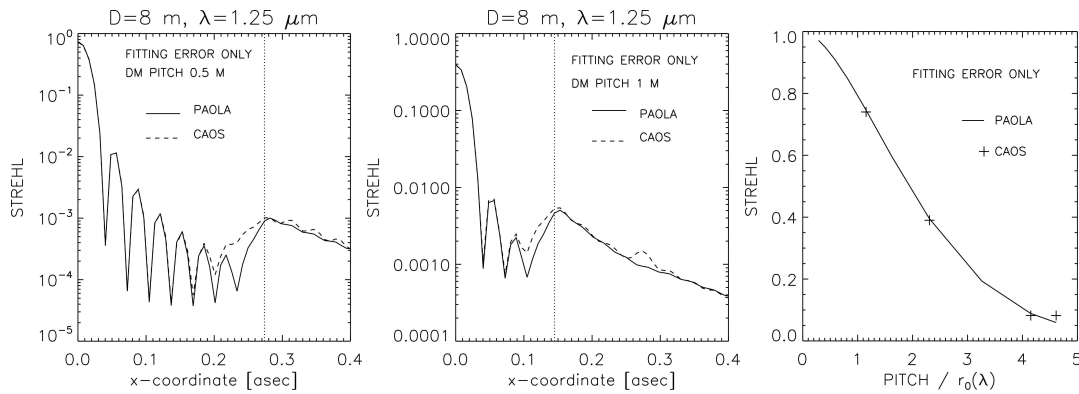
The turbulent atmosphere we consider is characterized by a Fried parameter  $r_0=14.4$  cm at 500 nm, and a wavefront outer scale  $L_0=25$  m, with 1000 independent  $128\times 128$  phase screens (with the addition of sub-harmonics) within the CAOS-based simulation. The DM pitches considered within PAOLA (0.5 m, 1 m, 1.8 m, and 2 m) correspond to sets of, respectively, 289, 81, 25, and again 25 IF within the CAOS model. Note that the IF basis considered for the present tests is a set of Xinetics-like IF<sup>2</sup> taken all over the square of side  $D$  containing the telescope pupil of diameter  $D$  (with  $D=8$  m here). One could already note that the approach we have adopted here has its limits:

1. because the CAOS-based simulation assumes here a perfect WFS (i.e. the measured phase *is* the phase, there is no spatial sampling), aberrations above the AO cutoff frequency can be somewhat affected by the DM correction, due to the IF structure and actuators geometry, while in a real system these high-order frequencies would remain as they are;
2. PAOLA assumes a perfect DM, fully correcting the phase within the AO cutoff frequency, which would need the IF to be *sinc*-like (the Fourier transform of a *sinc* being a door function), but of course the realistic IF we are considering here are not *sinc* functions.

These effects are clearly visible in the FE PSD shown in Fig. 3: the CAOS PSD (computed from the average PSD over each instantaneous residual phase) has features above the cutoff frequency, and shows a smooth transition to zero. Reversely, the PAOLA PSD shows a perfect Kolmogorov PSD above the cutoff frequency, and a perfect one-to-zero transition. Due to the structural relationship between the PSD and the PSF [1], these differences show up in the PSF too (see Fig.4), although the overall Strehl ratio is not really affected.

In order to make the PAOLA model closer to the CAOS result, we would need to implement into PAOLA a DM spatial transfer function model, a particularly interesting feature when the structure of the PSF within the cutoff frequency domain needs to be precisely known, as for instance when studying the performance of extreme AO with a possible coronagraph. Moreover, and in order to reproduce

<sup>2</sup> from a very realistic IF model based on laboratory measurements



**Fig. 4.** Fitting error PSF. Left: the case of a pitch of 0.5 m. Middle: the case of a pitch of 1 m. Right: comparative plot of the Strehl ratios obtained.

exactly the low–high spatial frequency transition, we would also need to better sample with respect to what is done by default: the sampling of the FE PSD appears to be too coarse and makes the PSF wings low–high frequency transition at a slightly different off-axis value than what is expected.

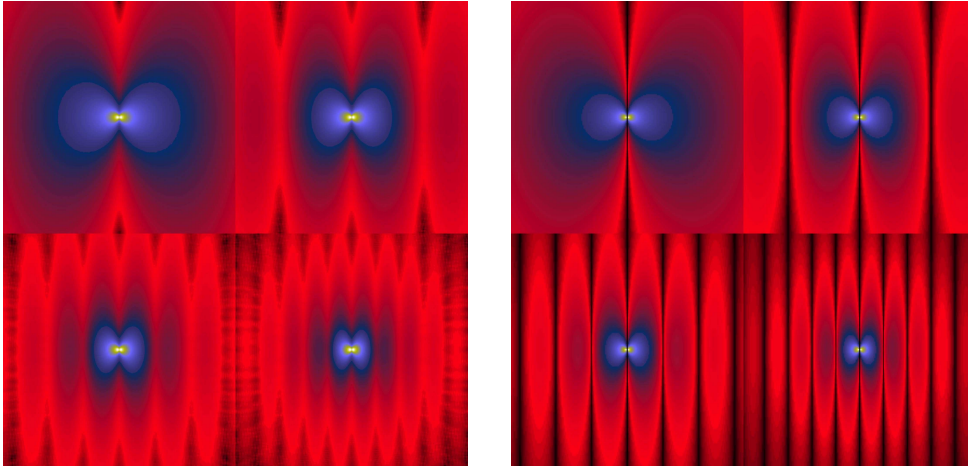
It is worth noting that we ran a comparison using *sinc* influence functions within CAOS, and found an excellent match between the PSF of the two models, particularly in the transition region. Besides, we also ran preliminary tests using Zernike modes — within CAOS — instead of influence functions, and in this case the CAOS PSD looked, as expected, totally different (circularly symmetric) from PAOLA’s PSD. In other words, the choice of the DM basis has a strong impact on the PSF structure modeling, and this has to be considered when using together the analytical approach and the MC approach for the study of a given system.

### 3.2 Anisoplanatic Error/Servo-Lag Error

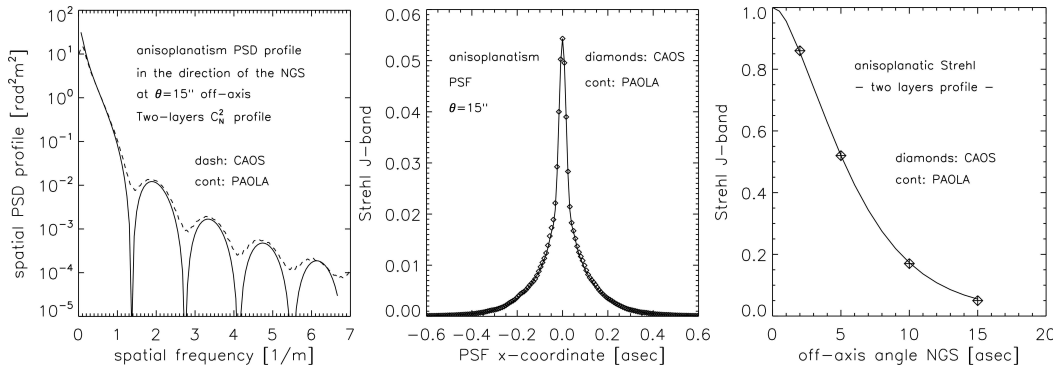
The second most frequent sources of AO error are angular anisoplanatism and servo-lag errors. These are somewhat correlated: a lateral shift of the turbulent layers during one loop period (responsible for the servo-lag error) is equivalent to an angular shift of the phase when looking in two different directions. In the spatial frequency domain, the computation of the servo-lag error PSD uses the same principles than the computation of the anisoplanatic error (except that we also have the averaging of the phase during the WFS exposure) and as a consequence the structure of the servo-lag and anisoplanatic PSD looks the same. We test here the simplest mode — the angular anisoplanatism — for a two-layers atmosphere (60% of the turbulence energy affected to an  $h=0$  km altitude layer and 40% to  $h=10$  km).

Theory [2] indicates that the anisoplanatic error PSD modulation period (in the spatial frequency domain) is proportional to  $1/(h\theta)$ , where  $\theta$  is the off-axis angle, and this is well apparent in our two models, as shown in Fig. 5, where both the CAOS and PAOLA PSD are represented. Fig. 6, left, shows a log profile comparison of the PSD: the fact that the CAOS-based PSD does not drop as deep as the theoretical PSD is certainly a sign of lack of numerical convergence (1000 independent realizations only). In any case, the PSF profile as well as the decrease of the Strehl with the off-axis angle are in excellent agreement, as shown in Fig. 6.

It must be noted that it is expected that for off-axis angles such that the on-axis and off-axis beams are totally separated, PAOLA should predict better Strehl ratios than CAOS: indeed, since the analytical approach neglects the finite beam width (infinite aperture approximation), there will always be some (although low) level of correlation between the two beams, at low spatial frequency. This is not apparent here because the off-axis angle are not large enough: a 15'' off-axis angle corresponds to a lateral shift of 0.73 m at the 10-km altitude layer, while 165'' would be necessary to separate the 8-m beams.



**Fig. 5.** Anisoplanatism PSDs. Left: bidimensional representations of the CAOS PSD for, from left to right and from top to bottom:  $\theta=2''$ ,  $5''$ ,  $10''$ , and  $15''$ . Right: idem for the PAOLA PSD.



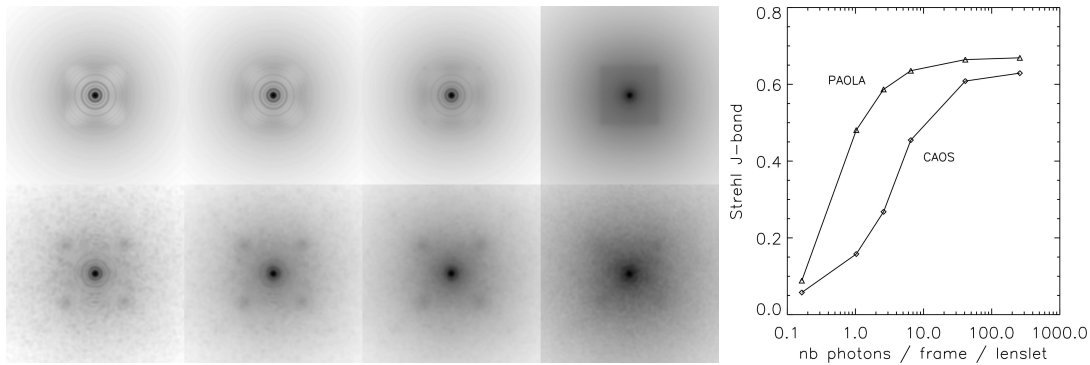
**Fig. 6.** Left: cut of the PSDs shown in Fig. 5. Middle: comparative cut of the PSFs for  $\theta=15''$ . Right: comparative plot of the obtained Strehl ratios for the various angles considered.

#### 4 (First Attempt Of) A Full Error Comparison, Featuring Wavefront Sensor Photon Noise

We are here entering the real exploration of expected differences between PAOLA and CAOS, each source of error being included: fitting error, WFS spatial aliasing, servo-lag error, and WFS noise (but angular anisoplanatism). AO system parameters adopted here are: wind velocities=8 m/s,  $16 \times 16$  subapertures SH WFS (with  $8 \times 8$  pixels of angular size  $0''.128$  per subaperture, sensing at 620 nm with a bandwidth of 245 nm, neither read-out noise nor dark-current noise considered), a 0.5 m-pitch DM (originally 289 IF for the CAOS model but filtered back to 206 modes after pseudo-inversion of the interaction matrix in order to eliminate modes which eigen-values were above a condition number of 10), and a global loop gain of 0.5.

Figure 7 shows the PSFs obtained and the resulting Strehl ratios. The results shown here are to be considered as they are: very preliminary. Several comments can be made, though:

1. impact of the waffle mode (the dots at the corners of the AO corrected domain) is missing in the PAOLA PSFs, which is not surprising as this effect is not modeled;
2. CAOS PSFs have not reached numerical convergence: we are generating 1000 iterations of 1 ms but only one realization of the turbulent atmosphere — we are here rather simulating time evolution than statistical averaging as in the two previous cases, hence the speckle noise is noticeable.



**Fig. 7.** The PSFs (top row from PAOLA, bottom row from CAOS with, for the rows and from left to right:  $m_V = 14, 15, 16,$  and  $18$ ), and plot of the predicted Strehl ratios.

We would probably need at least 10 times more independent realizations to have a minimum of statistical averaging in addition to time evolution;

3. one of the main unknown is of geometrical nature: the equivalence between the spatial frequency cutoff, defined by the DM pitch, and the number of modes actually corrected within the CAOS model has to be clearly established — the PAOLA model does not have any modal filtering (everything is perfectly corrected up to the cutoff frequency), while within CAOS a selection of the DM modes to correct is mandatorily performed, based on the modes' eigen values.
4. using a *sinc* influence function basis in CAOS would certainly improve the match with the PAOLA Strehl ratio, in the bright guide star case, where WFS noise is negligible (see Fig. 7, right);
5. both the number of modes to be corrected and the modal gain to be applied were absolutely not optimized within the CAOS model, while it should be done, in addition to the WFS time exposure, actually, in order to have a better evaluation of the attainable Strehl;
6. finally, it might be that the noise contribution is underestimated in PAOLA: indeed, the WFS noise PSD formula is proportional to  $f^{-2}$  where  $f$  is the modulus of the spatial frequency in the pupil plane, but the final spatial frequency sampling in the PSD matrix (i.e.  $\Delta f > 0$ ) makes that the noise PSD is necessarily underestimated near  $f = 0$ . The impact of this effect clearly needs to be explored further in the PAOLA model, and can well be the main explanation of the PAOLA–CAOS discrepancy in the moderate-to-low guide star intensity.

Implementing/considering the ideas/warnings discussed above should make the PAOLA prediction less optimistic, and the CAOS one less pessimistic, leading a priori to a better tuning between the two models. We can however remark that when the noise error dominates, as it is the case with  $m_V=18$  (corresponding to 0.2 photons/frame/lenslet), we have some convergence of the two models.

## 5 Concluding remarks

We realize that the effort of comparing the analytical and end-to-end approach has already allowed a better understanding of the importance of (1) the DM modes definition, and (2) the sensitivity to the noise modeling in the analytical model. These two aspects will be explored further in the near future. Generally, though, the convergence of the two approaches is promising, in particular for the anisoplanatism error.

It is worth noting finally that an effort of integrating the code PAOLA into the CAOS PSE is being carried out. The result of such an embedment is the Software Package PAOLAC (where “PAOLAC” stands for “PAOLA within CAOS”) for which a version 1.0 has already been released [7], but with the precedent open-loop version of PAOLA. A version integrating the recent close-loop feature of PAOLA is being built in parallel to this comparison study.

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