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Decentralized computation of charging controls for plug-in electric vehicles in the S-adapted information structure

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***Abstract*— We study the problem of charging an arbitrary number of plug-in electric vehicles (PEV) under a real-time electricity tariff that depends on the instantaneous grid load, with the addition of a stochastic process that affects the non-PEV demand. Each PEV is subjected to individual and coupling constraints. Formally, we are facing a Generalized Nash Equilibrium (GNE) seeking problem for stochastic aggregative games. The stochastic dynamics is modelized as an event tree and included according to the S-adapted information structure, which is suitable to describe stochastic processes that are independent of the players' control. The equilibrium is calculated by employing a decentralized scheme. We observe that the valley-filling behavior, which has been observed in previous studies concerning the PEV problem, can be significantly altered by the stochastic dynamics.**

I. INTRODUCTION

In the last years, plug-in electric vehicles (PEVs) have been subject to increasing market penetration, and are expected to achieve greater importance in the future [1]. A significative PEV share among the population raises new problems in the electrical grid management. Indeed, a new demand peak is expected to emerge during evening or by night, requiring additional electricity generation and grid capacity [2], [3]. The idea of modifying the users' behavior to obtain a shift in the PEV charging intervals, with the aim of reducing the grid load, plays a central role in the demand-side management for smart grids (see, for instance, [4], [5]). In the last decade, noncooperative game theory, that had previously mainly focused on supply-side management (see [6] for an example concerning electricity markets), has been employed in an increasing number of studies for regulating electricity demand. In particular, Ma, Callaway and Hiskens [7] proposed a game-theoretic, decentralized scheme based on a real-time electricity tariff depending on the instantaneous demand of electricity. Their approach converges to an unique Nash equilibrium in the large-population limit, and has been later extended to describe additional features, such as battery degradation [8] and grid overloads [9].

The PEV charging problem formulated by [7] falls within the class of aggregative games (see e.g. [10], [11]), according to which each player's cost function only depends on the other players' strategies through the sum of their respective controls. In the last years, decentralized and semi-decentralized methods to find Nash [12] or ϵ-Nash equilibria [13], [14], [15] in deterministic aggregative games with individual constraints have intensively been studied.

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Some attention has been further devoted to the study of Generalized Nash Equilibria (GNE), where players are also subjected to coupling constraints. The concept of GNE was firstly introduced in [16] and developed, to cite some of the main contributions, by [17], [18], [19]. Decentralized or semi-decentralized methods have been proposed to reach GNE equilibria for deterministic aggregative games [20], [21], [22] or aggregative equilibria, that correspond to GNEs in the large-population limit [23].

Among the above references, some variants of the PEV problem were discussed in [14], [15], [20], [23]. In this paper, we focus on the PEV problem as it is formulated in [7], but consider in addition a coupling constraint that imposes an upper limit to the aggregate PEV demand [20], [23]. Furthermore, we do not require the PEV population to be large, and are therefore concerned with a GNE problem similar to [20], with the difference that the price function we employ is convex but not necessarily linear. As a main contribution of this paper, we add a discrete-state stochastic dynamics that influences the non-PEV demand curve. To this purpose, we formulate the game according to the S-adapted information structure, a formalism introduced by Haurie, Zaccour and Smeers [24], and successively studied, to cite a few examples, in [25], [26], [27], [28], [29]. It is intended for games subject to an exogenous stochastic process that does not depend on the player's actions. During the game evolution, players are not allowed to observe the other players' actual controls, but can nevertheless inspect, at each time step, the realization of the exogenous stochastic process, and accordingly react by choosing a suitable control. We model the stochastic process by a decision tree and show that the extended game reformulation proposed by [20] can be generalized to our situation. We calculate a solution of the extended GNE problem by employing an extragradient algorithm [30] that solves the corresponding variational inequality.

We remark that some studies already exist concerning stochastic Nash equilibria seeking in aggregative games. Stochastic ϵ-Nash equilibria for unconstrained games, that converge towards Nash equilibria in the large population limit, are treated in [31], while different methods to calculate stochastic Nash equilibria are discussed in [32], [33]. Furthermore, [34] provides a distributed scheme to compute GNE in stochastic games, where they employ a penalty-based scheme to enforce the constraints to be satisfied. These references differ from our case in that they sample data to make inferences on the random distribution. In our model, by contrast, players are assumed to know in advance the properties of the underlying stochastic process. Moreover, as an important difference with respect to our paper, the above references are not concerned with the modelization of explicit time dynamics and nonanticipative behavior. Finally, we notice that our model presents some similarities with [35], which employ a model predictive control scheme to account for sudden deviations of the non-PEV demand curve. In this case, however, uncertainty is not modelled as an event tree: rather, the optimal control is recalculated at each period on the basis of new observation, by assuming a deterministic future evolution of the non-PEV demand curve.

This paper is organized as follows: in Section II, we introduce the basic model for the deterministic case, and review the main concepts concerning GNEs and variational inequalities. In Section III, we discuss the S-adapted information structure, add the stochastic dynamics, and formulate the problem as an extended variational inequality for $N+1$ players to allow the equilibrium to be calculated in a decentralized way. Finally, Section IV applies the developed formalism to a simple numerical example.

Notation: We use boldface to denote vectors, and calligraphic style to denote sets. To collect scalars or vectors into a larger vector we use a notation of the type $\boldsymbol{u} := (\boldsymbol{u}_n;\ n \in \{1,...,N\}) := (\boldsymbol{u}_1,...,\boldsymbol{u}_n)$, where $\boldsymbol{u} \in \mathbb{R}^{NT}$ and $\boldsymbol{u}_n \in \mathbb{R}^T$. For a scalar function $J(\boldsymbol{x},\boldsymbol{y})$, where $\boldsymbol{x} \in \mathbb{R}^T$, the gradient $\nabla_{\boldsymbol{x}} J(\boldsymbol{x},\boldsymbol{y})$ is defined as the vector $(\partial_{x_t} J(\boldsymbol{x},\boldsymbol{y});\ t \in \{1,...,T\}) \in \mathbb{R}^T$. The Euclidean scalar product is denoted by $\langle \cdot, \cdot \rangle$.

II. THE DETERMINISTIC PEV PROBLEM

A. Basic model

In this section, the PEV problem is formulated by closely following the formalism employed in [7]. We consider a population of N PEV's that must achieve a full battery charge before the end of a given time horizon. The whole time interval is divided into a set $\mathcal{T} := \{1,...,T\}$ of periods. During each period t, the n-th PEV applies a constant control

$$u_{n,t} \geq 0, \tag{1}$$

which describes the power according to which the battery is charged. The state-of-charge $x_{n,t}$ of PEV n at period t is a real number ranging between 0 (out of charge) and 1 (full charge). Starting from an initial state $x_{n,1} < 1$, its dynamics is governed by the equation $x_{n,t+1} = x_{n,t} + (\alpha_n/\beta_n)u_{n,t},\ t \in \mathcal{T}$, where $0 < \alpha_n \leq 1$ and $\beta_n > 0$ denote the charging efficiency and the battery size, respectively. In order to achieve a full charge, the controls must be chosen so that, at the end of the charging horizon,

$$x_{n,T+1} = 1, \quad \forall\, n \in \mathcal{N}. \tag{2}$$

We define a *strategy* for PEV n as the vector $\boldsymbol{u}_n := (u_{n,t}; t \in \mathcal{T}) \in \mathbb{R}^T$, and accordingly introduce the set $\mathcal{U}_n := \{\boldsymbol{u}_n \in \mathbb{R}^T : u_{n,t} \geq 0\ \forall\, t \in \mathcal{T},\ x_{n,T+1} = 1\}$ of all strategies that fulfill the local constraints (1) and (2). Furthermore, we consider the aggregate quantity $\bar{u}_t := N^{-1} \sum_{n=1}^{N} u_{n,t}$ describing the average control during each period. Unlike in [7], we require that the coupling constraint

$$\bar{u}_t \leq C_t \quad \forall\, t \in \mathcal{T}, \quad \text{where } C_t > 0, \tag{3}$$

must hold (see e.g. [20], [23]). This constraint imposes an upper bound to the power that can be delivered to the grid for charging PEVs. Denoting by $\boldsymbol{u} := (\boldsymbol{u}_n; n \in \mathcal{N}) \in \mathbb{R}^{NT}$ the collection of strategies for the entire PEV population, we finally define the set $\mathcal{U} := \{\boldsymbol{u} \in \mathbb{R}^{NT} : \bar{u}_t \leq C_t\ \forall\, t \in \mathcal{T},\ \ \boldsymbol{u}_n \in \mathcal{U}_n\ \forall\, n \in \mathcal{N}\}$ of feasible collections of strategies, to which we will simply refer as the set of feasible controls.

Proposition 1: For C_t large enough, the set $\mathcal{U}$ of feasible controls is nonempty, compact and convex. For $T > 1$, it further satisfies Slater's constraint qualification, i.e., its relative interior contains points for which $u_{n,t} > 0$ and $\bar{u}_t < C_t$.

The PEV problem can be formulated as a game by associating each PEV to a player, and by considering appropriate cost functions that players wish to minimize. The approach in [7] is that of employing a real-time electricity tariff depending on the total instantaneous electricity demand, and to introduce functions representing the cost for charging each PEV. We consider cost functions of the form

$$J_n(\boldsymbol{u}) := \sum_{t \in \mathcal{T}} p\left(\frac{1}{c}(d_t + \bar{u}_t)\right) u_{n,t}, \tag{4}$$

(see [7], [20]) where the functional $p(\cdot)$ describes the real-time electricity tariff, and takes as an argument the ratio between total average demand $d_t + \bar{u}_t$, where $d_t \geq 0$ denotes the average non-PEV demand per single player at period t, and average generation capacity $c > 0$ per single player. The above choice of the cost function is made for simplicity, although, in practice, the real-time pricing does not depend on the instantaneous demand, but is rather defined by auctions in the day-ahead or intra-day energy markets. We furthermore make the following standard assumption concerning the price function.

Assumption 1: The price function $p(\cdot)$ is convex, strictly increasing and twice differentiable.

The convexity assumption is realistic in electricity markets, where generation capacities are fixed in the short term, and where marginal costs are increasing. Notice that Assumption 1 is slightly stronger than the assumption made by Ma, Callaway and Hiskens [7], which employ continuously differentiable, strictly increasing price functions. Future research may be devoted to clarify whether the twice differentiability assumption can be relaxed to continuous differentiability.

B. Generalized Nash Equilibria and Variational Inequalities

In order to bring into evidence the controls of player n, let us reformulate the notation for the collection $\boldsymbol{u}$ of strategies according to $(\boldsymbol{u}_n, \boldsymbol{u}_{-n})$, where $\boldsymbol{u}_{-n} := (\boldsymbol{u}_m; m \in \mathcal{N},\ m \neq n)$ represents the collection of strategies of all players other than n. We are now in the position to define Generalized Nash Equilibria.

Definition 1: A collection $\boldsymbol{u}^* \in \mathcal{U}$ of strategies is a Generalized Nash Equilibrium (GNE) if, $\forall\, n \in \mathcal{N}$,

$$J_n(\boldsymbol{u}^*) \leq J_n\left(\boldsymbol{u}_n, \boldsymbol{u}_{-n}^*\right) \tag{5}$$

for any strategy $\boldsymbol{u}_n$ satisfying $\left(\boldsymbol{u}_n, \boldsymbol{u}_{-n}^*\right) \in \mathcal{U}$.

As shown by Rosen [17] (see also the more recent work of Facchinei and Kanzow [36]), the vector $\boldsymbol{u}^*$ is a GNE if it solves the variational inequality VI($\boldsymbol{F_r}$,$\mathcal{U}$), which is given by

$$\langle \boldsymbol{F_r}(\boldsymbol{u}^*), \boldsymbol{u} - \boldsymbol{u}^* \rangle \geq 0, \quad \forall\, \boldsymbol{u} \in \mathcal{U}, \tag{6}$$

where $\boldsymbol{F_r}(\boldsymbol{u}) := \Big(r_n \nabla_{\boldsymbol{u}_n} J_n(\boldsymbol{u});\;\; n \in \mathcal{N}\Big) \in \mathbb{R}^{NT}$ is a pseudogradient vector defined in terms of N parameters $r_n > 0$. A GNE of this type is called a normalized equilibrium. Notice that the above relation does not hold in the opposite direction: there may exist GNEs that do not solve the variational equality for any $r_n > 0$. Rosen also showed that a normalized equilibrium exists if the cost functions are convex in each variable and continuously differentiable, and if the set $\mathcal{U}$ of feasible controls satisfies the properties verified by Proposition 1. Moreover, for any choice of parameters $r_n > 0$, the corresponding normalized equilibrium is unique if the pseudogradient $\boldsymbol{F_r}$ is strictly monotone, i.e., if

$$\langle \boldsymbol{F_r}(\boldsymbol{u}) - \boldsymbol{F_r}(\boldsymbol{u}'), \boldsymbol{u} - \boldsymbol{u}' \rangle > 0,\; \forall\, \boldsymbol{u}, \boldsymbol{u}' \in \mathcal{U}. \tag{7}$$

Proposition 2: Suppose that Assumption 1 holds. Then, the pseudogradient $\boldsymbol{F_r}(\boldsymbol{u})$ of the cost functions defined in (4) is strictly monotone on $\mathcal{U}$.

Proof: Let $G_{\boldsymbol{r}}(\boldsymbol{u}) \in \mathbb{R}^{NT \times NT}$ be the Jacobian of the pseudogradient $\boldsymbol{F_r}(\boldsymbol{u})$. As proven in [17], Theorem 6, showing that $G_{\boldsymbol{r}}(\boldsymbol{u}) + G_{\boldsymbol{r}}^\top(\boldsymbol{u})$ is positive definite on $\mathcal{U}$ is sufficient to ensure that $\boldsymbol{F_r}(\boldsymbol{u})$ is strictly monotone. Since $p'(r_t) > 0$ and $p''(r_t) \geq 0$ (Assumption 1), and since controls are nonnegative (1), it is easy to verify that all elements of $G_{\boldsymbol{r}}(\boldsymbol{u})$ are strictly positive. From this fact, we conclude that $G_{\boldsymbol{r}}(\boldsymbol{u}) + G_{\boldsymbol{r}}^\top(\boldsymbol{u})$ is positive definite on $\mathcal{U}$, and therefore $\boldsymbol{F_r}(\boldsymbol{u})$ is strictly monotone. ■

From the aforementioned results of Rosen [17], Assumption 1, together with Proposition 1 and 2, guarantees existence and uniqueness of a normalized equilibrium of the PEV problem for every choice of parameters $r_n > 0$. In this paper, we focus on the normalized equilibrium defined by $r_n = 1\; \forall n \in \mathcal{N}$. For this peculiar case, we omit the subscript $\boldsymbol{r}$ and simply denote the pseudogradient by $\boldsymbol{F}(\boldsymbol{u})$.

III. THE STOCHASTIC CASE

A. The S-adapted information structure

In this section, we will redefine the quantities $\boldsymbol{u}_n$, $\boldsymbol{u}$, $\mathcal{U}_n$, $\mathcal{U}$, $J(\boldsymbol{u}_n, \boldsymbol{u}_{-n})$, and $\boldsymbol{F}(\boldsymbol{u})$ by generalizing them to a stochastic case. Let $(\Omega, \mathfrak{F}, \mathbb{P})$ be a probability space. We introduce a discrete-time, discrete-state stochastic process $\xi : \Omega \times \mathcal{T} \to \mathcal{S}$, where $\mathcal{S} = \{1, ..., S\}$ denotes the set of possible states. By convention, the initial state is fixed according to $\xi(\omega, 1) \equiv 1\; \forall \omega \in \Omega$. Let $\{\emptyset, \Omega\} \equiv \mathfrak{F}_1 \subseteq \mathfrak{F}_2 ... \subseteq \mathfrak{F}_T$, with $\mathfrak{F}_T \subseteq \mathfrak{F}$, be the natural filtration of the σ-algebra $\mathfrak{F}$ induced by the stochastic process ξ.

Definition 2: Given an $\omega \in \Omega$, the functional $\xi_t(\omega, \cdot) : \{1, ..., t\} \to \mathcal{S}$, defined by $\xi_t(\omega, t') := \xi(\omega, t')$, is called a sample path of the stochastic process up to period $t \in \mathcal{T}$. A sample path $\xi_T(\omega, \cdot)$ up to the last period T is also called, in short, a sample path.

We represent all sample paths by an event tree with T periods and K nodes. Let $\mathcal{K} := \{1, ..., K\}$ be the set of all nodes, and $\mathcal{K}(t) \subset \mathcal{K}$ the set of all nodes at period t. We denote by k_t an element of $\mathcal{K}(t)$, and by $\mathcal{A}(k_t)$ the set composed by k_t and by all of its ancestors.

To each node k_t, it corresponds a realization $\xi(k_t) \in \mathcal{S}$ of the random variable. We assume that the state $\xi(k_t)$ affects the exogenous, non-PEV demand d_t, and accordingly denote by $d(k_t)$ the non-PEV demand at node k_t. For each player n, a control $u_n(k_t)$ is associated to node k_t, and we accordingly define the control vectors $\boldsymbol{u}_n := \big(u_n(k_t);\; k_t \in \mathcal{K}\big) \in \mathbb{R}^K$.

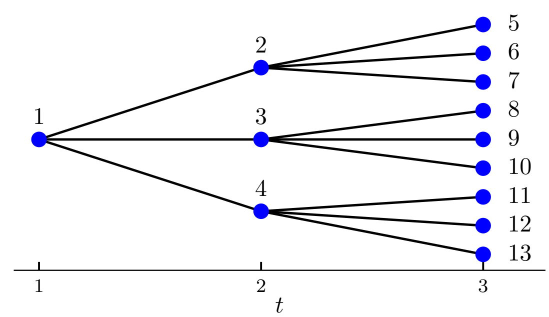

Fig. 1. An event tree with $S = 3$, $T = 3$ and $K = 13$. It holds, as an example, that $\mathcal{K}(2) = \{2, 3, 4\}$, $\mathcal{A}(5) = \{1, 2, 5\}$, $\xi(2) = 1$, $\xi(3) = 2$, and $\xi(4) = 3$.

According to the S-adapted information structure, in order to reflect nonanticipativity, the control $u_{n,t}$ chosen by player n at period t must be $\mathfrak{F}_t$-measurable; in other words, it can only depend on t and on the sample path $\xi_t(\omega, \cdot)$ of the stochastic process up to period t. It is easy to understand that an event tree offers a natural way to represent this structure (we refer to [37] for more details about event trees in the S-adapted information structure). Since each node k_t has an unique set of ancestors, there is indeed an unique sample path $\xi_t(\omega, \cdot)$ up to period t leading to node k_t. We may identify this sample path with the vector $\boldsymbol{\xi}(\mathcal{A}(k_t)) := (\xi(k_{t'});\; k_{t'} \in \mathcal{A}(k_t))$. We accordingly call the above-defined vector $\boldsymbol{u}_n$ of controls $u_n(k_t)\; \forall\, k_t \in \mathcal{K}$ an *S-adapted strategy* for player n. Analogously as before, the collection of S-adapted strategies will be denoted by $\boldsymbol{u} \in \mathbb{R}^{NK}$. The set of feasible controls is also generalized in a straightforward way. For the sake of completeness, let us first define $\mathcal{U}_n := \{\boldsymbol{u}_n \in \mathbb{R}^K : u_n(k_t) \geq 0\; \forall\, k_t \in \mathcal{K},\; \sum_{k_t \in \mathcal{A}(k_T)} u_n(k_t) = \gamma_n\; \forall\, k_T \in \mathcal{K}(T)\}$. Then, the whole set of feasible controls reads $\mathcal{U} := \{\boldsymbol{u} \in \mathbb{R}^{NK} : \boldsymbol{u}_n \in \mathcal{U}_n\, \forall\, n \in \mathcal{N},\; \bar{u}(k_t) \leq C_t\; \forall\, k_t \in \mathcal{K}\}$, where $\bar{u}(k_t) := N^{-1} \sum_{n \in \mathcal{N}} u_n(k_t)$.

We are now in the position to introduce the cost functions for the stochastic PEV problem. Consider at first a sample path $\boldsymbol{\xi}(\mathcal{A}(k_T))$ of the stochastic process, and associate to it, for player n, the (deterministic) cost function

$$J_n^{k_T}(\boldsymbol{u}_n, \boldsymbol{u}_{-n}) := \sum_{k_t \in \mathcal{A}(k_T)} p\left(\frac{d(k_t) + \bar{u}(k_t)}{c}\right) u_n(k_t). \tag{8}$$

The S-adapted, stochastic cost function $J_n(\boldsymbol{u}_n, \boldsymbol{u}_{-n})$ is defined as the expectation value of (8) over all sample paths. Denoting by $P(k_t)$ the probability for node k_t, we write

$$J_n(\boldsymbol{u}_n, \boldsymbol{u}_{-n}) := \sum_{k_t \in \mathcal{K}} P(k_t)\, p\left(\frac{d(k_t) + \bar{u}(k_t)}{c}\right) u_n(k_t). \tag{9}$$

We finally redefine the pseudogradient according to $\boldsymbol{F}(\boldsymbol{u}) := \Big(\nabla_{\boldsymbol{u}_n} J_n(\boldsymbol{u}_n, \boldsymbol{u}_{-n});\ n \in \mathcal{N}\Big) \in \mathbb{R}^{NK}$. Following the same argument used for Proposition 2, we easily conclude that, if Assumption 1 holds, the pseudogradient $\boldsymbol{F}(\boldsymbol{u})$ is strictly monotone.

B. Extended Game Reformulation

In [20], a reformulation of GNE problems as NE problems with $N+1$ players is made. Applying this reformulation to our case, the cost functions become

$$\bar{J}_n(\boldsymbol{u}, \boldsymbol{\lambda}) := J_n(\boldsymbol{u}) + \sum_{k_t \in \mathcal{K}} \lambda(k_t) u_n(k_t), \quad n \in \mathcal{N} \tag{10}$$

$$\bar{J}_{N+1}(\boldsymbol{u}, \boldsymbol{\lambda}) := -\sum_{k_t \in \mathcal{K}} \lambda(k_t) \left(\sum_n u_n(k_t) - NC_t\right), \tag{11}$$

where $\boldsymbol{\lambda} := (\lambda(k_t); k_t \in \mathcal{K}) \in \mathbb{R}^K$ denotes the strategy for the $(N+1)$-th player (we will also use the notation $\boldsymbol{u}_{N+1}$ to refer to $\boldsymbol{\lambda}$). The set of feasible controls is given by $\bar{\mathcal{U}} := \{(\boldsymbol{u}, \boldsymbol{\lambda}):\ \boldsymbol{u}_n \in \mathcal{U}_n,\ ;\forall\, n \in \mathcal{N};\ \lambda(k_t) \geq 0,\ \forall\, k_t \in \mathcal{K}\}$. Notice that, as an important difference with respect to the original formulation, all constraints are now individual. The original coupling constraint has been replaced by the action of the $(N+1)$-th player, which can be interpreted as a central operator establishing an additional price $\lambda(k_t)$ to incentivize the other N players to fulfill the original coupling constraint (3). Indeed, if the original constraint is violated, the central operator cost function becomes unbounded by below, so that an equilibrium is only possible when (3) is satisfied.

From Paccagnan et al. [20], Lemma 3, we conclude that, if the extended game on $\bar{\mathcal{U}}$ with cost functions (10) and (11) has an unique NE, this equilibrium corresponds to the unique solution of the variational inequality VI($\boldsymbol{F}$,$\mathcal{U}$). In turn, the extended game can be equivalently reformulated in terms of a variational inequality VI($\bar{\boldsymbol{F}}$,$\bar{\mathcal{U}}$) with pseudogradient $\bar{\boldsymbol{F}}(\boldsymbol{u}, \boldsymbol{u}_{N+1}) := \Big(\nabla_{\boldsymbol{u}_{\bar{n}}} \bar{J}_{\bar{n}}(\boldsymbol{u}, \boldsymbol{u}_{N+1});\ \bar{n} \in \mathcal{N} \cup \{N+1\}\Big) \in \mathbb{R}^{(N+1)K}$.

Proposition 3: The pseudogradient $\bar{\boldsymbol{F}}(\boldsymbol{u}, \boldsymbol{\lambda})$ of the cost functions (10) and (11) is strictly monotone on $\bar{\mathcal{U}}$.

Proof: It is easy to verify that

$$\begin{aligned}&\langle \bar{\boldsymbol{F}}(\boldsymbol{u}, \boldsymbol{\lambda}) - \bar{\boldsymbol{F}}(\boldsymbol{u}', \boldsymbol{\lambda}'), (\boldsymbol{u}, \boldsymbol{\lambda}) - (\boldsymbol{u}', \boldsymbol{\lambda}')\rangle \\ =&\langle \boldsymbol{F}(\boldsymbol{u}) - \boldsymbol{F}(\boldsymbol{u}'), (\boldsymbol{u} - \boldsymbol{u}')\rangle,\end{aligned} \tag{12}$$

so that Proposition 3 directly follows from the strict monotonicity of $\boldsymbol{F}(\boldsymbol{u})$. ■

Proposition 3 guarantees that the extended-game variational inequality VI($\bar{\boldsymbol{F}}$,$\bar{\mathcal{U}}$) has an unique solution [17]. From the aforementioned argument in Paccagnan et al. [20], we conclude that this solution is also the unique solution of the variational inequality VI($\boldsymbol{F}$,$\mathcal{U}$), and therefore, by Rosen [17], it solves the original GNE problem as well. The extended-game reformulation proves itself to be useful for constructing decentralized mechanisms to calculate the GNE. In this study, we apply the extragradient algorithm described by [30] to calculate the solution of VI($\bar{\boldsymbol{F}}$,$\bar{\mathcal{U}}$). Similarly to the APA algorithm in [20], the central operator communicates to the players, at each iteration step, the aggregate PEV demand $\bar{u}(k_t)$ and the price $\lambda(k_t)$ for all nodes $k_t \in \mathcal{K}$, while the controls update is performed individually, and then communicated back to the central operator. Then, in turn, the central operator updates the price $\lambda(k_{t+1})$ which will be used for the next iteration. Notice that we do not require strong monotonicity of the pseudogradient, which is used in [20] to guarantee convergence of the APA algorithm. While being sufficient for ensuring existence and uniqueness of the solution of VI($\boldsymbol{F}$,$\mathcal{U}$), strict monotonicity is less demanding in terms of the price function structure (we remark that [20] just considers quadratic games, and therefore linear price functions).

IV. NUMERICAL EXAMPLE

A. Setup

We consider a two-state ($S = 2$) Markov process for the non-PEV demand d_t, and a full-day time horizon with periods of one hour each ($T = 24$). Demand states are defined by a base-case demand curve $d_t^{(1)}$, equal to the one used by [7], and by a high demand curve $d_t^{(2)} = d_t^{(1)} + 0.5\,\text{kW}$. Due to external conditions, for instance weather, the actual non-PEV demand realization d_t can jump from one curve to the other. In the simple example discussed here, we limit the periods where jumps are allowed to the set $\tilde{\mathcal{T}} \equiv \{5, 9, 13, 17, 21\}$, and accordingly divide the day into six intervals $\mathcal{I}_{\tilde{t}} := \{\tilde{t}, ..., \tilde{t}+3\}$ of four hours each, where $\tilde{t} \in \{1\} \cup \tilde{\mathcal{T}}$. The tree describing the stochastic process is composed of 252 nodes, among which there are 32 leaf nodes, each corresponding to one sample path. Notice that an internal node k_t has two children nodes if $t \in \tilde{\mathcal{T}}$, and one child node otherwise. For the Markov process, we consider the dynamics described by the transition matrix $P_{ih} = 1/2$, for any $i, h \in \mathcal{S} = \{1, 2\}$. At every $\tilde{t} \in \tilde{\mathcal{T}}$, there is therefore an equal probability of remaining on the current demand curve or jumping to the other one.

We consider a population of $N = 10$ PEV's with properties equal to the heterogeneous-case example discussed in [7]. More specifically, the initial state-of-charge, charging efficiency, and capacity per single PEV amount to $x_{n,1} = 0.15$, $\alpha_n = 0.85$ and $c = 12$ kW for all $n \in \mathcal{N}$, respectively. The battery sizes β_n are equal to 10, 15 or 20 kWh, and are found with a proportion of 50%, 30% and 20% of the entire population, respectively. The price function is given by $p(r) = 0.15\, r^{3/2}$ \$/kWh. Finally, we consider the coupling constraint $C_t = 1.5\,\mathrm{kW}$ for any $t \in \mathcal{T}$.

B. Results

We implemented the extragradient algorithm in Octave, and used the "quadprog" solver to perform projections on the set $\bar{\mathcal{U}}$ of feasible controls. As a result, we find that the control vector $\boldsymbol{u}$ solving the GNE problem depends on the demand state $\xi(k_t)$ only for $t \in \mathcal{I}_9 \cup \mathcal{I}_{13} \cup \mathcal{I}_{17}$, while the demand realization during the other three time intervals turns out to be irrelevant. This can be explained i) by the large non-PEV demand during intervals $\mathcal{I}_1$ and $\mathcal{I}_5$, so that controls are zero in any case, and ii) by the fact that, during the last interval $\mathcal{I}_{21}$, a well-defined amount of power is left to be charged, and the control to achieve it cannot depend on a constant shift of the non-PEV demand. We are therefore left with eight different controls for each player, each being characterized by the non-PEV demand realization between $t = 9$ and $t = 20$. The average control $\bar{u}_t$ is plotted in Fig. 2 for the eight different cases. We may verify that controls are clearly nonanticipative, in the sense that any two sample paths being equal up to period $t \in \{12, 16, 20\}$, have controls that perfectly overlap up to that period. It is also interesting to notice that the effect of high non-PEV demand is that of postponing part of the charging load, in the hope that demand will eventually come back to its lower state at a later period.

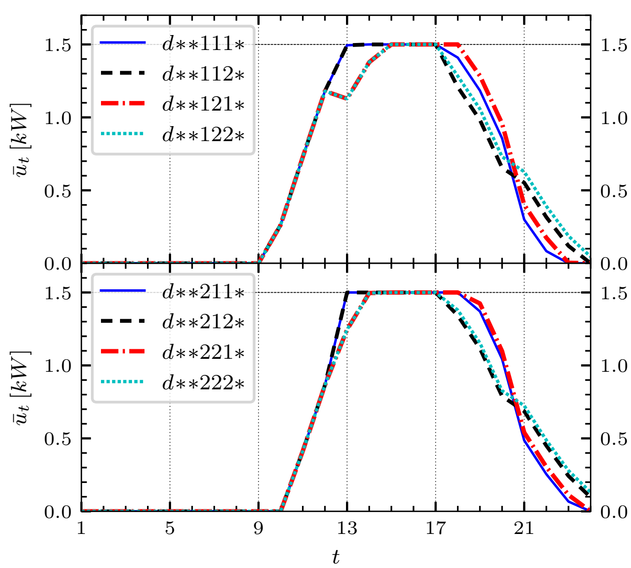

Fig. 2. Average control $\bar{u}_t$ for the eight different cases found in the solution. The six time intervals $\mathcal{I}_{\tilde{t}}$ are separated by dotted, vertical lines, while horizontal lines denote the coupling constraint C_t. In the legend, to describe the sample paths, the i-th character after the d denotes the state ("1": low, "2": high) during the i-th time interval, and asterisks denote the time intervals during which controls do not depend on the demand state.

The four controls shown in the upper part of Fig. 2 are also plotted in Fig. 3 in the case of some specific non-PEV demand realizations. Again, because of nonanticipativity, all curves are equal up to the third time interval ($t = 12$), while they are pairwise equal, for any two subplots on the same horizontal axis, up to the fourth time interval ($t = 16$). We may also remark that the total demand curves clearly deviate from the valley-filling property observed in [7], a first reason being the existence of the coupling constraint C_t, which is, for instance, active during the whole time interval $\mathcal{I}_{13}$ for the two upper subplots. A second, more interesting reason preventing perfect valley-filling is intrinsic of the S-adapted structure itself. For instance, the left subplots show a small "bump" in the charging controls after $t = 17$, which cannot be explained by the coupling constraint. Instead, the player is considering here the risk of having high demand during the last interval $\mathcal{I}_{21}$, and decides that it is better to increase the control during $\mathcal{I}_{17}$, so that less charging power is left for the last, still uncertain interval.

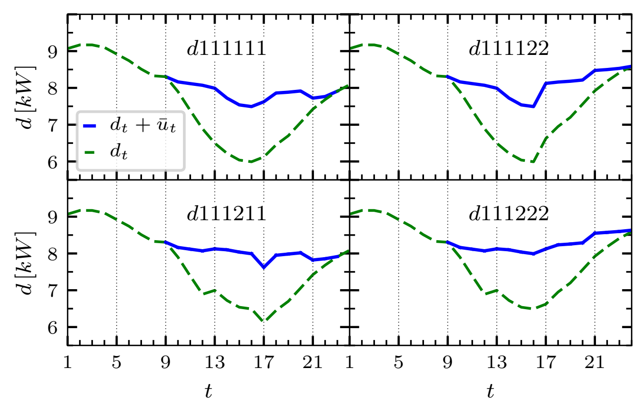

Fig. 3. Non-PEV demand and total demand for some specific sample paths. The six time intervals $\mathcal{I}_{\tilde{t}}$ are separated by dotted, vertical lines.

C. The cost of perfect information

Finally, we compare the expectation value of the average cost for charging a PEV when the GNE is calculated employing three different information structures: i) a model with perfect anticipative information, where the equilibrium is calculated by exactly knowing, from the very first period, the realization of the whole sample path; ii) the S-adapted information structure exposed in this paper; iii) a deterministic naïve approach which makes use of the expected demand, i.e., the average between low and high demand. For these three information structures, the expected costs amount to 1.1027 \$, 1.1044 \$ and 1.1066 \$, respectively. We can observe that, with respect to perfect information, the S-adapted information structure leads to a 0.15% increase of price, while the naïve approach leads to an increase of 0.35%. Therefore, the cost of the S-adapted model stands slightly closer to perfect information than to the naïve approach.

V. CONCLUSIONS

By employing an event tree, we have made use of the S-adapted information structure to describe an exogenous stochastic process that affects the non-PEV demand and, in turn, the real-time price of electricity. We have observed that, as expected, the effect of higher non-PEV demand is the one of delaying the charging controls of each PEV towards later periods. Furthermore, we have noticed that the valley-filling behavior observed in [7] can be compromised by the stochastic structure. Similarly to the deterministic model proposed in [20], our method works for an arbitrary number N of PEVs, and is able to deal with the presence of coupling constraints. As a difference, however, we employ a more general price function, since we do not require the cost function to be quadratic in the controls.

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REFERENCES

[1] Y. Zhou, M. Wang, H. Hao, L. Johnson, H. Wang, and H. Hao, "Plug-in electric vehicle market penetration and incentives: a global review," *Mitigation and Adaptation Strategies for Global Change*, vol. 20, no. 5, pp. 777–795, Jun 2015.

[2] D. M. Lemoine, D. M. Kammen, and A. E. Farrell, "An innovation and policy agenda for commercially competitive plug-in hybrid electric vehicles," *Environmental Research Letters*, vol. 3, no. 1, p. 014003, Jan 2008.

[3] S. Rahman and G. B. Shrestha, "An investigation into the impact of electric vehicle load on the electric utility distribution system," *IEEE Transactions on Power Delivery*, vol. 8, no. 2, pp. 591–597, 1993.

[4] A. Mohsenian-Rad, V. W. S. Wong, J. Jatskevich, R. Schober, and A. Leon-Garcia, "Autonomous demand-side management based on game-theoretic energy consumption scheduling for the future smart grid," *IEEE Transactions on Smart Grid*, vol. 1, no. 3, pp. 320–331, Dec 2010.

[5] W. Saad, Z. Han, H. V. Poor, and T. Basar, "Game-theoretic methods for the smart grid: an overview of microgrid systems, demand-side management, and smart grid communications," *IEEE Signal Processing Magazine*, vol. 29, no. 5, pp. 86–105, Sep 2012.

[6] C. Metzler, B. F. Hobbs, and J.-S. Pang, "Nash-Cournot equilibria in power markets on a linearized DC network with arbitrage: Formulations and properties," *Networks and Spatial Economics*, vol. 3, no. 2, pp. 123–150, Jun 2003.

[7] Z. Ma, D. S. Callaway, and I. A. Hiskens, "Decentralized charging control of large populations of plug-in electric vehicles," *IEEE Transactions on Control Systems Technology*, vol. 21, no. 1, pp. 67–78, 2013.

[8] Z. Ma, S. Zou, and X. Liu, "A distributed charging coordination for large-scale plug-in electric vehicles considering battery degradation cost," *IEEE Transactions on Control Systems Technology*, vol. 23, no. 5, pp. 2044–2052, Sep 2015.

[9] Z. Ma, S. Zou, L. Ran, X. Shi, and I. A. Hiskens, "Efficient decentralized coordination of large-scale plug-in electric vehicle charging," *Automatica*, vol. 69, pp. 35 – 47, 2016.

[10] N. S. Kukushkin, "Best response dynamics in finite games with additive aggregation," *Games and Economic Behavior*, vol. 48, no. 1, pp. 94 – 110, 2004.

[11] M. K. Jensen, "Aggregative games and best-reply potentials," *Economic Theory*, vol. 43, no. 1, pp. 45–66, 2010.

[12] J. Koshal, A. Nedić, and U. V. Shanbhag, "A gossip algorithm for aggregative games on graphs," in *2012 IEEE 51st IEEE Conference on Decision and Control (CDC)*. IEEE, 2012, pp. 4840–4845.

[13] F. Parise, B. Gentile, S. Grammatico, and J. Lygeros, "Network aggregative games: distributed convergence to Nash equilibria," in *2015 54th IEEE Conference on Decision and Control (CDC)*. IEEE, 2015, pp. 2295–2300.

[14] D. Paccagnan, M. Kamgarpour, and J. Lygeros, "On aggregative and mean field games with applications to electricity markets," in *2016 European Control Conference (ECC)*, June 2016, pp. 196–201.

[15] S. Grammatico, F. Parise, M. Colombino, and J. Lygeros, "Decentralized convergence to Nash equilibria in constrained deterministic mean field control," *IEEE Transactions on Automatic Control*, vol. 61, no. 11, pp. 3315–3329, Nov 2016.

[16] G. Debreu, "A social equilibrium existence theorem," *Proceedings of the National Academy of Sciences*, vol. 38, no. 10, pp. 886–893, 1952.

[17] J. B. Rosen, "Existence and uniqueness of equilibrium points for concave n-person games," *Econometrica*, vol. 33, no. 3, pp. 520–534, 1965.

[18] P. T. Harker, "Generalized Nash games and quasi-variational inequalities," *European Journal of Operational Research*, vol. 54, no. 1, pp. 81 – 94, 1991.

[19] F. Facchinei and C. Kanzow, "Generalized Nash equilibrium problems," *4OR*, vol. 5, no. 3, pp. 173–210, 2007.

[20] D. Paccagnan, B. Gentile, F. Parise, M. Kamgarpour, and J. Lygeros, "Distributed computation of generalized Nash equilibria in quadratic aggregative games with affine coupling constraints," *2016 IEEE 55th Conference on Decision and Control (CDC)*, pp. 6123–6128, 2016.

[21] G. Belgioioso and S. Grammatico, "Semi-decentralized Nash equilibrium seeking in aggregative games with separable coupling constraints and non-differentiable cost functions," *IEEE Control Systems Letters*, vol. 1, pp. 400–405, 2017.

[22] D. Paccagnan, B. Gentile, F. Parise, M. Kamgarpour, and J. Lygeros, "Nash and Wardrop equilibria in aggregative games with coupling constraints," *IEEE Transactions on Automatic Control*, vol. 64, no. 4, pp. 1373–1388, April 2019.

[23] S. Grammatico, "Dynamic control of agents playing aggregative games with coupling constraints," *IEEE Transactions on Automatic Control*, vol. 62, no. 9, pp. 4537–4548, Sep. 2017.

[24] A. Haurie, G. Zaccour, and Y. Smeers, "Stochastic equilibrium programming for dynamic oligopolistic markets," *Journal of Optimization Theory and applications*, vol. 66, no. 2, pp. 243–253, 1990.

[25] E. Altman, A. Haurie, F. Moresino, and O. Pourtallier, "Approximating Nash-equilibria in nonzero-sum games," *International Game Theory Review*, vol. 2, no. 2-3, pp. 155–172, 2000.

[26] A. Haurie and F. Moresino, "Computation of S-adapted equilibria in piecewise deterministic games via stochastic programming methods," *Annals of the International Society of Dynamic Games*, vol. 6, pp. 225–252, 2001.

[27] ——, "S-adapted oligopoly equilibria and approximations in stochastic variational inequalities," *Annals of Operations Research*, vol. 114, no. 1, pp. 183–201, Aug 2002.

[28] L. Drouet, A. Haurie, F. Moresino, J.-P. Vial, M. Vielle, and L. Viguier, "An oracle based method to compute a coupled equilibrium in a model of international climate policy," *Computational Management Science*, vol. 5, no. 1, pp. 119–140, Feb 2008.

[29] P.-O. Pineau, H. Rasata, and G. Zaccour, "Impact of some parameters on investments in oligopolistic electricity markets," *European Journal of Operational Research*, vol. 213, no. 1, pp. 180 – 195, 2011.

[30] F. Facchinei and J.-S. Pang, *Finite-dimensional variational inequalities and complementarity problems*. Springer Science & Business Media, 2007.

[31] M. Huang, P. E. Caines, and R. P. Malhamé, "Large-population cost-coupled LQG problems with nonuniform agents: individual-mass behavior and decentralized ε-Nash equilibria," *IEEE transactions on automatic control*, vol. 52, no. 9, pp. 1560–1571, 2007.

[32] J. Lei, U. V. Shanbhag, J.-S. Pang, and S. Sen, "On synchronous, asynchronous, and randomized best-response schemes for computing equilibria in stochastic Nash games," *arXiv:1704.04578*, 2017.

[33] J. Lei and U. V. Shanbhag, "Distributed variable sample-size gradient-response and best-response schemes for stochastic Nash games over graphs," *arXiv:1811.11246*, 2018.

[34] C. Yu, M. van der Schaar, and A. H. Sayed, "Distributed learning for stochastic generalized Nash equilibrium problems," *IEEE Transactions on Signal Processing*, vol. 65, no. 15, pp. 3893–3908, Aug 2017.

[35] Z. Ma, I. Hiskens, and D. Callaway, "A decentralized MPC strategy for charging large populations of plug-in electric vehicles," *IFAC Proceedings Volumes*, vol. 44, no. 1, pp. 10 493 – 10 498, 2011, 18th IFAC World Congress.

[36] F. Facchinei and C. Kanzow, "Penalty methods for the solution of generalized Nash equilibrium problems," *SIAM Journal on Optimization*, vol. 20, no. 5, pp. 2228–2253, 2010.

[37] A. Haurie and G. Zaccour, *S-adapted equilibria in games played over event trees: an overview*. Boston, MA: Birkhäuser Boston, 2005, pp. 417–444.