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Key Points:
- *E* field during the rocket ascent prior to the lightning initiation is analyzed
- Ground corona and wire corona sheath are included in the model
- Performance of the developed analytical model is better compared to the previous models

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A model for the evaluation of the electric field associated with the lightning-triggering rocket wire and its corona

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Abstract In this paper, we analyze the electric field at ground level during the first stage of triggered lightning experiments, i.e., during the rocket ascent and prior to the lightning initiation. At distances of some tens of meters from the triggering wire, the electric field decreases significantly, while at distances of several hundred meters, there is only a very small decrease of the electric field. Two effects determine the level of the electric field reduction: the corona layer at ground level and the corona sheath around the triggering wire. We present an analytical solution based on the charge simulation method to study the phenomenon. The model is validated by comparing its results to those obtained by numerical simulations using the finite element method. A ground space charge layer and a corona sheath around the rocket-triggered lightning wire are included in the simulation. It is shown that, depending on the charge distribution, the change of the sign of the electric field is correctly predicted by our model. The obtained reductions of the electric field are consistent with simulations and experiments presented in the literature. Moreover, the proposed analytical solution is faster, and it allows studying the influence of several parameters simultaneously, i.e., the radius of the corona sheath and the space charge layer parameters. The described analytical model allows the estimation of the corona sheath radius if the parameters of the space charge layer are known from experiment.

1. Introduction

Lightning discharges triggered from a thundercloud by using a rocket trailing a conductive wire are widely used in lightning research [*Rakov and Uman*, 2007]. The main advantages of this technique are (1) similarity to subsequent strokes in natural lightning discharges, (2) known time and location of the triggered lightning, and (3) possibility of simultaneous measurement of lightning parameters prior to, during, and after the lightning discharge. Triggered lightning is therefore often used to study lightning properties and to test the accuracy of lightning models and the performance of lightning location systems. It is also possible to trigger discharges exhibiting some of the characteristics of first return strokes by replacing part of the conducting wire by an insulator; but this technique is rarely used since the strike point of the discharge cannot be predetermined. In this paper we analyze triggered experiments with rocket trailing a conductive wire.

Electric field measurements at ground level have been reported for many triggered lightning experiments [*Standler*, 1975; *Fieux et al.*, 1978; *Nakamura et al.*, 1987; *Liu et al.*, 1994; *Willett et al.*, 1999; *Biagi et al.*, 2011]. These experimental data give insights into the various stages in the development of rocket-triggered lightning. In this paper, only the electric field measurements during the first stage are considered, i.e., starting from the rocket launch until the lightning initiation. A recent review of these data obtained during several triggered lightning experiments is presented in *Baba and Rakov* [2011].

The lightning is usually initiated when the wire pulled by the rocket extends to a height of about 200 to 300 m above ground. While the wire extends upward, the electric field at ground level measured at close distances from the wire decreases. At distances of 30 to 100 m, a reduction of more than 30% with respect to the prelaunch value of the electric field has been measured. At distances exceeding several hundred meters, only a small field reduction has been observed. It was also observed that, at a given site, the electric field reduction varies from one discharge to another. A summary of the maximum values of the electric field reduction measured in different studies is given in Table 1. In some cases, a decrease of more than 100% was reported, which corresponds to a change of sign of the electric field.

Table 1. Experimental Data on the Maximum Reduction of the Electric Field Observed at Ground Level

N	Reference	Distance From the Launch Site, m	Rocket Altitude, m	Maximum Electric Field Reduction, %
1	*Willett et al.* [1999]	30	300	110
2	*Nakamura et al.* [1987]	40	100	140
3	*Biagi et al.* [2011]	60	300	75
4	*Liu et al.* [1994]	75	380	75
5	*Fieux et al.* [1978]	100	150	50
6	*Standler* [1975]	100	500	50

Two processes lead to the reduction of the electric field at ground level. The first one is the space charge generated by corona effect on the ground. Indeed, under the charged thundercloud, charges of opposite polarity are induced in the conductive ground. Corona is formed at ground level due to the increase of the electric field at sharp protrusions and edges of irregularities, such as buildings, trees, bushes, and grass [*Cooray*, 2003]. The layer of ions created by corona effect can extend to an altitude of several hundreds of meters [*Chauzy and Soula*, 1999; *Willett et al.*, 1999]. However, the charge density is lower at higher altitudes.

The second process involves the movement of charges from the ground toward the triggering wire during its vertical expansion. Almost immediately after the rocket leaves the ground, corona space charge is created around the wire [*Rizk*, 2011].

As a result, the reduction of the electric field at ground level depends on (1) the amount of charge accumulated on the wire and around it and (2) on the amount of charge and its distribution in the ground corona layer.

Note that this dynamic process can be modeled as a series of steady states represented by a wire of varying length [*Cooray et al.*, 2007], allowing the use of an electrostatic approach to study the problem.

The corona sheath radius can be estimated from simulations. Two models have been proposed to study this phenomenon. A summary of the essential elements of these models is presented in Table 2, and a more complete description of each of them is given hereafter.

In the model by *Baba and Rakov* [2011], the lightning-triggering wire extends upward with a speed of about 150 m/s [*Rakov and Uman*, 2007]. In this model, due to computational limitations, the vertical conductor has a radius of 0.27 m instead of the actual 0.1 m radius used during the rocket-triggered lightning experiments. The corona space charge that surrounds the wire is represented as a cylindrical sheath of outer radius of several meters. Both corona sheath and wire are assumed to be perfectly conductive (the influence of the corona conductivity was shown to be not significant [*Baba and Rakov*, 2011]). Finally, the ground corona is modeled by multiple concentric, perfectly conducting cylindrical tubes placed above a flat, perfectly conducting ground plane. The tubes have a thickness of 2 m and three different heights, 40, 50, and 100 m. The finite difference time domain (FDTD) method is used to solve the problem. The only variable in this model is the radius r of the corona sheath around the wire. In order to predict electric field changes that are in agreement with experimental data, values of r on the order of 4 to 16 m were required in the FDTD simulations.

In the model by *Biagi et al.* [2011], an axis-symmetrical model centered on the vertical ascending lightning triggering wire was used to study the electric field distribution and the wire shielding effect. The wire was assumed to be infinitely thin, even though the actual radius of the wire used in their experiments was 0.1 mm. The presence of a corona sheath around the wire was neglected by Biagi et al. However, they did consider a space charge layer with a charge density exponentially decaying with altitude. The finite element method was used for the field computation.

Table 2. Comparison of the Computational Models Used to Study the Reduction of the Electric Field at Ground Level During Triggered Lightning Experiments

Study	Triggering Wire	Corona Around Triggering Wire	Space Charge Layer	Computational Method
Biagi et al. [2011]	Infinitely thin wire	Neglected	Any distribution (analytical expression or measured data)	FEM
Baba and Rakov [2011]	Cylinder	Increased radius of cylinder	Approximated as concentric cylinders	FDTD
This study	Cylinder	Increased radius of cylinder	Any distribution (analytical expression or measured data)	Analytical

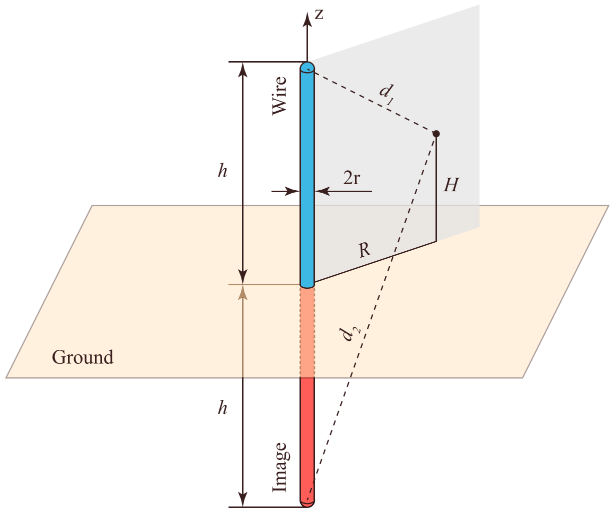

Figure 1. Model geometry.

In this paper, we present a new computational model derived analytically and based on the charge simulation method (CSM). The proposed model takes into account both the space charges generated on the ground and those around the wire.

The paper is organized as follows: in section 2, an analytical solution of the problem is derived based on the charge simulation method. In section 3, the results of the developed model are compared with numerical simulations obtained using finite element method (FEM) simulations, making reference to available experimental data. In section 4, the field reduction effect is analyzed. In section 5, the proposed model is compared with FEM simulation results from *Biagi et al.* [2011] and with FDTD simulations from *Baba and Rakov* [2011]. A summary and conclusions are given in section 6.

2. Proposed Analytical Model

In this section, we present analytical formulas to calculate the electric field and the potential around a thin metallic rod placed on a conductive ground. To derive these expressions, we use the charge simulation method (CSM) and the method of images widely used in electrostatic field calculations [*Kolechitskiy*, 1983; *Malik*, 1989].

The lightning-triggering rocket and wire with surrounding corona sheath are represented as a cylinder with a hemispherical cap as shown in Figure 1. The cylinder is placed on an infinite, perfectly conducting plane. According to the charge simulation method, the cylinder can be modeled as a charged line with linear distribution of charge τ on it. The hemisphere is modeled by a point charge Q. The method of images allows us to represent the problem as a charged line of length $2h$ with two charges of opposite polarities located at the extremities of the line.

The first step is to find the relation between the point charges at the extremities, Q; the linear charge density, τ; and the background potential. Due to the symmetry of the problem, we will work, without loss of generality, in the *x-z* plane. Two points are selected on the surface of the cylinder: its middle point and its top.

The potential at any given point (x_0, z_0) around the charged line with τ varying from τ_1 at the bottom to τ_2 at the top as shown in Figure 2 is defined as

$$U = \frac{\tau_1}{4\pi\varepsilon(z_2 - z_1)}\left((z_2 - z_0)\cdot ln\left(\frac{(z_2 - z_0) + r_2}{(z_1 - z_0) + r_1}\right) + r_1 - r_2\right) + \\ + \frac{\tau_2}{4\pi\varepsilon(z_2 - z_1)}\left((z_0 - z_1)\cdot ln\left(\frac{(z_2 - z_0) + r_2}{(z_1 - z_0) + r_1}\right) + r_2 - r_1\right) \quad (1)$$

where

$$r_1 = \sqrt{(z_1 - z_0)^2 + x_0^2},\ r_2 = \sqrt{(z_2 - z_0)^2 + x_0^2}$$

and epsilon is the permittivity of the surrounding medium.

To calculate the potential U_{mid} of the charged line on the surface of the charged line at midheight $(r, h/2)$, we make the following substitutions in equation (1):

$$\tau_1 = -\tau, \tau_2 = \tau, z_1 = -h, z_2 = h, z_0 = \frac{h}{2}, x_0 = r$$

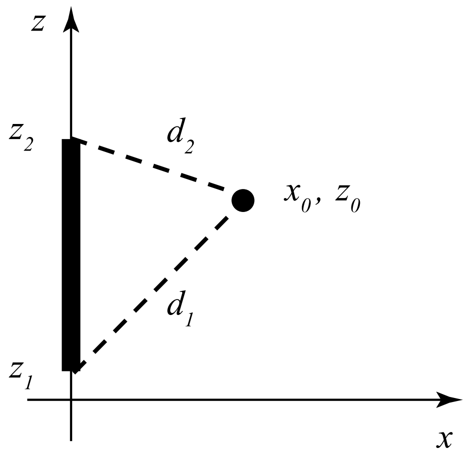

Figure 2. Definition of the variables for evaluating the electric potential of the line.

After simplifications, we obtain

$$U_{\text{mid}} = \frac{\tau}{4\pi\varepsilon} \ln\left(\frac{h\sqrt{3}}{r \cdot e}\right) \tag{2}$$

The potential U_{top} of the charged line at the top of the cylinder (0, $h + r$) can be obtained by performing the following substitutions in equation (1):

$$\tau_1 = -\tau, \tau_2 = \tau, z_1 = -h, z_2 = h, z_0 = h + r, x_0 = 0$$

It can be shown that U_{top} can be simplified to

$$U_{\text{top}} = \frac{\tau}{4\pi\varepsilon} \ln\left(\frac{2 \cdot h}{r \cdot e^2}\right) \tag{3}$$

Since the potential on the surface of the cylinder is zero, the following system of equations for two selected points can be written.

$$\begin{cases} \dfrac{\tau}{4\pi\varepsilon} \ln\left(\dfrac{h\sqrt{3}}{r \cdot e}\right) - U_1 = 0 \\ \dfrac{\tau}{4\pi\varepsilon} \ln\left(\dfrac{2 \cdot h}{r \cdot e^2}\right) + \dfrac{Q}{4\pi\varepsilon r} - U_2 = 0 \end{cases} \tag{4}$$

where we have called the background potentials at the middle point and the top U_1 and U_2, respectively. At the first point, in the middle of the cylinder, we have neglected the potential from the charge $+Q$ and its image $-Q$. At the second point, on the top hemispherical charge, we have assumed that the potential from the bottom charge $-Q$ is negligible but the presence of the top charge $+Q$ cannot be neglected.

By solving this system of equations, the unknown variables Q and τ can be expressed as functions of U_1 and U_2:

$$\begin{aligned} Q &= \frac{4\pi\varepsilon r\left(U_1 \ln\left(\dfrac{r^2}{4h^2}\right) + U_2 \ln\left(\dfrac{3h^2}{r^2}\right) + 4U_1 - 2U_2\right)}{\ln\left(\dfrac{3h^2}{r^2}\right) - 2} \\ \tau &= \frac{8\pi\varepsilon}{\ln\left(\dfrac{3h^2}{r^2}\right) - 2} U_1 \end{aligned} \tag{5}$$

For a given electric background field, assumed to depend only on the height, distribution above the ground E (z), the space charge distribution can be found as

$$\rho = \varepsilon_0 \frac{\partial E}{\partial z} \tag{6}$$

and the distribution of the background electric potential can be found as

$$U = -\int_0^z E dz \tag{7}$$

Finally, the potential at any point (R, H) around the linearly charged wire with a charge Q on its top shown in Figure 1 can be defined as the sum of the background potential U_3 at that point, the potential U_τ from the charged line, and the potential U_Q from the charge $+Q$ and its image $-Q$:

$$U_{\text{total}} = U_3 + U_\tau + U_Q \tag{8}$$

Potential U_τ can be found by performing the following substitutions in equation (1):

$$\tau_1 = -\tau, \tau_2 = \tau, z_1 = -h, z_2 = h, z_0 = H, x_0 = R$$

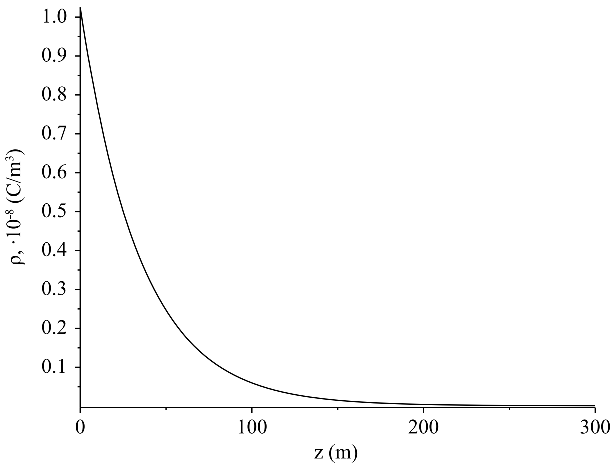

Figure 3. Variation of space charge density with altitude. Adapted from *Becerra et al.* [2007].

where the value of τ is determined from equation (5), which results in

$$U_\tau = \frac{U_1 \ln\left(\frac{-h+H-r_1}{h+H-r_2}\right)H}{\ln\left(\frac{h\sqrt{3}}{r\cdot e}\right)h} \tag{9}$$

where

$$r_1 = \sqrt{(H-h)^2 + R^2},$$
$$r_2 = \sqrt{(H+h)^2 + R^2}$$

The distances r_1 and r_2 are also used to express the potential U_Q as follows:

$$U_Q = \frac{Q}{4\pi\varepsilon r_1} - \frac{Q}{4\pi\varepsilon r_2} \tag{10}$$

By substituting the expression for Q from equation (5), we obtain

$$U_Q = \frac{r(r_1 - r_2)}{r_1 r_2}\frac{\left(U_1 \ln\left(\frac{r^2}{4h^2}\right) + U_2 \ln\left(\frac{3h^2}{r^2}\right) + 4U_1 - 2U_2\right)}{\ln\left(\frac{3h^2}{r^2}\right) - 2} \tag{11}$$

All components of U_{total} have been described above. For this study, we are interested only in the values of the electric field at ground level in order to compare them with available experimental data. Only the vertical component of the electric field exists on the ground since it is assumed to be perfectly conducting. This field component can be calculated from

$$E_z = -\frac{\partial U_{\text{total}}}{\partial H} \tag{12}$$

3. Validation of the Model

To validate our analytical model based on the charge simulation method, we have used the finite element method to study the same simulated rocket-triggered experiment and we have compared the results of both methods.

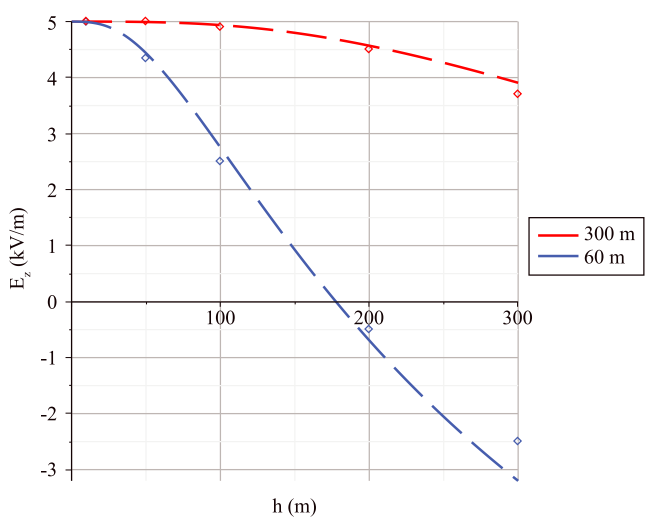

Figure 4. Comparison of the field variation at ground level observed at 60 m (shown in blue) and 300 m (shown in red) from the rocket-triggered lightning launch site obtained by using the derived analytical CSM-based equations (dashed line) and numerical FEM (dots) simulations.

The selected setup consists of an ascending wire modeled as a cylinder with a radius of 0.5 m, representing the 0.1 mm triggering wire and the corona sheath around it. Corona from the ground objects is introduced in the model as a space charge layer ρ, with numerical values approximated from the measurements reported by *Becerra et al.* [2007]. The variation of ρ (C/m^3) with altitude was approximated by the following relationship, and it is represented in Figure 3:

$$\rho = 1.0457\cdot 10^{-11}\cdot e^{0.00002919z} + 1.0238\cdot 10^{-8}\cdot e^{-0.02858z} \tag{13}$$

With known electric field at the ground level ($E = 5$ kV/m) and at high altitude ($E = 46.7$ kV/m), we can derive the

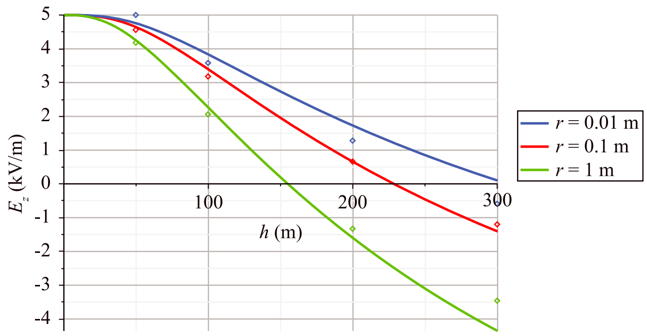

Figure 5. Electric field at ground level at a distance $D = 60$ m from the launching site during the rocket ascent. The solid lines represent the results obtained with our analytical CSM equations, while the dots represent the numerical FEM results.

following expression for the background electric field $E(z)$ (kV/m):

$$E(z) = 5 + 40.48 \cdot e^{0.00002919z} - 40.48 \cdot e^{-0.02858z} \quad (14)$$

Prior to the rocket launch, the electric field at ground level is assumed to have a value of 5 kV/m. As in rocket-triggered experiments, we are interested in studying the variation of the electric field at ground level at close and far distances from the launching pad during the rocket ascent prior to the lightning initiation. The calculated field reductions at 60 m and 300 m as a function of the height of the rocket are shown in Figure 4.

It can be seen from Figure 4 that the analytical model developed using the CSM provides accurate results very close to the values obtained by solving the problem with FEM. An inversion of the electric field polarity can also be observed in Figure 4. It is worth noting that the change of polarity is due to the charges induced in the conductor, which results in an electric field that overcomes the background electric field at close distances to the conductor, producing therefore an inversion of the polarity which has been experimentally observed in rocket-triggered lightning experiments as reported by *Baba and Rakov* [2011]. When the rocket reaches a height of 170 m, the vertical electric field at 60 m changes sign and it remains negative as the rocket continues its ascent. This effect has been observed in some experiments as can be seen from Table 1 [*Nakamura et al.*, 1987; *Willett et al.*, 1999] and only at close distances of 30 m and 40 m, respectively.

4. Analysis of Field Reduction Effect

Several parameters determine the rate of field reduction at ground level, including the space charge density distribution and the radius of the wire with its surrounding corona sheath. In the previous models presented in section 1, only one variable was analyzed at a time and the other remained constant. In our analytical model, it is possible to study the influence of these two parameters simultaneously. In this section, we discuss the influence of the corona sheath radius, and in section 5.1, we analyze the influence of multiple parameters within the same model.

We have calculated the variation of the electric field at a distance $D = 60$ m for three different radii r of the conductor representing the wire and the corona sheath around it. We have used two methods for the analysis: CSM and FEM. Figure 5 shows the vertical electric field E_z at ground level ($H = 0$) as a function of the altitude of the launched rocket until its height reaches $h = 300$ m.

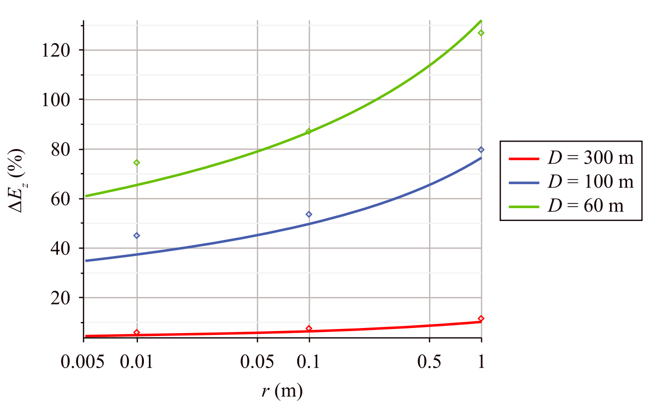

Figure 6. Reduction of the electric field at ground level at several distances from the launch site calculated for different radii of the conductor. The solid lines represent the results obtained with the analytical CSM expressions, while the dots represent the numerical FEM results.

The decrease in the electric field at ground level, at a given distance from the wire, and for a given conductor radius r representing the wire with its corona sheath, can be expressed as

$$\Delta E_z, \% = \frac{E_0 - E_z(h = 300\ \text{m})}{E_0} \cdot 100 \quad (15)$$

in which E_0 is the background electric field and E_z ($h = 300$ m) is the vertical electric field determined when the rocket reaches a height of 300 m above ground.

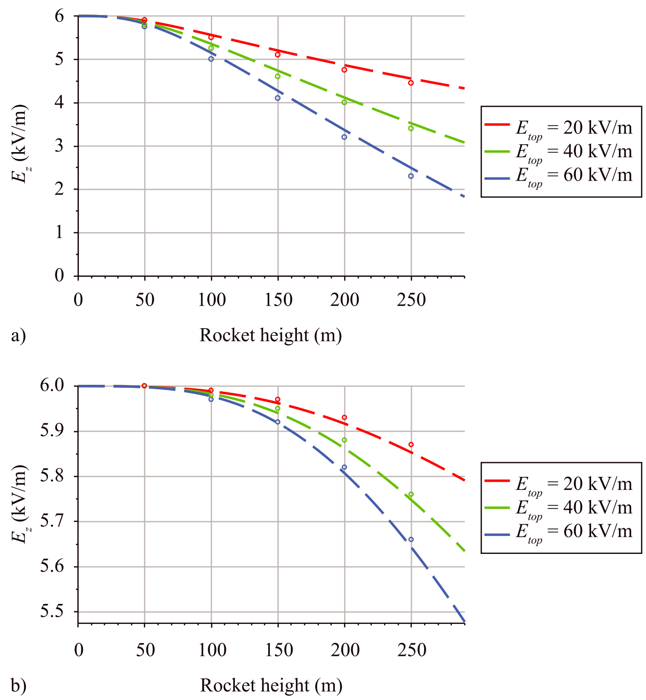

Figure 7. Electric field at ground level during the rocket ascent at distances of (a) 60 m and (b) 300 m. The dashed lines correspond to the analytical results from our model. The dots correspond to the FEM model from *Biagi et al.* [2011].

Figure 6 presents the reduction in the electric field defined by equation (15) as a function of the conductor radius r representing the wire and its corona sheath. The values of E_z shown in Figure 6 are calculated using the CSM-based expressions and FEM at the moment when the rocket reaches an altitude $h = 300$ m and prior to the lightning initiation at distances D varying from 30 to 300 m.

5. Comparison With Previous Models

5.1. Comparison With the Model of *Biagi et al.* [2011]

In the model of *Biagi et al.* [2011], the triggering wire itself was modeled as an infinitely thin conductor. An exponentially decaying space charge density profile versus altitude was used, given in equation (16) below, which depends on the rate of charge decrease with height d and on the electric field magnitude far above the space charge layer E_{top}:

$$\rho = \varepsilon_0 \frac{\exp(-z/d)}{d}\left(E_{\text{top}} - E_0\right) \tag{16}$$

The finite element method was used by Biagi et al. to calculate the variation of the electric field at ground level during the rocket ascent, and the following parameters were used:

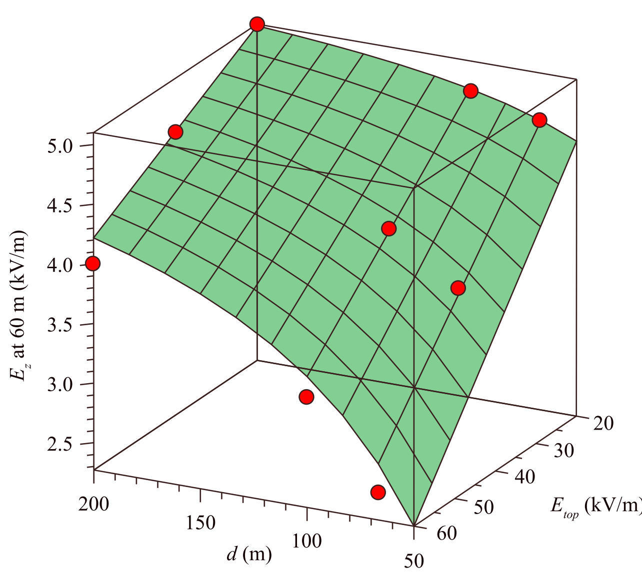

Figure 8. Electric field at ground level E_z at a distance of 60 m from the launching site at the moment when the rocket is at 200 m altitude. The green surface represents the solution of the analytical model; the red dots represent the results obtained by *Biagi et al.* [2011].

$$\begin{aligned} E_0 &= 6 \ \text{kV/m}, \\ E_{\text{top}} &= 20, 40, 60 \ \text{kV/m}, \\ d &= 100 \ \text{m} \end{aligned}$$

We have introduced these parameters into equation (16) and have performed equations (6)–(12) for all of them, assuming a wire radius of 0.1 m. A comparison between our analytical approach and the model of *Biagi et al.* [2011] is shown in Figure 7. It can be seen from this figure that analysis based on the charge simulation method can be used for this study and that it is in good agreement with the model of *Biagi et al.* [2011].

As it was mentioned earlier, the model of *Biagi et al.* [2011] allows the analysis of the electric field reduction only due to the space charge layer above the ground, neglecting the presence of the corona sheath around the wire. Essentially, two variables were used in

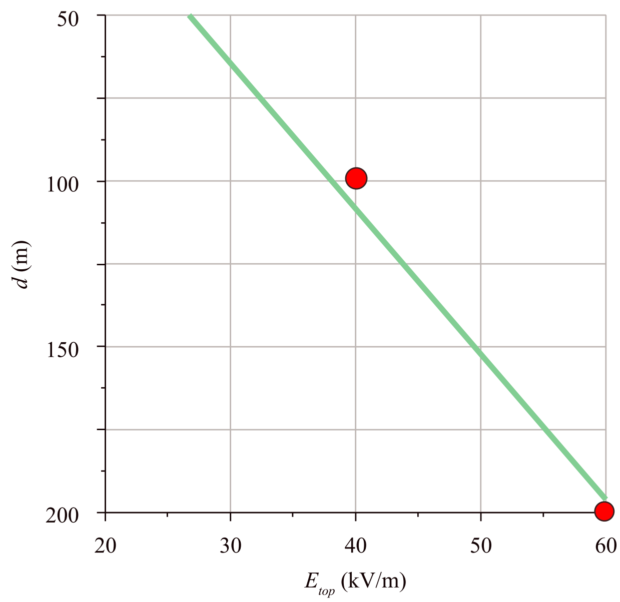

Figure 9. Same as Figure 8; slice is shown for $E_z = 4.2$ kV/m.

their model: the rate of electric field reduction d and the electric field far above the ground E_{top} (E_∞ in their paper). In their paper, Biagi et al. analyzed nine pairs of values for these parameters, and they calculated their influence on the field reduction. Their results are shown in Figure 8, together with the results obtained using our analytical model. From that figure, it is clear that a given value of electric field can be obtained from a multitude of points with coordinates (E_{top}, d, E_z) that belong to the surface representing the solution.

Therefore, the solution is not unique as illustrated in Figure 9, where a slice of the surface from Figure 8 is represented for $E_z = 4.2$ kV/m. From Figure 7 in *Biagi et al.* [2011], it follows that, when a rocket is at 200 m altitude, two sets of parameters E_{top} and d would result in a field reduction down to 4.2 kV/m. These sets of parameters are ($E_{top} = 40$ kV/m, $d = 100$ m) and ($E_{top} = 60$ kV/m, $d = 200$ m). However, in reality, there are many other combinations of these parameters resulting in the same field reduction as illustrated in Figure 9.

Our analytical model allows the study of the influence of both parameters of the model proposed by *Biagi et al.* [2011], E_{top} and d and, at the same time, the study of the influence of the corona sheath radius r, therefore increasing the number of variables to three. The solution to equation (12) for $E_z = 4.2$ kV/m is illustrated in Figure 10 and can be defined as

$$
\begin{aligned}
E_{top} &= f(r, d) \text{ when} \\
E_z &= 4.2 \text{ kV/m}, \\
h &= 200 \text{ m}, \quad H = 0 \text{ m}, \\
R &= 60 \text{ m}
\end{aligned}
\tag{17}
$$

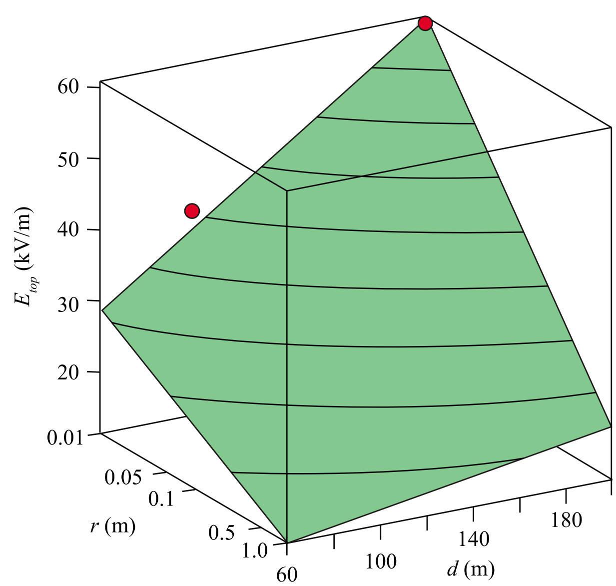

Figure 10. Illustration of equation (16) representing the possible values of the electric field far above the ground E_{top}, the corona sheath radius r, and the parameter d for the observed value of 4.2 kV/m for the electric field at ground level at the distance of 60 m from the launching site at the moment when the rocket is at 200 m altitude. The green surface represents the result derived from the analytical model; the red dot was taken from the results obtained by *Biagi et al.* [2011].

The result shown in Figure 9 can be viewed as a slice of Figure 10 for the case when the corona sheath has a fixed radius of 0.01 m. A full analysis performed with the developed analytical CSM model shows that the same reduction in the electric field at ground level can be explained not only by the different parameters of the space charge layer but also by different values of the corona sheath radius. Without additional experimental data on the space charge layer parameters, it is impossible to attribute the observed field reduction only to the corona sheath radius or to the space charge layer parameters.

5.2. Comparison With the Model of *Baba and Rakov* [2011]

In the paper by *Baba and Rakov* [2011], another model of space charge layer

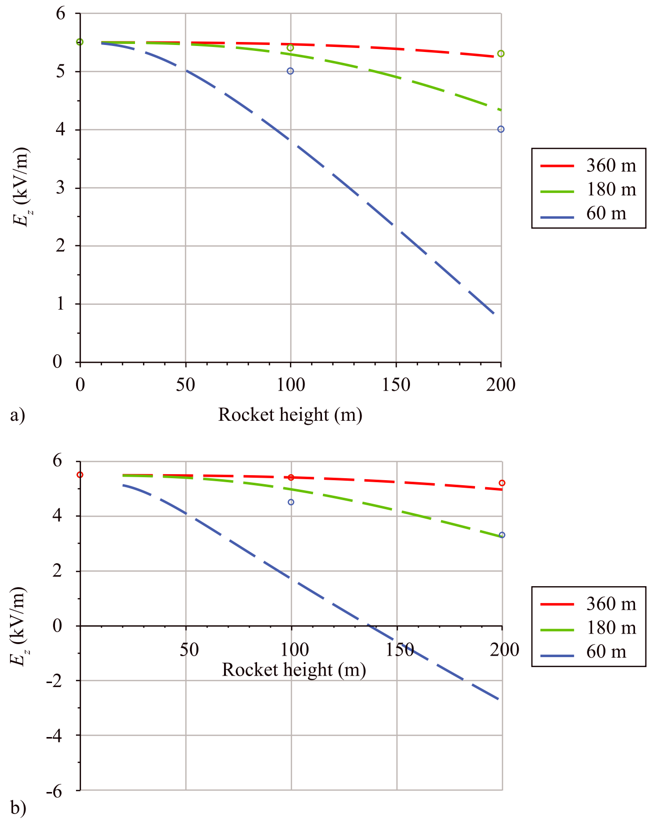

Figure 11. Electric field at ground level during the rocket ascent for the wire radius of (a) 2 m and (b) 8 m. The dashed lines correspond to the analytical results, and the dots correspond to the FDTD model from *Baba and Rakov* [2011]. Several distances from the wire were considered: 60, 180, and 360 m.

has been proposed. The ground space charge layer is modeled with concentric conductive tubes, whereas in both our approach and in *Biagi et al.* [2011], the variation of space charge density versus altitude is integrated directly into the model.

To perform the comparison with our analytical simulation, we have approximated the computed distribution of the background electric field shown in Figure 11a in *Baba and Rakov* [2011] as

$$E = 712.3 \cdot \exp\left(\frac{-z}{45.11}\right) - 758 \cdot \exp\left(\frac{-z}{48.02}\right) + 51.2 \qquad (18)$$

Consequently,

$$\rho = -1.3974 \cdot 10^{-10} \cdot e^{-0.0222z} + 1.3969 \cdot 10^{-10} \cdot e^{-0.0208z} \qquad (19)$$

These expressions have been then used to perform equations (7)–(12). In *Baba and Rakov* [2011], the influence of the corona sheath radius was analyzed, and it was found to play a major role in the electric field reduction. Cylinder radii of 2 and 8 m were introduced in our model to compare the results of both simulations. The electric field reduction is shown in Figure 11 at several distances from the rocket launching site.

From Figure 11, it can be concluded that the model of *Baba and Rakov* [2011] correctly estimates the values of the electric field reduction at far distances from the rocket-triggered lightning wire. At close distances, however, a significant difference between the results of the two models can be observed. This can be explained essentially by the presence of the concentric conductive tubes used in their model to represent the space charge layer. Such configuration influences significantly the electric field at ground level. In *Baba and Rakov* [2011] it was shown that only a very large corona sheath (meters or tens of meters in diameter) can produce significant electric field reduction. However, from our analytical calculations, we can see that even a corona sheath of relatively small radius can result in the inversion of the field polarity observed experimentally.

6. Conclusions

In this paper, we have analyzed the electric field at ground level during the first stage of triggered lightning experiments, i.e., during the rocket ascent and prior to the lightning initiation. At distances of some tens of meters from the triggering wire, the electric field decreases significantly, while at distances of several hundred meters, there is only a very small decrease of the electric field. Two effects determine the level of the electric field reduction: the corona layer at ground level and the corona sheath around the triggering wire.

We have developed an analytical solution based on the charge simulation method to study the phenomenon. A ground space charge layer and a corona sheath around the rocket-triggered lightning wire were included in the simulation. It was shown that, depending on the charge distribution, the change of the sign of the electric field is correctly predicted by our model. Moreover, the proposed analytical solution is faster, and it allows studying the influence of several parameters simultaneously, i.e., the radius of the corona sheath and the space charge layer parameters. The described analytical model allows the estimation of the corona sheath radius if the parameters of the space charge layer are known from experiment. The model does not suffer from limiting assumptions which were made in previously available models, and the predicted electric fields are consistent with experimental data.

The developed model can also be used to study the electric field reduction around other symmetrical structures, such as antenna masts and tall buildings. Future developments include the introduction of a dynamic variation of the corona radius as a function of the electric field.

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