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Incentive and Welfare Effects of an Idiosyncratic Interest Rate

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Abstract

We provide a microeconomic analysis of saving behavior with idiosyncratic interest rates, for which a transfer rate determines the spread between the interest rate in the good and the bad state. A positive (negative) transfer rate reinforces the decision to save for correlation loving (averse) individual. We also define a critical transfer rate that induces agents to save who would not do so otherwise, and determine its comparative statics. We find sufficient conditions for a larger transfer rate to increase savings and show that an idiosyncratic interest rate is welfare increasing for individuals with non-trivial correlation attitudes. The welfare benefits of idiosyncratic interest rates are related to their insurance effects.

Keywords: correlation attitude · multivariate risk · risk preferences · saving · idiosyncratic interest rate

JEL-Classification: D14 · D15 · D60 · D81 · E21 · E43

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1 Introduction

Saving is one way to prepare for the future. The motives for saving are numerous and include in particular consumption smoothing over the life cycle (e.g., Ando and Modigliani, 1963; Modigliani, 1986), intergenerational transfers in the form of bequests (e.g., Kotlikoff and Summers, 1981) and anticipation of future risks (e.g., Kimball, 1990; Hubbard et al., 1995). Our paper contributes to this last strand of literature on risk-induced saving behavior.

Leland (1968) and Sandmo (1970) were the first to point out that income risk creates a precautionary demand for saving. Kimball (1990) called such behavior "prudent" and showed that it is equivalent to a positive third derivative of utility in the expected utility model. These works paved the way to study the effect of risks other than income risk on optimal saving behavior. In that respect, non-financial risks, and in particular health risks, have been shown to affect individual saving decisions (see e.g. Jappelli et al. (2007), Nocetti and Smith (2010), Denuit et al. (2011)). Interest rate risk has also been addressed as a driver of saving decisions. Indeed, Eeckhoudt and Schlesinger (2008) and Chiu et al. (2012) study the effect of interest rate risk on optimal saving and determine sufficient conditions on preferences for an increase in interest rate risk to raise savings. Jouini et al. (2013), among other things, define necessary and sufficient conditions on preferences for Nth-degree risk changes in interest rate risk to increase saving.

The above literature on interest rate risk consider that such risk is not specific to the individual and represents an aggregate shock to the individual. However, in many circumstances, at the time the saving decision is made, the interest rate is uncertain but in a way that is specific to the individual who decides about her consumption, i.e. the interest rate is idiosyncratic to the individual. Numerous financial tools can illustrate an idiosyncratic interest rate, the most obvious being conventional insurance contracts. Indeed, an insurance contract only returns something if the bad state of the world occurs for an individual. Other forms of an idiosyncratic interest rate can be illustrated with the concept of health savings accounts (HSAs)[1], individual retirement accounts, standard annuities or enhanced annuities. In the U.S., for example, savings via an HSA are tax-exempt if utilized for medical expenditures, implying a higher realized return on each before-tax dollar saved through the HSA in case of sickness.[2] For retirement accounts, individuals can only access their savings if they reach retirement age. In case of premature death, the accumulated savings can be subject to taxation, reducing the effective return, or they might be completely forfeited in the absence of heirs.

[1] Specific forms of HSAs can be found in the U.S., South Africa, Canada, Singapore and China (see Peter et al., 2016). In the U.S., HSAs were introduced in 2004 and have been attracting a growing number of individuals since then covering 20.2 million Americans as of January 2016 (see the AHIP 2016 HSA census available at `https://www.ahip.org/`).

[2] If savings are withdrawn for non-medical consumption, individuals have to repay the tax advantage and an additional penalty resulting in a lower return on saving. In this sense, the realized interest rate on saving depends on the individual's health state.

Standard annuities offer a higher return to individuals when they live long than when they die young. So-called enhanced annuities (see Steinorth, 2012) provide an even higher level of income than standard annuities for qualifying purchasers. Qualifying conditions are typically related to impaired health so that from the perspective of an individual in the accumulation phase the effective interest rate on saving depends on her/his future health circumstances. Other forms of idiosyncratic interest rates include income-contingent loans as well as shocks to human capital. Idiosyncratic interest rate is therefore a way to identify these commonplace practices together because they serve economically-similar functions, i.e. a tool which return is uncertain and depends on realizations specific to the individual.

The aim of this paper is to develop a simple model to investigate the incentives and welfare effects of such idiosyncratic interest rates at the individual level. Specifically, we use a standard two-period consumption-saving model with time-separable utility. To model an interest rate whose level is specific to the individual, i.e. idiosyncratic, we introduce two states of nature and require for the expected interest rate to coincide with the prevailing market interest rate for reasons of comparability. This allows us to rewrite an idiosyncratic interest rate with the help of a transfer rate, which is defined as the spread between the interest rate in the good and the bad state. Hence, we model an idiosyncratic interest rate as a mean-preserving spread of the market interest rate (see Rothschild and Stiglitz, 1970), contributing to the literature on risk-induced saving behavior. The idiosyncratic interest rate in our model depends on certain states of nature that are related to the individual's endowment which is considered as non-financial to allow for further generality. As a result, interest rate risk and risk over the individual's non-financial endowment are correlated in our model, which distinguishes our work from the previous literature on interest rate risk.

We address three specific items in this paper. First, we study the conditions for which the individual decides to save anything at all or not in the presence of an idiosyncratic interest rate. We call such analysis the incentive effects of an idiosyncratic interest rate at the extensive margin. Second, we investigate the optimal level of saving under an idiosyncratic interest rate and analyze how this optimal level of saving reacts to a change in the transfer rate. We name such analysis the incentive effects of an idiosyncratic interest rate at the intensive margin. Third, we investigate the impact of an idiosyncratic interest rate on individuals welfare and relate that impact on the insurance effects of an idiosyncratic interest rate.

Our results show that our proposed system of an idiosyncratic interest rate reinforces the decision to save if and only if the transfer rate is of the same sign as the consumer's correlation attitude (see Epstein and Tanny, 1980). We derive a critical transfer rate above which consumers save who would not do so under the market interest rate depending on their correlation attitudes. We provide comparative statics of this critical transfer rate and uncover the intuition for the associated trade-offs. In a further step, we show that measures of partial prudence in wealth and partial cross-prudence in the non-financial attribute allow to sign the effects of a change in the transfer rate on the optimal level of saving. Finally, we show that an idiosyncratic interest rate increases consumer welfare as long as individuals are not correlation neutral. This result is related to the insurance mechanism of an idiosyncratic interest rate by discussing how and when in the presence of insurance an idiosyncratic interest still raises consumer welfare.

The paper is organized as follows. In the next section we introduce the benchmark model of an idiosyncratic interest rate and show how it modifies the individual's saving trade-off. Section 3 defines the critical transfer rate that induces individuals to save who would not do so under the market interest rate, and studies its comparative statics. In Section 4, we investigate how the optimal level of saving reacts to changes in the idiosyncratic interest rate. In Section 5, we discuss the welfare consequences of an idiosyncratic interest rate and study the optimal transfer rate. A final section concludes.

2 The benchmark model

We consider a simple two-period model and an individual whose preferences are characterized by bivariate vNM utility functions $u(w, H)$ and $v(w, H)$ in period one and two, where w denotes income and H a non-financial variable (e.g., health, human capital, or any other exogenous shock). Throughout the analysis we assume the level of the non-financial variable to be certain in the first period, which is why we suppress it to simplify notation. We denote by $v^{(i,j)}$ the (i, j)th cross derivative of v with respect to its first and second argument,

$$v^{(i,j)}(w, H) = \frac{\partial^{i+j} v(w, H)}{\partial w^i \partial H^j},\ i, j \geq 0. \tag{1}$$

Whenever $j = 0$ or $i = 0$, we obtain unidirectional derivatives with respect to only w or only H. We make standard assumptions on individual preferences: u is strictly increasing and concave, $u' > 0$ and $u'' < 0$, and v is strictly increasing and concave in each argument, $v^{(1,0)} > 0$, $v^{(0,1)} > 0, v^{(2,0)} < 0$ and $v^{(0,2)} < 0$. In other words, the individual's preferences are non-satiated and risk-averse with respect to each argument.

For now, we impose no restriction on the cross derivative $v^{(1,1)}$. In the terminology of Epstein and Tanny (1980), the individual is said to be correlation loving (neutral, averse) if $v^{(1,1)} > 0$ $(= 0,\ < 0)$.[3] For such an individual, the marginal utility of wealth increases (remains constant, decreases) in the non-financial variable. If the non-financial variable is health, existing results suggest that marginal utility of wealth is increasing or constant in health status for severe injuries and decreasing in health status for minor injuries (see Viscusi and Evans, 1990; Evans and Viscusi, 1991; Sloan et al., 1998; Carthy et al., 1998). Recently, Finkelstein et al. (2013) estimate that a one-standard deviation increase in the number of chronic diseases leads to a 10-25% decrease in marginal utility of consumption, consistent with correlation loving. Ebert and van de Kuilen (2017) instead find experimental evidence in favor

[3] Richard (1975) speaks of multivariate risk aversion, multivariate risk neutrality and multivariate risk seeking instead, a terminology which is less common nowadays.

of correlation aversion for the economic domains of time preferences, social preferences and waiting time. In other words, the non-financial variable can be a complement or a substitute for wealth in the Edgeworth sense depending on whether preferences are correlation loving or averse. The notion of correlation attitude has recently regained attention in the economics literature (e.g., Eeckhoudt et al., 2007; Denuit et al., 2010; Crainich et al., 2016).

The individual has a certain income of w_1 in the first period. In the second period, he has a certain income of w_2 and faces a binary non-financial risk taking value H_g with probability $(1-p)$ and H_b with probability p such that $H_g > H_b$. Subscripts g and b are shorthand for the good and the bad state of nature. Our specification with a non-financial risk of loss corresponds to Cook and Graham's (1977) setting who study insurance demand for irreplaceable commodities, and we will point out some commonalities when discussing welfare effects in Section 5. Expected utility in the second period is discounted by the utility discount factor $\beta \leq 1$ to allow for impatience. The individual decides how much to save in the first period. As a benchmark, we first assume a certain interest rate r. The individual maximizes his expected lifetime utility according to the following objective function:

$$\max_s \Big\{ u(w_1 - s) + \beta \left[pv(w_2 + (1+r)s, H_b) + (1-p)v(w_2 + (1+r)s, H_g) \right] \Big\}. \tag{2}$$

A purely monetary loss is a special case of a bivariate utility function with $v(w, H)$ being additive, $v(w, H) = v(w + H)$. In this case, risk aversion over income implies that marginal utility of wealth is decreasing in H. As a result, a purely monetary loss is an application of the bivariate model with $v^{(1,1)} < 0$. The first-order condition for the optimal level of saving, denoted by s_0, is given by

$$\begin{aligned} -u'(w_1 - s_0) + \beta(1+r) \Big[& pv^{(1,0)}(w_2 + (1+r)s_0, H_b) \\ & + (1-p)v^{(1,0)}(w_2 + (1+r)s_0, H_g) \Big] \; = \; 0. \end{aligned} \tag{3}$$

The second-order condition holds because the objective function is globally concave in s due to the concavity of u and v in their first argument.

Consider now that the return on saving is no longer certain at the time the individual decides about consumption. In other words, we now study a risky interest rate. Existing research has mainly focused on cases where interest rate risk represents an aggregate shock to the individual (e.g., Eeckhoudt and Schlesinger, 2008; Chiu et al., 2012; Jouini et al., 2013). We in turn analyze an *idiosyncratic interest rate*, whose levels depend on the individual's endowment. In terms of our model, we therefore assume an interest rate of r_b in the bad state ($H = H_b$) and an interest rate of r_g in the good state ($H = H_g$). We use the term *transfer rate* for the difference or spread between the two levels of the interest rate, $\Delta = r_g - r_b$. For reasons of comparability, we assume that the expected value of the idiosyncratic interest rate coincides with the prevailing market interest rate, $pr_b + (1-p)r_g = r$. Under this assumption, an idiosyncratic interest rate is a mean-preserving spread of the prevailing market interest rate in the sense of Rothschild and Stiglitz (1970).

This comparability assumption also allows us to rewrite the two levels of the idiosyncratic interest rate as $r_b = r - (1-p)\Delta$ and $r_g = r + p\Delta$. If $\Delta = 0$, the interest rate is the same in both states of nature ($r_b = r_g = r$), and the agent's decision problem is the one in Eq. (2). But if $\Delta > 0$ (< 0), then $r_b < r < r_g$ ($r_b > r > r_g$), and there is perfect positive (negative) correlation between the interest rate and the individual's non-financial endowment. In this case, the interest rate is higher (lower) in the good state of nature than the bad state of nature. If we require that $r_b \geq -1$ and $r_g \geq -1$, then the transfer rate is contained in the compact interval $[-(1+r)/p, (1+r)/(1-p)]$.

We illustrate an existing case of saving under an idiosyncratic interest rate by the concept of HSAs, for which the return on saving depends on the individual's health state. If the individual stays healthy, savings are forfeited corresponding to $r_g = -1$ in our notation.[4] From that we can easily infer the value of Δ that equates the expected idiosyncratic interest rate to the market interest rate. It is given by $\Delta = -(1+r)/p < 0$, and the interest rate when sick is then $r_b = (r + (1-p))/p > r$. This uncovers the rationale behind HSAs, which is to allow people to benefit from a higher return on savings when they become sick.

Conventional insurance can also be illustrated by an idiosyncratic interest rate. Consider for simplicity the case of a purely monetary loss with $H_b = -L$ and $H_g = 0$. Then wealth in the bad state of the world writes as $w_2 + s - s(1-p)\Delta - L$ and wealth in the good state of the world as $w_2 + s + sp\Delta$. If $s = pL$ and $\Delta = -1/p$, wealth in both states of the world becomes w_2. This indeed corresponds to conventional insurance, i.e. to the case where an individual confronted with a loss L with probability p pays an upfront actuarial premium, $s = pL$, to have the loss fully reimbursed. Naturally, the case $\Delta = 0$ corresponds to ordinary savings, i.e., saving to smooth consumption or to redistribute across time. Other values of Δ than $\Delta = -1/p$ and $\Delta = 0$ represent a mix between insurance and saving. Hence, larger, respectively lower, values of the transfer rate in absolute terms imply that the idiosyncratic interest rate serves an insurance function, respectively a saving function, to a larger degree.

With an idiosyncratic interest rate, the individual's optimal level of saving solves the following maximization problem:

$$\max_s \left\{ u(w_1 - s) + \beta \left[pv(w_2 + (1+r_b)s, H_b) + (1-p)v(w_2 + (1+r_g)s, H_g) \right] \right\}. \qquad (4)$$

We denote by $U(s; \Delta)$ the agent's expected intertemporal consumption utility as a function of saving and the transfer rate. We use subscripts for derivatives with respect to model

[4] This is the case, for example, for flexible spending accounts where savings do not roll over and are therefore lost if they cannot be spent on medical consumption.

parameters. The optimal level of saving, denoted by s^*, is then determined implicitly by the first-order condition $U_s(s^*;\Delta) = 0$, which is given by

$$\begin{aligned} -u'(w_1 - s^*) + \beta(1+r)\left[pv^{(1,0)}(B^*) + (1-p)v^{(1,0)}(G^*)\right] \\ +\beta p(1-p)\Delta\left[v^{(1,0)}(G^*) - v^{(1,0)}(B^*)\right] \;=\; 0. \end{aligned} \tag{5}$$

B^* and G^* abbreviate the combination of wealth at the optimal level of saving s^* and the non-financial variable in the bad and the good state of nature, respectively. That is, $B^* = (w_2 + (1+r_b)s^*, H_b)$ and $G^* = (w_2 + (1+r_g)s^*, H_g)$. Under our assumptions on u and v, the individual's intertemporal objective function is globally concave in s for any transfer rate. In the remainder of this paper, we will analyze the incentive effects of an idiosyncratic interest rate at the extensive and the intensive margin as well as its effects on individual welfare.

3 Incentive effects at the extensive margin

We first investigate the incentive effects of an idiosyncratic interest rate at the extensive margin. As explained in the introduction, we refer to the extensive margin when it comes to the individual's decision whether to save anything at all or not. The individual finds it optimal to engage in saving ($s^* > 0$) if and only if $U_s(0;\Delta) > 0$, that is,

$$\begin{aligned} -u'(w_1) + \beta(1+r)\left[pv^{(1,0)}(B_0) + (1-p)v^{(1,0)}(G_0)\right] \\ +\beta p(1-p)\Delta\left[v^{(1,0)}(G_0) - v^{(1,0)}(B_0)\right] \;>\; 0, \end{aligned} \tag{6}$$

where $B_0 = (w_2, H_b)$ and $G_0 = (w_2, H_g)$ are shorthand for the outcomes if $s = 0$. The equivalence between condition (6) and the optimality of a positive amount of saving follows from the concavity of the objective function. If the individual finds it optimal to save a positive amount, any amount less than that would be suboptimally low and *a fortiori* no savings at all so that (6) is satisfied. Likewise, if (6) holds, the optimal level of saving must be to the right of $s = 0$ per concavity of the objective function. With this simple argument in mind, we can use condition (6) as a litmus test for whether an individual can be classified as a saver or a borrower.

Condition (6) also shows that the transfer rate interacts with the individual's correlation attitude. The last term in (6) is positive if $\text{sgn}(\Delta) = \text{sgn}\left(v^{(1,1)}\right)$ and negative if $\text{sgn}(\Delta) = -\text{sgn}\left(v^{(1,1)}\right)$. So whenever the individual is correlation neutral ($v^{(1,1)} = 0$), the transfer rate is irrelevant at the extensive margin. In all other cases, we can rearrange (6) to find the value of the transfer rate that separates savers from borrowers. This critical level is given by

$$\Delta_{crit} = \frac{u'(w_1) - \beta(1+r)\left[pv^{(1,0)}(B_0) + (1-p)v^{(1,0)}(G_0)\right]}{\beta p(1-p)\left[v^{(1,0)}(G_0) - v^{(1,0)}(B_0),\right]}, \tag{7}$$

and a simple rearrangement of condition (6) yields the following result.

Proposition 1. *Consider an idiosyncratic interest rate with transfer rate* Δ*.*

(i) A correlation lover saves (borrows) if $\Delta > (<)\Delta_{crit}$*.*

(ii) A correlation averter saves (borrows) if $\Delta < (>)\Delta_{crit}$*.*

We inspect Eq. (7) more closely to develop some intuition for this result. Δ_{crit} compares the individual's saving decision under the market interest rate to the effect of the transfer rate on saving incentives. The numerator is positive if the individual borrows under the market interest rate and negative if he saves under the market interest rate. The denominator depends on correlation attitude. Under correlation loving, the marginal utility of wealth is higher in the good state than in the bad state of nature, while the reverse is true under correlation aversion. So the denominator of Δ_{crit} is positive under correlation loving and negative under correlation aversion. Hence, for a correlation lover who does not save under the market interest rate (i.e., $s_0 \leq 0$), Δ_{crit} is non-negative and an idiosyncratic interest rate needs to have a sufficiently high transfer rate to provide sufficient incentives for the individual to start saving. If a correlation lover already saves under the market interest rate (i.e., $s_0 > 0$), then Δ_{crit} is negative and the individual would still find it optimal to save under any idiosyncratic interest rate with a non-negative transfer rate. Similar reasoning applies to correlation averters.

We can use these arguments to determine the incentive effects of an idiosyncratic interest rate at the extensive margin in an economy of heterogeneous agents. Assume that agents differ in their financial and non-financial endowments, risk and time preferences, etc. but assume that everybody is correlation loving. Such heterogeneity will generate a distribution over Δ_{crit} with one value for each individual. Those who already save under the market interest rate will have a negative Δ_{crit}, those who borrow under the market interest rate will have a positive Δ_{crit}. If we now introduce an idiosyncratic interest rate and raise Δ continuously, then Proposition 1 predicts that those individuals with a negative Δ_{crit} will still save and those with a positive Δ_{crit} will start to save as soon as Δ is large enough. The reverse argument holds for an economy of correlation averters. Therefore, an idiosyncratic interest rate with a positive (negative) transfer rate incentivizes saving behavior at the extensive margin in an economy of correlation lovers (averters).

In a next step we provide comparative statics of the critical transfer rate. An increase in the absolute value of the transfer rate constitutes a mean-preserving spread of the idiosyncratic interest rate. That is, more extreme levels of the critical transfer rate further away from zero indicate that a stronger distortion of the market interest rate is required to turn a borrower into a saver. By differentiating Δ_{crit} with respect to the exogenous parameters of our model, we identify factors that correlate with a stronger need to provide saving incentives.

An individual who does not save under the market interest rate is characterized by $U_s(0;0) \leq 0$. This means that his marginal rate of substitution of first-period income for second-period income, $\mu_{1,2}$, does not exceed $1/(1+r)$.[5] Indeed, for such an individual, a marginal reduction of first-period income, $\mathrm{d}w_1 < 0$, needs to be compensated by an increase in second-period income of $\mathrm{d}w_2 = (-\mathrm{d}w_1)/\mu_{1,2}$ to keep his expected lifetime utility constant. If $\mu_{1,2}$ is bounded by $1/(1+r)$, such a change can only be effectuated if $-\mathrm{d}w_2/\mathrm{d}w_1 \geq (1+r)$. As a result, the available interest rate on the market is not sufficiently attractive for the individual to engage in saving.

We are now in a position to state comparative statics of Δ_{crit}. To keep the presentation simple, we only cover the case of a correlation lover and relegate the technical proof to Appendix A.1. The results for a correlation-averse agent are quite similar.

Proposition 2. *Assume a correlation lover who does not save under the market interest rate. Then, the critical transfer rate to induce saving is:*

a) decreasing in the utility discount factor, the market interest rate, and first-period income,

b) increasing in second-period income if $v^{(2,1)} \leq 0$, or if $v^{(2,1)} > 0$ and $\mu^g_{1,2} \geq 1/(1+r)$,[6]

c) decreasing in the high non-financial endowment,

d) increasing in the low non-financial endowment if and only if $\mu^g_{1,2} < 1/(1+r)$,

e) increasing in the probability of loss if $p \geq 1/2$, or if $p < 1/2$ and $\mu^g_{1,2} \geq 1/(1+r)$.

We will explain these effects by providing some economic intuition. For *a)*, we note that a higher utility discount factor, a higher market interest rate, and increased first-period income all have the effect to reinforce the baseline incentive to save under the market interest rate and/or to strengthen the incentive effect of a given transfer rate. Therefore, the critical transfer rate decreases because it takes less to "persuade" the individual to engage in saving. Result *c)* follows a similar logic. To understand *b)*, notice that higher second-period income lowers the baseline incentive to save. If the individual is not cross-prudent in the non-financial variable ($v^{(2,1)} \leq 0$), then also the incentive effect is lower for higher second-period income because the difference between marginal utility of wealth is decreasing in second-period income. Under cross-prudence in the non-financial variable ($v^{(2,1)} > 0$), the effect of higher

[5] The technical definition is given by

$$\mu_{1,2} = \frac{U_{w_2}(0;0)}{U_{w_1}(0;0)} = \frac{\beta\left[pv^{(1,0)}(w_2,H_b) + (1-p)v^{(1,0)}(w_2,H_g)\right]}{u'(w_1)}, \tag{8}$$

which does not exceed $1/(1+r)$ if and only if $U_s(0;0) \leq 0$.

[6] $\mu^g_{1,2}$ and $\mu^b_{1,2}$ are the individual's marginal rate of substitution of first-period income for second-period income if the the non-financial variable is H_g or H_b with certainty, i.e.,

$$\mu^g_{1,2} = \frac{\beta v^{(1,0)}(w_2,H_g)}{u'(w_1)} \quad \text{and} \quad \mu^b_{1,2} = \frac{\beta v^{(1,0)}(w_2,H_b)}{u'(w_1)}. \tag{9}$$

second-period income on the baseline incentive to save is still negative but the effect of higher second-period income on the additional incentive provided by the idiosyncratic interest rate is now positive. In this case, a sufficient condition for the first effect to dominate the second one is given by $\mu^g_{1,2} \geq 1/(1+r)$. To explain *d)*, we remark that the low level of the non-financial variable entails a trade-off because it is positively associated with the baseline incentive to save but negatively with the incentive effect of the transfer rate. The second effect predominates if $\mu^g_{1,2} < 1/(1+r)$, and the critical transfer rate increases. Statement *e)* follows because an increase in the probability of loss makes the state of nature more likely in which marginal utility of wealth is low. This reduces the baseline incentive to save. The incentive effect of a given transfer rate is related to the probability of loss via the riskiness of the idiosyncratic interest rate, whose variance is given by $p(1-p)\Delta^2$. As a result, the incentive effect increases for $p < 1/2$ and decreases for $p > 1/2$. This explains the sufficient condition in the first case and the unambiguous overall effect in the second case. Notice that under correlation loving $\mu_{1,2} < \mu^g_{1,2}$ so that $U_s(0;0) \geq 0$ or equivalently, $\mu_{1,2} \leq 1/(1+r)$, does *not* impose any restrictions on the size of $\mu^g_{1,2}$ relative to $1/(1+r)$. Overall the comparative statics reveal that the critical transfer rate is jointly determined by individual risk and time preferences, financial and non-financial endowments and the market environment.

4 Incentive effects at the intensive margin

In the previous section we investigated how an idiosyncratic interest rate affects the incentive to engage in saving (i.e., whether $s^* > 0$). Proposition 1 shows that a critical level of the transfer rate separates savers from borrowers. Therefore, an idiosyncratic interest rate can provide stronger saving incentives than the market interest rate at the extensive margin. Proposition 2 identifies correlates of the strength of this incentive based on the exogenous parameters of our model. We now turn to the optimal level of saving and provide conditions under which an idiosyncratic interest rate has clear-cut effects on the amount that individuals save from one period to another.

To do this, we differentiate the first-order condition for optimal saving under an idiosyncratic interest rate with respect to the transfer rate:

$$\begin{aligned} U_{s\Delta}(s^*;\Delta) \; &= \; \beta p(1-p)\Big\{-v^{(1,0)}(B^*) + v^{(1,0)}(G^*) \\ &\quad -(1+r_b)s^* v^{(2,0)}(B^*) + (1+r_g)s^* v^{(2,0)}(G^*)\Big\}. \end{aligned} \tag{10}$$

The four terms in the curly bracket denominate different economic effects that individuals trade off in their saving decision as the transfer rate changes. The first one is negative because a higher transfer rate reduces the return on saving in the bad state of nature (substitution effect). The second one is positive because a higher transfer rate increases the return on saving in the good state of nature (substitution effect). The third one is positive for $s^* > 0$ because a higher transfer rate reduces the individual's wealth in the bad state of nature, which increases the marginal utility of wealth (wealth effect). The fourth one is negative for $s^* > 0$ because a higher transfer rate increases the individual's wealth in the good state of nature, which reduces the marginal utility of wealth (wealth effect). So there is a substitution and a wealth effect in each state of nature, which differ in sign. Moreover, the two substitution effects and the two wealth effects differ in sign across states. Therefore, a change in the transfer rate introduces complex effects into the consumption-saving trade-off.

The risk literature has identified partial risk aversion (see Menezes and Hanson, 1970) as a determinant of the comparative statics of optimal saving when the (certain) interest rate changes (see for example Chiu et al., 2012). It is defined as follows.

Definition 1. The agent's *partial risk aversion in wealth* is $\mathcal{R}(x+y,H) = -y\frac{v^{(2,0)}(x+y,H)}{v^{(1,0)}(x+y,H)}$.

We can then rewrite the curly bracket in Eq. (10) in terms of partial risk aversion as follows:

$$v^{(1,0)}(B^*)\big[\mathcal{R}(w_2+(1+r_b)s^*,H_b)-1\big] - v^{(1,0)}(G^*)\big[\mathcal{R}(w_2+(1+r_g)s^*,H_g)-1\big]. \tag{11}$$

We obtain a clear comparative statics effect if the two square brackets differ in sign. So if partial risk aversion is less than unity in the bad state and larger than unity in the good state, the optimal level of saving increases as the transfer rate increases. The approach based on partial risk aversion is not as satisfactory in case of an idiosyncratic interest rate because a common assumption on partial risk aversion is that it is either uniformly above unity or below unity (e.g., Chiu et al., 2012), in which case we cannot sign (11). Therefore, we introduce two other intensity measures of the agent's risk preferences that will allow us to obtain definitive comparative statics.

Definition 2.

a) The agent's *partial prudence in wealth* is $\mathcal{P}(x+y,H) = -y\frac{v^{(3,0)}(x+y,H)}{v^{(2,0)}(x+y,H)}$.

b) The agent's *partial cross-prudence in the non-financial variable* is $\mathcal{C}(x+y,H) = -y\frac{v^{(2,1)}(x+y,H)}{v^{(1,1)}(x+y,H)}$.

We decompose possible values of the transfer rate into intermediate levels between 0 and Δ_{crit}, that is, $I = (\min\{0,\Delta_{crit}\},\max\{0,\Delta_{crit}\})$, and extreme levels that are not between 0 and Δ_{crit}, that is, $J = [-(1+r)/p, \min\{0,\Delta_{crit}\}) \cup (\max\{0,\Delta_{crit}\},(1+r)/(1-p)]$.[7] Based on this decomposition, we can formulate the following proposition.

Proposition 3. *Consider an idiosyncratic interest rate with transfer rate Δ and assume that the agent's partial cross-prudence in the non-financial variable is bounded by unity. If $\Delta \in I$ and the agent's partial prudence in wealth is bounded by 2, or if $\Delta \in J$ and the agent's partial prudence in wealth exceeds 2, then:*

[7] If $\Delta_{crit} < -(1+r)/p$ or $\Delta_{crit} > (1+r)/(1-p)$, then J will be a single interval. In all other cases, it consists of two intervals.

(i) Correlation lovers will save more after a marginal increase of the transfer rate.

(ii) Correlation averters will save more after a marginal decrease of the transfer rate.

The proof is given in Appendix A.2. To dissect these conditions, we provide Table 1. Consider a correlation lover; on the diagonal are those situations where he saves due to a sufficiently large positive transfer rate or where he borrows due to a sufficiently small negative transfer rate (i.e., $\Delta \in J$). In this case, partial prudence in wealth above 2 ensures more saving or less borrowing as the transfer rate increases. On the off-diagonal are situations where the individual saves despite a negative transfer rate or where he borrows despite a positive transfer rate (i.e., $\Delta \in I$). In those cases, partial prudence in wealth below 2 ensures more saving or less borrowing as the transfer rate increases. In either scenario, the restriction on partial prudence in wealth needs to be coupled with a condition on partial cross-prudence in the non-financial variable.

$\mathcal{C} < 1$	$\Delta > \Delta_{crit}$	$\Delta < \Delta_{crit}$
$\Delta > 0$	$\mathcal{P} > 2$	$\mathcal{P} < 2$
$\Delta < 0$	$\mathcal{P} < 2$	$\mathcal{P} > 2$

Table 1: Sufficient conditions to sign $\mathrm{d}s^*/\mathrm{d}\Delta$

To develop some intuition, we first formulate the two knife-edge cases that are not covered in Proposition 3 (i.e., $\Delta \in \{0, \Delta_{crit}\}$ as corollaries. They will allow for simpler conditions and help make the role of our different assumptions transparent. We state their proof in Appendix A.3.

Corollary 1 ($\Delta = 0$)**.** *Starting at the market interest rate, if $\Delta_{crit} < 0$ and the agent is cross-prudent in the non-financial variable, or if $\Delta_{crit} > 0$ and the agent is cross-imprudent in the non-financial variable, then:*

(i) A correlation lover will save more after a marginal increase of the transfer rate.

(ii) A correlation averter will save more after a marginal decrease of the transfer rate.

Corollary 2 ($\Delta = \Delta_{crit}$)**.** *Starting at the critical transfer rate, a marginal increase (decrease) of the transfer rate increases saving if the individual is correlation loving (averse).*

Corollary 2 is actually identical to Proposition 1. If the transfer rate is at the critical level, the individual does neither save nor borrow because his endowed intertemporal consumption profile is already optimal. But if $s^* = 0$, the two wealth effects in Eq. (10) disappear and the change in the transfer rate boils down to the comparison between both substitution effects. For $s^* = 0$, wealth levels in the second period do not depend on the state of nature, and the comparison between the two substitution effects depends entirely on the individual's correlation attitude. As we know from Proposition 1, the statement in Corollary 2 does not only hold at the margin but globally for any increase or decrease of the transfer rate.

When starting at the market interest rate (i.e., $\Delta = 0$), we also obtain a simpler result as stated in Corollary 1. The reason is that final wealth levels coincide under a certain interest rate because $r_g = r_b = r$. Then, correlation attitude allows us to compare the two substitution effects whereas cross-prudence or cross-imprudence in the non-financial variable ranks the two wealth effects in Eq. (10). In this case, assumptions on the agent's risk attitudes instead of on the intensity of his risk attitudes suffice to obtain clear results.

In the general case of Proposition 3, we proceed by aggregating the substitution and the wealth effect in each state of nature. If we can then show that the incentive effect in the good state of nature exceeds the incentive effect in the bad state of nature, a marginal increase of the transfer rate will lead to more savings. Each incentive effect depends on the wealth level in that state as well as on the value of the non-financial variable in that state. For correlation lovers, the restriction on partial prudence in wealth ensures that the incentive effect increases when going from the wealth level in the bad state to the wealth level in the good state. Likewise, for correlation lovers the restriction on partial cross-prudence in the non-financial variable ensures that the incentive effect increases when going from the low level of the non-financial variable to its high level. For correlation averters instead, the preference conditions induce the opposite ranking of the incentive effect in the good state versus the bad state of nature. This intuition also reveals why the prudence measure needs to be bounded from below in some cases while it needs to be bounded from above in other cases whereas the cross-prudence measure is always bounded from above. The reason is that it can be the wealth level in the good state or the bad state, which is larger, depending on whether the individual saves or borrows and on the sign of the transfer rate (see Proposition 1), while it is always the value of the non-financial variable in the good state which is larger.

The preference conditions in Proposition 3 may appear complex at first sight, but they are well known in the literature. Chiu et al. (2012) show that partial prudence in wealth exceeds 2 for all $y > 0$ and $x \geq 0$ if and only if $-yv^{(3,0)}(y, H)/v^{(2,0)}(y, H) > 2$ for all $y > 0$, where the latter is relative prudence in wealth. The comparison of relative prudence with 2 often appears in the literature, for example in the comparative statics of the demand for a risky asset with respect to changes in its return distribution (Hadar and Seo, 1990; Choi et al., 2001), to sign the effect of uncertainty on bargaining outcomes (White, 2008), the effect of changes in interest rate risk on optimal saving (Eeckhoudt and Schlesinger, 2008), and in more general saving, portfolio choice and output choice problems under uncertainty (Chiu et al., 2012). Jouini et al. (2013) provide an overarching analysis and identify conditions under which the restriction on relative prudence is also necessary for clear comparative statics.

We point out that the expression $-yv^{(2,1)}(x + y, H)/v^{(1,1)}(x + y, H)$ has not been introduced into the literature yet. Consistent with calling $v^{(2,1)}$ cross-prudence in the non-financial variable (Eeckhoudt et al., 2007), we refer to this coefficient as an intensity measure of partial cross-prudence in the non-financial variable.[8] As in Chiu et al. (2012), this measure exceeds 1 for all $y > 0$ and $x \geq 0$ if and only if $-yv^{(2,1)}(y, H)/v^{(1,1)}(y, H) > 1$ for all $y > 0$, where the latter is an intensity measure of relative cross-prudence in the non-financial variable. The comparison of a prudence index to unity is less prevalent in the literature. Still, two recent papers demonstrate the usefulness of this threshold to determine the impact of inequality and economic convergence on the efficient discount rate (Gollier, 2015) and to explain a decision-maker's attitude to an increase in initial wealth when he faces two interdependent multiplicative risks (Denuit and Rey, 2014).

There is little - if not to say no - empirical guidance to judge how restrictive the conditions in Proposition 3 are. Rey and Rochet (2004) discuss several specifications for bivariate preferences that can help shed some light on this issue.[9] We stress that these conditions are sufficient but not necessary, so among those individuals who do not satisfy them, some will increase saving (or reduce borrowing) and some will react in the opposite direction as the transfer rate changes in a particular direction.

In the following, we will also derive some other comparative statics of saving behavior at the intensive margin. Unlike in case of the transfer rate, many of these results are straightforward. We will use some of them in the next section to determine the welfare effects of an idiosyncratic interest rate.

Remark 1. *The optimal level of saving under an idiosyncratic interest rate is:*

a) increasing in the utility discount factor and first-period income,

b) decreasing in second-period income,

c) always increasing in the market interest rate for borrowers; for savers it is increasing in the market interest rate if partial risk aversion in wealth is less than unity,

c) increasing (decreasing) in the high and the low non-financial endowment for correlation lovers (averters).

The proof is given in Appendix A.5. These effects follow directly from how the respective change affects the marginal benefit or the marginal cost of saving. Result *c)* contains the

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[8] We justify this terminology in Appendix A.4 with the help of the partial prudence premium. See Trautmann and van de Kuilen (2018) for a recent survey of the broader evidence on higher order risk preferences.

[9] Under additive separability ($v^{(1,1)} = 0$), an idiosyncratic interest rate does not affect saving behavior at the extensive margin (see Proposition 1) but it still has an effect at the intensive margin. The restriction on $\mathcal{C}$ is irrelevant in this case, and individuals save more (borrow less, resp.) for more extreme transfer rates if partial prudence in wealth exceeds 2 (is bounded by 2, resp.). In Noussair et al. (2013), 62% of their demographically representative sample have relative prudence above 2. For multiplicative separability, the utility of the non-financial variable cancels out of the preferences coefficients in Definition 2. Then more extreme transfer rates increase saving (reduce borrowing, resp.), if partial prudence in wealth exceeds 2 (is bounded by 2, resp.) whereas partial risk aversion in wealth needs to be bounded by unity. In case of saving this rules out iso-elastic utility and constant absolute risk aversion, but is compatible with increasing relative risk aversion (see Ogaki and Zhang, 2001). In case of borrowing the conditions are satisfied for log-utility or any other iso-elastic utility function, which is less concave than log-utility.

usual trade-off that a higher interest rate has two conflicting effects when individuals save, a positive substitution effect because the return on saving is higher and a negative wealth effect because the individual's wealth in the second period increases. We point out that partial risk aversion is uniformly below unity if and only if relative risk aversion is (see Chiu et al., 2012), and remark that this sufficient condition is well-known in the consumption-saving literature.[10] The comparative statics properties in Remark 1 hold independent of whether interest rate risk represents an idiosyncratic or an aggregate shock.

5 Welfare effects at the individual level

We have seen that an idiosyncratic interest rate has an impact on saving behavior at the extensive and the intensive margin. In a final step, we will investigate how it affects a consumer's welfare. This will allow us to answer the question to what extent individuals actually benefit from an idiosyncratic interest rate. Our results show that the welfare effects of an idiosyncratic interest rate are related to its insurance effects. For this reason, we proceed in two steps and first analyze the individually-optimal transfer rate and then the relationship between idiosyncratic interest rates and insurance.

5.1 The individually-optimal transfer rate

For a transfer rate of Δ, the individual's indirect utility function is given by $U(s^*;\Delta)$, where s^* is defined in Eq. (5). It measures the individual's welfare at his optimal level of saving, taking the transfer rate into account. The envelope theorem yields

$$\frac{\mathrm{d}U}{\mathrm{d}\Delta} = \frac{\partial U}{\partial \Delta} + \frac{\partial U}{\partial s}\frac{\mathrm{d}s}{\mathrm{d}\Delta} = \frac{\partial U}{\partial \Delta} = \beta p(1-p)s^* \left[v^{(1,0)}(G^*) - v^{(1,0)}(B^*)\right], \tag{12}$$

because $\partial U/\partial s = 0$ from the optimality of s^*. We utilize Eq. (12) to investigate the impact of the transfer rate on the consumer's welfare. This allows us to rule out certain levels of the transfer rate as suboptimal. We summarize our results in the following proposition and provide a proof in Appendix A.6.

Proposition 4. *Consider a consumer who is not correlation neutral and take $\Delta \in I$. If $\Delta_{crit} < (>) 0$, a marginal increase (decrease) of the transfer rate raises the consumer's welfare.*

We conclude that transfer rates between 0 and Δ_{crit} are not optimal because the consumer can be made better off. We point out the two knife-edge cases as separate corollaries and will use them to develop some intuition. Proofs are given in Appendix A.7.

[10] See, for example, Eeckhoudt and Schlesinger (2008) and Chiu et al. (2012) who prove the sufficiency of this condition for first-order stochastic changes in the interest rate, and Courbage and Rey (2007) and Menegatti (2009) for optimal saving in the presence of a non-financial risk. Jouini et al. (2013) provide conditions under which the restriction on relative risk aversion is also necessary for definitive comparative statics.

Corollary 3 ($\Delta = 0$)**.** *Starting at the market interest rate, a marginal increase (decrease) of the transfer rate raises the consumer's welfare if* $\Delta_{crit} < (>) 0$.

Corollary 4 ($\Delta = \Delta_{crit}$)**.** *The consumer's welfare has a local minimum at the critical transfer rate.*

Corollary 4 rules out the critical transfer rate as a maximizer of the consumer's welfare. We know from Proposition 1 that the individual neither saves nor borrows when $\Delta = \Delta_{crit}$. For both correlation lovers and correlation averters, marginal utility of consumption differs between the two states of nature, which is why a transfer rate other than the critical one would improve on welfare because it allows to transfer wealth from the low to the high marginal utility state via saving or borrowing. A similar rationale underlies Corollary 3. When saving under the market interest rate, wealth in the second period does not depend on the state of nature. In this case, a correlation attitude other than neutral drives a wedge between marginal utility of consumption in both states. Therefore, a marginal departure from the market interest rate improves the consumer's consumption profile, which increases his intertemporal welfare. Proposition 4 shows that this rationale extends to any transfer rate between 0 and Δ_{crit}. Furthermore, our reasoning reveals that the potential welfare benefits of an idiosyncratic interest rate are related to its insurance effects because it allows to redistribute wealth from the low to the high marginal utility state.

Corollary 3 also informs us about who would opt into a savings plan with an idiosyncratic interest rate. Assume a small positive transfer rate $\Delta > 0$. According to Corollary 3, those consumers with $\Delta_{crit} < 0$ benefit from such a plan. According to Proposition 1, these are correlation lovers who save under the market interest rate and correlation averters who borrow under the market interest rate. Similarly, an idiosyncratic interest rate with a small negative transfer rate would attract correlation loving borrowers and correlation averse savers. So a consumer's correlation attitude interacts with his saving behavior under the market interest rate to determine his preference of an idiosyncratic interest rate over the status-quo.

We make another observation related to the change in saving behavior of those individuals who decide to opt in. Consider a savings plan with an idiosyncratic interest rate and a small positive transfer rate $\Delta > 0$. We know from Corollary 3 that it attracts correlation loving savers and correlation averse borrowers. We know from Corollary 1 that correlation loving savers who are cross-prudent in the non-financial variable will save more under the savings plan with an idiosyncratic interest rate. However, correlation averse borrowers who are cross-imprudent in the non-financial variable would save more, that is, borrow less, under the market interest rate. Both groups of consumers benefit from the idiosyncratic interest rate but only one group saves more, whereas the other one saves less and borrows more compared to their behavior under the market interest rate. We conclude that changes in saving behavior do not inform about changes in welfare and vice versa.

In the specific example of HSAs, the return on saving is higher in the bad state than the good state, corresponding to $\Delta < 0$. When the non-financial variable is health, many papers suggest correlation loving preferences ($v^{(1,1)} > 0$), see Finkelstein et al. (2013). This appears to be in contrast to Proposition 4 and Corollary 3 because correlation loving savers would benefit from a marginally positive, not negative, transfer rate. We suggest a possible reconciliation based on Liu's (2004) approach of endogenous health care spending. If individuals experiencing a negative health shock can spend money to (partially) restore their health status, a negative health shock becomes a negative wealth shock. As explained in Section 2, a monetary loss is a special case of a bivariate utility function with $v^{(1,1)} < 0$, in which case savers exhibit $\Delta_{crit} > 0$ and benefit from a marginally negative transfer rate.[11]

Starting from Eq. (12), we will now analyze the transfer rate that maximizes the consumer's welfare. First, if we require $r_b \geq -1$ and $r_g \geq -1$, the admissible values for the transfer rate lie in $[-(1+r)/p, (1+r)/(1-p)]$, which is a compact interval of $\mathbb{R}$. We then know from the extreme value theorem that $U(s^*; \Delta)$ attains a maximum because it is a continuous function of Δ. Proposition 4 and Corollaries 3 and 4 show that any maximizer must lie in J. In the sequel, we focus on interior solutions. A prerequisite for an interior solution is that the effect of the non-financial risk on marginal utility of consumption can be offset monetarily, that is, we can find wealth levels w_{2g} and w_{2b} in the good and the bad state respectively such that $v^{(1,0)}(w_{2g}, H_g) = v^{(1,0)}(w_{2b}, H_b)$. We summarize some comparative statics in the next proposition and provide a proof in Appendix A.8.

Proposition 5. *Let Δ^* be an interior maximizer of the consumer's welfare as a function of the transfer rate, $U(s^*; \Delta)$. Such an individually-optimal transfer rate is:*

a) increasing in the high and decreasing in the low non-financial variable for correlation loving savers and correlation averse borrowers,

b) decreasing in the high and increasing in the low non-financial variable for correlation loving borrowers and correlation averse savers,

c) increasing in the utility discount factor and first-period income if and only if partial risk aversion is higher in the bad state than the good state.

To develop some intuition for these effects, we explain how a change in the parameters under consideration affects the trade-off that an individually-optimal transfer rate solves. For the high and the low non-financial variable, the direct channel dominates in the following sense. At $\Delta = \Delta^*$, marginal utility of consumption is equal across states of nature. If the high non-financial variable increases, this raises the marginal utility of consumption in the good state for a correlation lover. To counterbalance this effect, the transfer rate needs to be adjusted in such a way as to increase wealth, which lowers marginal utility. If the individual saves, this is achieved by a higher transfer rate, while the reverse is true if he borrows. A similar reasoning applies to a correlation averter and the low non-financial variable.

[11] Steinorth (2012) and Peter et al. (2016) develop models of saving behavior that are explicitly catered to HSAs and take some of their institutional features into account.

The utility discount factor and first-period income do not affect marginal utility of consumption directly in the second period but indirectly via their effect on saving. According to Remark 1, individuals save more when the utility discount factor or first-period income increase. This results in two wealth effects, one in each state of nature, both of which lower the marginal utility of consumption. If the effect is equally strong in both states, the optimality condition for Δ^* remains unaffected and no adjustment is required. However, if both effects differ in their relative strength, the individually-optimal transfer rate changes. For example, if the effect is stronger in the bad state than the good state, marginal utility in the bad state drops by more than in the good state, and the individually-optimal transfer rate increases to redistribute wealth from the low to the high marginal utility state.

These comparative static results reveal that the transfer rate which maximizes a particular individual's intertemporal welfare is jointly determined by a variety of factors, including the agent's preferences and endowments. In an economy of heterogeneous agents, policymakers will have to trade off these various determinants when setting a transfer rate that applies uniformly across agents. Proposition 5 suggests that this is a highly complex task, which we leave for further research.

5.2 Insurance effects

To bring out the insurance effects of an idiosyncratic interest rate more explicitly and to relate our work more closely to Cook and Graham's (1977), we present a slightly modified model that includes both saving and insurance. As such we generalize the model pioneered by Dionne and Eeckhoudt (1984) and studied more recently by Hofmann and Peter (2016), who focus on a purely monetary risk. Given that the non-financial variable is not directly insurable, we assume now that the non-financial risk is flanked by monetary risk. A good example for such a situation is the case of a health risk where deteriorated health is accompanied by treatment expenses, or the risk of disability which results in lower productivity on the labor market. Health insurance can reimburse treatment expenses and long-term disability insurance can replace a portion of the individual's income.

In terms of our model, we assume a financial loss of T associated with the low outcome of the non-financial variable H_b in the second period. Individuals can purchase insurance against payment of a premium π in the first period, which reimburses a fraction $\alpha \in [0,1]$ of the financial loss. We assume that the price of insurance is proportional to its discounted actuarial value, that is, $\pi = m\alpha pT/(1+r)$, where m is a loading factor. The insurance contract is called actuarially favorable (fair, unfair) if $m < (=,>)1$. The agent's objective function is then given by

$$U(s,\alpha;\Delta) = u(w_1 - s - \pi) + \beta\left[pv(w_2 + (1+r_b)s - (1-\alpha)T, H_b) + (1-p)v(w_2 + (1+r_g)s, H_g)\right], \tag{13}$$

and he chooses saving and a level of insurance coverage to maximize his intertemporal expected utility. We show in Appendix A.9 that the objective function is globally concave in (s,α) if both u and v are concave functions of wealth, independent of the individual's correlation attitude. We focus only on the welfare effects of an idiosyncratic interest rate in our modified model. Here is our last result, which we prove in Appendix A.10.

Proposition 6. *Assume that U admits an interior solution and consider the consumer's welfare at the optimal levels of saving and insurance as a function of the transfer rate.*

(i) If insurance is actuarially unfair, a marginal decrease (increase) of the transfer rate raises the welfare of savers (borrowers).

(ii) If insurance is actuarially fair, the transfer rate does not affect the consumer's welfare.

(iii) If insurance is actuarially favorable, a marginal increase (decrease) of the transfer rate raises the welfare of savers (borrowers).

We first remark that Proposition 1 generalizes to the combined saving-insurance problem in the sense that a critical threshold on the transfer rate, which only depends on exogenous parameters and preferences, determines whether individuals are to be classified as savers or borrowers.[12] Proposition 6 tells us that an idiosyncratic interest raises a consumer's welfare if and only if there are cost differences between the saving mechanism and the insurance mechanism. Indeed, our comparability assumption that $pr_b+(1-p)r_g = r$ is the analog to the assumption of actuarial fairness in insurance. It states that saving via an idiosyncratic interest rate is ex-ante budget neutral. If actuarially fair insurance is available (i.e., $m = 1$), then saving under the market interest rate combined with an optimal level of insurance coverage can perfectly smooth differences in marginal utility across states and time. In such a case, there is no reason to deviate from the status quo. In all other cases (i.e., $m \neq 1$), the individual's optimal behavior when saving under the market interest rate leaves some difference between marginal utility in the bad versus the good state, which allows for an idiosyncratic interest rate to add value.

Another avenue for an idiosyncratic interest rate to be beneficial are institutional constraints on the insurance market. Proposition 6 is valid for interior solutions and specifically for levels of insurance coverage that do not exceed full coverage (i.e., $\alpha \leq 1$). In case of a purely monetary risk, we know from Mossin (1968) that full coverage is optimal if insurance is actuarially fair and that partial insurance is optimal if insurance is actuarially unfair. This result was extended to a two-period model with endogenous saving by Dionne and Eeckhoudt (1984) but still under the assumption of a purely monetary risk. Cook and Graham (1977) use an atemporal framework to show that Mossin's result does not generalize when a non-financial risk is present. Their result also holds in our two-period version of the problem with

[12] With the techniques in Courbage et al. (2017) one can show that, if $\alpha_0 = \arg\max_{\alpha\in[0,1]} U(0, \alpha; \Delta)$, then the individual saves if and only if $U_\alpha(0, \alpha_0; \Delta) > 0$. Therefore, the formulation of Proposition 6 in terms of savers and borrowers is for reasons of compactness and could be rewritten based on exogenous parameters.

endogenous saving. Indeed, starting at the market interest rate, one can show that individuals would like to overinsure if and only if the loading factor is below a threshold value. This threshold value is equal to 1 for correlation neutral agents, consistent with Dionne and Eeckhoudt (1984), below 1 for correlation lovers and above 1 for correlation averters, extending Cook and Graham's result to two periods. So a correlation averter will prefer an idiosyncratic interest rate with a small positive or negative transfer rate, depending on whether he is a saver or borrower, over the market interest rate even if actuarially fair insurance is available because insurance contracts typically do not reimburse more than the actual loss amount.

6 Conclusion

This paper studies the incentives and welfare effects of an idiosyncratic interest rates at the individual level. The central idea of an idiosyncratic interest rate is that the interest rate depends on states of nature that are specific to the individual and is implemented via a transfer rate. This transfer rate is a mean-preserving spread of the market interest rate (see Rothschild and Stiglitz, 1970), making it possible to relate our work to the literature on risk-induced saving behavior. An idiosyncratic interest rate is motivated by existing examples such as conventional insurance, HSAs, individual retirement accounts, standard annuities in general and enhanced annuities specifically, but our analysis abstracts from institutional details and studies the incentive and welfare effects of an idiosyncratic interest rate in a general economic environment. The underlying commonality is that the realized return on saving depends on the occurrence of specific states of nature that are typically related to a non-financial attribute of the individual's utility function. As a consequence and unlike previous research which considers the interest rate risk as an aggregate shock, the resulting interest rate depends on the individual's endowment in order to provide explicit saving incentives, making interest rate risk and risk over the individuals non-financial endowment correlated.

We find that the individual's correlation attitude then determines whether a positive or a negative transfer rate increases the incentive to save. This has direct implications for the share of savers in an economy. We also define a critical transfer rate above which individuals save who would not do so under the market interest rate depending on their correlation attitudes. In that sense, an idiosyncratic interest rate with a positive, respectively negative, transfer rate incentivizes saving behavior at the extensive margin in an economy of correlation lovers, respectively correlation averters. In a comparative statics analysis, we determine how the critical transfer rate depends on exogenous parameters and show how income levels, non-financial endowments, the intensity of the present bias and the probability of loss correlate with a stronger need to provide saving incentives. We also show how the optimal level of saving reacts to changes in the transfer rate, which depends on measures of partial prudence and cross-prudence. These conditions therefore determine the incentive effects of an idiosyncratic interest rate at the intensive margin. A final section is devoted to the optimal idiosyncratic interest rate that best suits the consumer's needs in the sense that it maximizes his intertemporal welfare. We show that our system of an idiosyncratic interest rate is a source of welfare gains as long as individuals are not correlation neutral. Lastly, we relate the impact of an idiosyncratic interest rate on individuals welfare on its insurance effects. We show that in presence of insurance, there are at least two avenues for an idiosyncratic interest to raise consumer welfare. One is cost differences between the saving and the insurance mechanism. Another one are institutional constraints on the insurance market.

Our results show that the incentives to save can be modified in a way that increases consumer welfare, and it is this welfare effect that corresponds directly with the consumer's correlation attitude, not the incentive effect. The size of potential welfare gains is an important follow-up question, which we leave for future research. Also, several extensions such as multiple periods or infinite horizons as well as economies with heterogeneous agents or overlapping generations seem promising. An important limitation of our analysis is that we abstract from transaction and information cost to focus on the link between an idiosyncratic interest rate and individual preferences. In practice, the implementation of such a savings plan is costly and not all types of non-financial outcomes might be perfectly verifiable. Such costs and imperfections need to be offset against the monetary equivalent of any resulting welfare gains to maintain a balanced view. Still, our results may serve as the starting point for the analysis of individual-specific selection and incentive effects for existing forms of saving under an idiosyncratic interest rate, and may also provide a prelude for the development of new forms of idiosyncratic interest rates in the future.

A Mathematical proofs

A.1 Proof of Proposition 2

We differentiate the critical transfer rate with respect to the exogenous parameters. We repeat its definition, which is

$$\Delta_{crit} = \frac{u'(w_1) - \beta(1+r)\left[pv^{(1,0)}(B_0) + (1-p)v^{(1,0)}(G_0)\right]}{\beta p(1-p)\left[v^{(1,0)}(G_0) - v^{(1,0)}(B_0)\right]}. \tag{14}$$

The numerator $N = u'(w_1) - \beta(1+r)\left[pv^{(1,0)}(B_0) + (1-p)v^{(1,0)}(G_0)\right]$ of Δ_{crit} is non-negative because we assume that the individual does not save under the market interest rate (i.e., $U_s(0;0) \leq 0$). The denominator $D = \beta p(1-p)\left[v^{(1,0)}(G_0) - v^{(1,0)}(B_0)\right]$ of Δ_{crit} is positive for correlation lovers due to $v^{(1,1)} > 0$.

The derivative of Δ_{crit} with respect to the utility discount factor β is

$$\frac{\mathrm{d}\Delta_{crit}}{\mathrm{d}\beta} = -\frac{u'(w_1)}{\beta D} < 0. \tag{15}$$

The derivative of Δ_{crit} with respect to the market interest rate r is

$$\frac{\mathrm{d}\Delta_{crit}}{\mathrm{d}r} = -\frac{\beta\left[pv^{(1,0)}(B_0) + (1-p)v^{(1,0)}(G_0)\right]}{D} < 0. \tag{16}$$

If we differentiate Δ_{crit} with respect to first-period income w_1, we obtain

$$\frac{\mathrm{d}\Delta_{crit}}{\mathrm{d}w_1} = \frac{u''(w_1)}{D} < 0. \tag{17}$$

This proves *a)*.

To show *b)* to *e)*, we first state the corresponding four derivatives and provide sufficient conditions to sign them in the following paragraphs. Differentiating Δ_{crit} with respect to second-period income w_2 yields

$$\begin{aligned}\frac{\mathrm{d}\Delta_{crit}}{\mathrm{d}w_2} \quad = \quad & \frac{\beta p(1-p)}{D^2}\Big\{-\beta(1+r)\left[v^{(1,0)}(G_0) - v^{(1,0)}(B_0)\right]\left[pv^{(2,0)}(B_0) + (1-p)v^{(2,0)}(B_0)\right] \\ & - N\left[v^{(2,0)}(G_0) - v^{(2,0)}(B_0)\right]\Big\},\end{aligned} \tag{18}$$

which we rewrite as follows:

$$\begin{aligned}\frac{\mathrm{d}\Delta_{crit}}{\mathrm{d}w_2} = \frac{\beta p(1-p)}{D^2}\Big\{&-\beta(1+r)v^{(2,0)}(G_0)\left[v^{(1,0)}(G_0) - v^{(1,0)}(B_0)\right] \\ &+ \left[v^{(2,0)}(G_0) - v^{(2,0)}(B_0)\right]\left[\beta(1+r)v^{(1,0)}(G_0) - u''(w_1)\right]\Big\}.\end{aligned} \tag{19}$$

The derivative of Δ_{crit} with respect to the high value of the non-financial variable H_g is

$$\frac{\mathrm{d}\Delta_{crit}}{\mathrm{d}H_g} = \frac{\beta p(1-p)}{D^2} \left\{\beta(1+r)v^{(1,0)}(B_0) - u'(w_1)\right\} v^{(1,1)}(G_0). \tag{20}$$

The derivative of Δ_{crit} with respect to the low value of the non-financial variable H_b is

$$\frac{\mathrm{d}\Delta_{crit}}{\mathrm{d}H_b} = -\frac{\beta p(1-p)}{D^2} \left\{\beta(1+r)v^{(1,0)}(G_0) - u'(w_1)\right\} v^{(1,1)}(B_0). \tag{21}$$

Finally, we compute the derivative of Δ_{crit} with respect to the probability of the bad state p:

$$\frac{\mathrm{d}\Delta_{crit}}{\mathrm{d}p} = \frac{1}{p(1-p)D} \left\{\beta(1+r)p(1-p)\left[v^{(1,0)}(G_0) - v^{(1,0)}(B_0)\right] - N(1-2p)\right\}, \tag{22}$$

which we rearrange to

$$\begin{aligned}\frac{\mathrm{d}\Delta_{crit}}{\mathrm{d}p} &= \frac{1}{p(1-p)D} \left\{\beta(1+r)p^2\left[v^{(1,0)}(G_0) - v^{(1,0)}(B_0)\right]\right. \\ &\quad \left. +(1-2p)\left[\beta(1+r)v^{(1,0)}(G_0) - u'(w_1)\right]\right\}.\end{aligned} \tag{23}$$

All of these four derivatives involve either the term $\left[\beta(1+r)v^{(1,0)}(G_0) - u'(w_1)\right]$ or the term $\left[\beta(1+r)v^{(1,0)}(B_0) - u'(w_1)\right]$. To sign these terms we need to compare $\mu^g_{1,2}$ and $\mu^b_{1,2}$ as defined in Fn. 6 to $1/(1+r)$. When $v^{(1,1)} > 0$, we obtain

$$\mu^b_{1,2} = \frac{\beta v^{(1,0)}(w_2, H_b)}{u'(w_1)} < \frac{\beta v^{(1,0)}(w_2, H_g)}{u'(w_1)} = \mu^g_{1,2}, \tag{24}$$

so that $\mu^b_{1,2} < \mu_{1,2} < \mu^g_{1,2}$. As we assume the individual not to engage in saving under the market interest rate, we have $U_s(0;0) \leq 0$, which is equivalent to $\mu_{1,2} \leq 1/(1+r)$. Therefore, $\mu^b_{1,2} < 1/(1+r)$ while $\mu^g_{1,2}$ may or may not exceed $1/(1+r)$.

Inspection of Eq. (18) reveals that the first term in the curly bracket is always positive whereas the second term is non-negative if $v^{(2,1)} \leq 0$; then, $\mathrm{d}\Delta_{crit}/\mathrm{d}w_2$ is positive. If $v^{(2,1)} > 0$, we obtain a sufficient condition for $\mathrm{d}\Delta_{crit}/\mathrm{d}w_2$ to be positive from Eq. (19) in the form of $\mu^g_{1,2} \geq 1/(1+r)$. As explained above, $\mu^b_{1,2}$ is bounded by $1/(1+r)$ so the curly bracket in Eq. (20) is negative, which renders $\mathrm{d}\Delta_{crit}/\mathrm{d}H_g$ negative as well. The sign of $\mathrm{d}\Delta_{crit}/\mathrm{d}H_b$ coincides with the sign of the curly bracket in Eq. (21), and an equivalent condition for $\mathrm{d}\Delta_{crit}/\mathrm{d}H_b$ to be positive is for $\mu^g_{1,2}$ to be bounded by $1/(1+r)$. Finally, inspection of Eq. (22) reveals that $\mathrm{d}\Delta_{crit}/\mathrm{d}p$ is positive if $p \geq 1/2$. In those cases where $p < 1/2$, we derive a sufficient condition for $\mathrm{d}\Delta_{crit}/\mathrm{d}p$ to be positive from Eq. (23) in the form of $\mu^g_{1,2} \geq 1/(1+r)$.

A.2 Proof of Proposition 3

We first define $F(x+y, H) = v^{(1,0)}(x+y, H) + yv^{(2,0)}(x+y, H)$ for any x, y, H. Then, $F(w_2+(1+r_b)s^*, H_b)$ and $F(w_2+(1+r_g)s^*, H_g)$ denote the net effect between the substitution and the wealth effect in the bad and the good state of nature, respectively. With this notation, we can rewrite the curly bracket in Eq. (10) as $F(w_2+(1+r_g)s^*,H_g)-F(w_2+(1+r_b)s^*,H_b)$. So whenever the incentive effect in the good state exceeds the incentive effect in the bad state, a marginal increase in the transfer rate will increase saving and vice versa.

We first analyze the case of a correlation lover ($v^{(1,1)}>0$). Assume that $\Delta \in I$, that is, $\min\{0,\Delta_{crit}\}<\Delta<\max\{0,\Delta_{crit}\}$. Then, either $0<\Delta<\Delta_{crit}$ so that $r_g>r_b$ and $s^*<0$ per Proposition 1(i), or $\Delta_{crit}<\Delta<0$ so that $r_b>r_g$ and $s^*>0$ per Proposition 1(i). In both cases we obtain $(1+r_b)s^*>(1+r_g)s^*$ so that $F(w_2+(1+r_b)s^*,H)<F(w_2+(1+r_g)s^*,H)$ if $F(x+y,H)$ is decreasing in y (i.e., if $\mathrm{d}F(x+y,H)/\mathrm{d}y<0$). The last condition is equivalent to $-yv^{(3,0)}(x+y,H)/v^{(2,0)}(x+y,H)<2$. Now if $F(x+y,H)$ is increasing in H (i.e., if $\mathrm{d}F(x+y,H)/\mathrm{d}H>0$), which is equivalent to $-yv^{(2,1)}(x+y,H)/v^{(1,1)}(x+y,H)<1$ due to $v^{(1,1)}>0$, the following chain of inequalities holds:

$$F(w_2+(1+r_b)s^*,H_b)<F(w_2+(1+r_g)s^*,H_b)<F(w_2+(1+r_g)s^*,H_g). \tag{25}$$

If $\Delta \in J$, then $(1+r_b)s^*<(1+r_g)s^*$ so that $F(x+y,H)$ being decreasing in y ensures that the incentive effect increases when replacing wealth in the bad state by wealth in the good state.

The case of a correlation averter ($v^{(1,1)}<0$) is similar. For $\Delta\in I$, we have $(1+r_b)s^*<(1+r_g)s^*$ and for $\Delta\in J$ we have $(1+r_b)s^*>(1+r_g)s^*$ due to Proposition 1(ii). The respective restriction on partial prudence in wealth then ensures that the incentive effect decreases when replacing wealth in the bad state by wealth in the good state of nature. Also, $-yv^{(2,1)}(x+y,H)/v^{(1,1)}(x+y,H)<1$ is now equivalent to $F(x+y,H)$ being decreasing in H (i.e., $\mathrm{d}F(x+y,H)/\mathrm{d}H<0$) due to $v^{(1,1)}<0$. This yields the following chain of inequalities:

$$F(w_2+(1+r_b)s^*,H_b)>F(w_2+(1+r_g)s^*,H_b)>F(w_2+(1+r_g)s^*,H_g). \tag{26}$$

Hence, saving increases following a marginal decrease of the transfer rate.

A.3 Proof of Corollaries 1 and 2

For $\Delta=0$, we obtain $r_b=r_g=r$ so that G^* and B^* only differ in the non-financial variable. The curly bracket in Eq. (10) then simplifies to

$$\left[v^{(1,0)}(G^*)-v^{(1,0)}(B^*)\right]+(1+r)s^*\left[v^{(2,0)}(G^*)-v^{(2,0)}(B^*)\right], \tag{27}$$

and the sign of the first square bracket coincides with that of $v^{(1,1)}$ whereas the sign of the second square bracket coincides with that of $v^{(2,1)}$.

For $\Delta=\Delta_{crit}$, the optimal level of saving is zero ($s^*=0$) and the curly bracket in Eq. (10) simplifies to $v^{(1,0)}(G_0)-v^{(1,0)}(B_0)$. This is positive (negative) if the agent is correlation loving (averse).

A.4 On the intensity of partial cross-prudence in the non-financial variable

Let $\widetilde{\varepsilon}$ denote a small zero-mean risk on wealth, $\mathbb{E}\widetilde{\varepsilon} = 0$. If the individual is not cross-prudence neutral ($v^{(2,1)} \neq 0$), the introduction of this wealth risk will affect the marginal utility of the non-financial variable. If the individual is cross-prudent in the non-financial variable, it will increase expected marginal utility of the non-financial variable. If the financial outcome is given by $x + y$ and y is replaced by $y(1 + \widetilde{\varepsilon})$, we can ask about the certain reduction ϕ of y that has the same effect, that is,

$$\mathbb{E}v^{(0,1)}(x + y(1 + \widetilde{\varepsilon}), H) = v^{(0,1)}(x + y(1 - \phi), H). \tag{28}$$

We call ϕ a *partial cross-prudence premium.* It measures by how much the risk $\widetilde{\varepsilon}$ affects the marginal utility of the non-financial variable in units of the financial variable. Analogous to Pratt (1964), we use a second- and first-order Taylor approximation for the left- and the right-hand side of this equation to obtain

$$\phi \simeq -\frac{1}{2} \cdot \mathrm{Var}(\widetilde{\varepsilon}) \cdot \frac{yv^{(2,1)}(x + y, H)}{v^{(1,1)}(x + y, H)}, \tag{29}$$

where $\mathrm{Var}(\widetilde{\varepsilon})$ denotes the variance of the $\widetilde{\varepsilon}$ risk. This approximation allows us to interpret the coefficient $-yv^{(2,1)}(x + y, H)/v^{(1,1)}(x + y, H)$ as an intensity measure of partial cross-prudence in the non-financial variable. For small risks it is proportional to the size of the partial cross-prudence premium.

A.5 Proof of Remark 1

The proof follows by taking the derivative of the first-order expression (5) with respect to the exogenous parameters. For the utility discount factor β this yields

$$U_{s\beta} = (1 + r_b)pv^{(1,0)}(B^*) + (1 + r_g)(1 - p)v^{(1,0)}(G^*) > 0. \tag{30}$$

For first-period income w_1 we obtain

$$U_{sw_1} = -u''(w_1 - s^*) > 0. \tag{31}$$

For second-period income w_2, we derive

$$U_{sw_2} = \beta(1 + r_b)pv^{(2,0)}(B^*) + \beta(1 + r_g)(1 - p)v^{(2,0)}(G^*) < 0. \tag{32}$$

For the market interest rate r, we obtain

$$\begin{aligned} U_{sr} \;=\; & \beta pv^{(1,0)}(B^*) + \beta(1 - p)v^{(1,0)}(G^*) \\ & + \beta(1 + r_b)s^* pv^{(2,0)}(B^*) + \beta(1 + r_g)s^*(1 - p)v^{(2,0)}(G^*). \end{aligned} \tag{33}$$

For individuals who borrow,[13] this is always positive indicating that a higher market interest rate leads to less borrowing (i.e., more saving). For individuals who save, we can rearrange U_{sr} as follows:

$$\begin{aligned} U_{sr} &= \beta p v^{(1,0)}(B^*)\left(1-\left(-(1+r_b)s^*\frac{v^{(2,0)}(B^*)}{v^{(1,0)}(B^*)}\right)\right) \\ &\quad +\beta(1-p)v^{(1,0)}(G^*)\left(1-\left(-(1+r_g)s^*\frac{v^{(2,0)}(G^*)}{v^{(1,0)}(G^*)}\right)\right). \end{aligned} \tag{34}$$

If relative risk aversion in wealth is less than unity, both round brackets will be positive indicating that an increase in the market interest rate raises saving. Finally, we obtain

$$U_{sH_b} = \beta(1+r_b)(1-p)v^{(1,1)}(B^*) \text{ and } U_{sH_g} = \beta(1+r_g)pv^{(1,1)}(G^*) \tag{35}$$

for the two non-financial variables in the second period. The sign of either expression coincides with the sign of $v^{(1,1)}$.

A.6 Proof of Proposition 4

For $\Delta \in (\Delta_{crit}, 0)$, we have $r_b > r_g$, $s^* > 0$ for correlation lovers and $s^* < 0$ for correlation averters (see Proposition 1). Therefore, $w_2 + (1+r_b)s^* > w_2 + (1+r_g)s^*$ for correlation lovers, which renders the square bracket in (12) positive, and $w_2+(1+r_b)s^* < w_2+(1+r_g)s^*$ for correlation averters, which renders the square bracket in (12) negative. In either case, we obtain $\mathrm{d}U/\mathrm{d}\Delta > 0$ because the square bracket in (12) and the agent's saving choice have the same sign.

Similarly, if $\Delta \in (0, \Delta_{crit})$, we have $r_g > r_b$, $s^* < 0$ for correlation lovers and $s^* > 0$ for correlation averters (see Proposition 1). Then, $w_2+(1+r_b)s^* > w_2+(1+r_g)s^*$ for correlation lovers, which renders the square bracket in (12) positive, and $w_2+(1+r_b)s^* < w_2+(1+r_g)s^*$ for correlation averters, which renders the square bracket in (12) negative. In either case, $\mathrm{d}U/\mathrm{d}\Delta < 0$ because the square bracket in (12) and the agent's saving choice have the opposite sign.

A.7 Proof of Corollaries 3 and 4

For $\Delta = 0$, we have $r_b = r_g = r$, and the square bracket in Eq. (12) is positive (negative) for correlation lovers (averters). If $\Delta_{crit} < 0$, correlation lovers save while correlation averters borrow under the market interest rate. In either case, $\mathrm{d}U/\mathrm{d}\Delta > 0$. If $\Delta_{crit} > 0$, correlation lovers borrow while correlation averters save under the market interest rate, and $\mathrm{d}U/\mathrm{d}\Delta < 0$.

At $\Delta = \Delta_{crit}$, behavior switches from borrowing to saving for correlation lovers and from saving to borrowing for correlation averters. In a neighborhood of Δ_{crit}, the square bracket

[13] That is, if $\Delta < \Delta_{crit}$ for correlation lovers or $\Delta > \Delta_{crit}$ for correlation averters, see Proposition 1.

in Eq. (12) is strictly positive (negative) for correlation lovers (averters). In either case, $U(s^*;\Delta)$ switches from strictly decreasing to strictly increasing at Δ_{crit}.

A.8 Proof of Proposition 5

If Δ^* is an interior maximizer of $U(s^*;\Delta)$, then $\mathrm{d}U(s^*;\Delta^*)/\mathrm{d}\Delta = 0$ and $\mathrm{d}^2U(s^*;\Delta^*)/\mathrm{d}\Delta^2 < 0$. Taking into account that s^* depends on Δ, we obtain

$$\frac{\mathrm{d}^2U}{\mathrm{d}\Delta^2} = U_{\Delta\Delta} + U_{s\Delta}\frac{\mathrm{d}s^*}{\mathrm{d}\Delta}, \tag{36}$$

which is negative for $\Delta = \Delta^*$ due to the second-order condition.

We denote by k an exogenous variable of our model and consider the following parameters: $k \in \{H_g, H_b, \beta, w_1\}$. To examine how Δ^* depends on k, we need to sign $\mathrm{d}\Delta^*/\mathrm{d}k$. We acknowledge the effect of k on Δ^* by writing $\Delta^*(k)$; then, the effect of k on the optimal level of saving is twofold, directly via the first-order condition (5), and indirectly through its effect on the optimal transfer rate, so that we denote $s^*(\Delta^*(k), k)$. Finally, the effect of k on the first-order optimality condition for the individually-optimal transfer rate, $U_\Delta = 0$, is threefold; there is a direct effect if k appears directly in this condition, there is an indirect effect through the optimal transfer rate and another indirect effect through the optimal level of saving. We capture all of these effects in our notation as follows:

$$U_\Delta(s^*(\Delta^*(k), k), \Delta^*(k), k) = 0. \tag{37}$$

The net effect of a marginal variation in k must be such that the first-order optimality condition remains satisfied, i.e.,

$$U_{\Delta s}\left(\frac{\mathrm{d}s^*}{\mathrm{d}\Delta}\cdot\frac{\mathrm{d}\Delta^*}{\mathrm{d}k} + \frac{\mathrm{d}s^*}{\mathrm{d}k}\right) + U_{\Delta\Delta}\cdot\frac{\mathrm{d}\Delta^*}{\mathrm{d}k} + U_{\Delta k} = 0. \tag{38}$$

Solving for $\mathrm{d}\Delta^*/\mathrm{d}k$ renders

$$\frac{\mathrm{d}\Delta^*}{\mathrm{d}k} = -\frac{U_{\Delta k} + U_{\Delta s}\cdot\frac{\mathrm{d}s^*}{\mathrm{d}k}}{U_{\Delta\Delta} + U_{\Delta s}\cdot\frac{\mathrm{d}s^*}{\mathrm{d}\Delta}}. \tag{39}$$

The denominator is negative due to the second-order condition for Δ^*. Therefore, the sign of $\mathrm{d}\Delta^*/\mathrm{d}k$ coincides with the sign of its numerator. We apply the implicit function rule and rearrange to obtain:

$$U_{\Delta k} + U_{\Delta s}\cdot\frac{\mathrm{d}s^*}{\mathrm{d}k} = U_{\Delta k} - U_{\Delta s}\frac{U_{sk}}{U_{ss}} = -\frac{1}{U_{ss}}\cdot\Big[U_{\Delta s}U_{sk} - U_{\Delta k}U_{ss}\Big]. \tag{40}$$

So the sign of $\mathrm{d}\Delta^*/\mathrm{d}k$ coincides with the sign of the square bracket in Eq. (40). In the sequel, we will determine this sign for $k \in \{H_g, H_b, \beta, w_1\}$, taking into account that both $U_s(s^*;\Delta^*) = 0$ and $U_\Delta(s^*;\Delta^*) = 0$ hold at an interior maximizer of $U(s^*;\Delta)$.

For $k = H_g$, we obtain

$$U_{\Delta s}U_{sH_g} - U_{\Delta H_g}U_{ss} = \beta p(1-p)v^{(1,1)}(G^*)s^* \underbrace{\left[-u''(w_1 - s^*) - \beta(1+r_b)(1+r)v^{(2,0)}(B^*)\right]}_{>0}, \tag{41}$$

and the sign is jointly determined by the individual's correlation attitude and saving behavior. For $k = H_b$, we find

$$U_{\Delta s}U_{sH_b} - U_{\Delta H_b}U_{ss} = \beta p(1-p)v^{(1,1)}(B^*)s^* \underbrace{\left[u''(w_1 - s^*) + \beta(1+r_g)(1+r)v^{(2,0)}(G^*)\right]}_{<0}, \tag{42}$$

and the sign is also jointly determined by the individual's correlation attitude and saving behavior. This proves *a)* and *b)*. Notice that $\operatorname{sgn}(\mathrm{d}\Delta^*/\mathrm{d}H_g) = -\operatorname{sgn}(\mathrm{d}\Delta^*/\mathrm{d}H_b)$ whatever the agent's correlation attitude and saving behavior. In other words, the comparative statics of the two non-financial variables always go in opposite direction.

To show *c)*, we set $k = \beta$ and calculate

$$\begin{aligned} U_{\Delta s}U_{s\beta} - U_{\Delta\beta}U_{ss} &= \beta(1+r)p(1-p)v^{(1,0)}(G^*)v^{(1,0)}(B^*) \\ &\quad \cdot \left[\mathcal{R}(w_2 + (1+r_b)s^*, H_b) - \mathcal{R}(w_2 + (1+r_g)s^*, H_g)\right], \end{aligned} \tag{43}$$

which is positive if and only if relative risk aversion is higher in the bad state than the good state. Similarly, for $k = w_1$ we find

$$\begin{aligned} U_{\Delta s}U_{sw_1} - U_{\Delta w_1}U_{ss} &= -\beta p(1-p)u''(w_1 - s^*)v^{(1,0)}(G^*) \\ &\quad \cdot \left[\mathcal{R}(w_2 + (1+r_b)s^*, H_b) - \mathcal{R}(w_2 + (1+r_g)s^*, H_g)\right] \end{aligned} \tag{44}$$

and obtain the same equivalent condition for a positive sign.

A.9 Global concavity of objective function (13)

We suppress the argument of utility in the first period and use B and G to abbreviate the pairs of consumption and the non-financial variable in the bad and the good state, respectively, for a given amount of saving and insurance coverage. We obtain the following derivatives:

$$\begin{aligned} U_{ss} &= u'' + \beta(1+r_b)^2 p v^{(2,0)}(B) + \beta(1+r_g)^2(1-p)v^{(2,0)}(G) < 0, \\ U_{\alpha\alpha} &= \left(\frac{m}{1+r}pT\right)^2 u'' + \beta p T^2 v^{(2,0)}(B) < 0, \\ U_{s\alpha} &= \frac{m}{1+r}pTu'' + \beta pT(1+r_b)v^{(2,0)}(B) < 0. \end{aligned} \tag{45}$$

After some algebra, we calculate the determinant of the Hessian of U to be

$$\begin{aligned} U_{ss}U_{\alpha\alpha} - U_{s\alpha}^2 &= \beta p T^2 \left\{ \left(\frac{mp(1+r_g)}{1+r} - 1 \right)^2 u'' v^{(2,0)}(B) \right. \\ &\quad \left. + (1-p)(1+r_g)^2 v^{(2,0)}(G) \left(p \left(\frac{m}{1+r} \right)^2 u'' + v^{(2,0)}(B) \right) \right\}, \end{aligned} \tag{46}$$

which is positive. Therefore, U is globally concave in (s, α) for any transfer rate as long as u and v are concave functions of wealth.

A.10 Proof of Proposition 6

An interior solution (s^*, α^*) is characterized by the following pair of first-order conditions,

$$\begin{aligned} U_s &= -u'(w_1^*) + \beta(1+r) \left[p v^{(1,0)}(B^*) + (1-p) v^{(1,0)}(G^*) \right] \\ &\quad + \beta p (1-p) \Delta \left[v^{(1,0)}(G^*) - v^{(1,0)}(B^*) \right] = 0, \end{aligned} \tag{47}$$

$$U_\alpha = -\frac{m}{1+r} p T u'(w_1^*) + \beta p T v^{(1,0)}(B^*) = 0, \tag{48}$$

where w_1^*, B^* and G^* are shorthand for the arguments in the first and second period when evaluated at (s^*, α^*). We solve $U_\alpha = 0$ for $u'(w_1^*)$, substitute it into in $U_s = 0$, and rearrange to obtain

$$\beta(1-p)(1+r_g) \left[v^{(1,0)}(G^*) - v^{(1,0)}(B^*) \right] + \frac{m-1}{m} v^{(1,0)}(B^*) = 0. \tag{49}$$

The consumer's indirect utility function is given by $U(s^*, \alpha^*; \Delta)$, and an application of the envelope theorem yields

$$\frac{\mathrm{d}U}{\mathrm{d}\Delta} = \beta p (1-p) s^* \left[v^{(1,0)}(G^*) - v^{(1,0)}(B^*) \right] = -\frac{p s^*}{(1+r_g)} \frac{m-1}{m} v^{(1,0)}(B^*), \tag{50}$$

where the last equality holds by substituting Eq. (49). If insurance is actuarially fair (i.e., $m = 1$), then $\mathrm{d}U/\mathrm{d}\Delta$ is zero, which proves (*ii*). If insurance is actuarially unfair (i.e., $m > 1$), then $\mathrm{d}U/\mathrm{d}\Delta$ is negative for savers and positive for borrowers, proving (*i*). If insurance is actuarially favorable (i.e., $m < 1$), then $\mathrm{d}U/\mathrm{d}\Delta$ is positive for savers and negative for borrowers, proving (*iii*).

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