Optimizing the marriage market through the reallocation of partners:

An application of the linear assignment model

by

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Abstract:

Research shows that success of marriages and other intimate partnerships depend on objective attributes such as differences of age, cultural background or educational levels between partners. This article proposes a mathematical approach of marriage which intends to optimally allocate spouses in order to reduce the likelihood of divorce within the set of structural constraints defining a marriage market. Based on a representative and longitudinal sample of 1074 cohabitating and married couples living in Switzerland, we estimate various objective functions corresponding to age, education, ethnicity and previous divorce experience concerning every possible combination of men and women. Our results show that the current state of marriages or partnerships is well below the social optimum. About 7 individuals over 10 (68%) are reallocated to a couple with a higher chance of survival than the actual couple that they belong to. This reallocation leads the initial non optimal situations to the final optimal situations with a reduction of the objective function by 21% of its initial value.

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1. Introduction

Although at first sight men and women "choose" their mates on the basis of feelings of love, physical attraction and similarity of tastes, beliefs, attitudes and shared values which are supposed to help them be happy together (Berscheid and Reis, 1998; Kalmijn, 1998, Yela and Sangrador, 2001; Hohmann-Marriott, 2006), research shows that the longevity of marriages or partnerships also depends on objective attributes such as differences of age, family history and educational levels between partners. In this regard, one may wonder what would be an optimal attribution of partners, and how far from the optimum the current situation lies?

A straightforward criterion for asserting this optimum is constituted by the minimization of divorces and separations. Many scholars have emphasized the costs of divorce, either economic, social or psychological (Amato, 2000). The negative influence of marriage breakups strongly affect future generations because it has significant negative effects on childrens' educational trajectories and thus on human capital (Amato and Keith, 1991; Amato, 2000; Cherlin et al., 1998). Thus, even a slight increase of the average strengths of couples might strongly reduce the social costs of family breakdown. This article proposes a mathematical approach trying to optimally allocate spouses in order to reduce the likelihood of divorce within the set of demographic constraints defining the marriage market (Blau et al., 1967).

This paper is organized as follows. In Section 2, we present academic findings related to the marriage market and divorces. In Section 3, we describe the sociological survey that was used to build the database as well as optimization parameters developed through econometrics models. In Section 4, we describe the optimization model we have developed. In Sections 5 and 6, we present, interpret and discuss the results of our case study. We finally conclude this paper by indicating limitations of our approach and future research directions.

2. The marriage market and the propensity of couples to split

The marriage market is constrained by objective factors. Studies show that the tendency of similar individuals to match is overly dominant, a well-known fact described by sociologists as "homogamy" (Blackwell and Lichter, 2004; Kalmijn, 1998). Spouses tend to be similar with respect to social characteristics such as social origin, education, race, ethnicity, religion and age. Interaction opportunities in mate selection can be studied in a macro-level perspective. First, they depend on the relative size of the various groups present in a population, and second, on the distributions of social groups across space. Homogamy is stronger in large groups and in areas in which groups are dominant. The rate of heterogamy is directly related to the heterogeneity of the population (Blau et al., 1982; Blau and Schwartz, 1984). Individuals belonging to small minorities are forced to marry out of their groups because few potential spouses are available in their own groups. Homogamy and heterogamy thus depend to a large extent on the homogeneity or heterogeneity of the society considered, in terms of cohort sizes, educational levels, religious and ethnic composition, etc.

Homogamy has been hypothesized to have an impact on conjugal satisfaction and divorce. Overall, it was found that heterogamous marriages or partnerships increase the probability of break-up. Indeed, cultural dissimilarities increase the likelihood to divorce (Price-Bonham and Balswick, 1980; Felmlee et al, 1990; Janssen, 2002). For age heterogamy, it was found that the likelihood of break-up increases when the wife is older (Janssen, 2002). Couples in which one partner divorced in a former relationship have a higher probability to experience breakup (Cherlin & Furstenberg, 1994). Results about educational are more mixed. Some studies show that educationally homogamous marriages are more stable than educationally mixed marriages (Bumpass, Castro Martin & Sweet, 1991; Jalovaara, 2003; Tzeng, 1992).

To summarize, research underlines that marriage choices are constrained by the sizes of social groups present in any population, and that heterogamous couples, on average, have a larger chance to be ended by a divorce or a separation. Based on this set of evidence, our study

intends to reduce the likelihood of divorce in reallocating individuals in more optimal couples based on objective criteria such as those mentioned above.

3. Data and parameters

We use data from the Swiss contemporary family survey done in 1999 (Widmer et al., 2003). This is a representative random sample of 1'534 couples, married or unmarried, aged from 18 to 75, residing in the three linguistic areas of Switzerland. A follow up of 1'074 of those couples was done in 2004 and allows knowing how many of them separated between 1999 and 2004 (Widmer et al., 2006).

For each couple belonging to the dataset indicators of homogamy were constructed. In order to evaluate the specific influence of those variables on the break-up, we ran a logistic regression analysis that uses the indicator "separation occurring between the two waves of the study" as a dependant dummy variable. A series of variables, used as predictors, measure homogamy in terms of difference of age (five categories), nationality as a proxy for cultural origin (six categories), level of education (four categories) and divorce from a previous partner as a proxy for homogamy of family trajectories (four categories). The parameter estimates produced by the logistic regression expressed as odds ratios are used to weight the different criteria retained for the optimization model. A large weight associated with a given predictor indicates a strong statistical influence towards couple dissolution during the period of observation (see Table 1).

Effect	Odds ratio							
Age homogamy								
Wife is 5+ years older than husband	3.361*							
Wife is 4-2 years older than husband	1.462							
Wife is as old as husband (REF)	-							
Wife is 4-2 years younger than husband	0.694							
Wife is 5+ years younger than husband	0.678							
Homogamy of family trajectory								
No previous divorce (REF)	-							
Wife has previously divorced	2.507							
Husband has previously divorced	6.401**							
Wife & husband have previously divorced	1.286							
Educational homogamy								
Wife & husband have a low level (REF)	-							
Wife has a higher level than husband	0.125*							
Wife has a lower level than husband	0.456*							
Wife & husband have a high level	0.586							
Cultural homogamy (Citizenship of origin) (JAG CORRECT?)								
Wife & husband are Swiss (REF)	-							
One is Swiss the other from western country	1.505							
Both from Western countries	3.451*							
One is Swiss the other from non-Western country	4.300**							
Both from non-Western country	11.314*							
One from Western country, the other from non-Western country	1.416							

Global significance Likelihood Ratio test: Chi²: 55.7507; df: 15; p <.0001 p<.05; ** p<.01

Table 1: Weights for predictors (logistic regression)

Overall, the odds ratios follow the literature on the effects of homogamy on marital stability. Couples in which the wife is older than the man have greater odds to split, as well as those in which the two partners do not have the same family trajectory (with one partner only having experienced divorce in a previous relationship). Couples in which both partners are Swiss citizens have much lower odds of divorcing than couples in which one is Swiss and the other comes from a non-Western country, i.e. have a quite distinct cultural background. Other couples such as those in which two partners come from other countries have high divorce rates but this is likely to be caused by migration. Results for education, however, contradict the overall prediction of a positive effect of homogamy as the two cases in which partners have dissimilar educational levels have greater odds not to divorce.

4. The optimization model

The optimization model we have developed is based on the assumption that a central matrimonial agency knows exactly the characteristics of all potential mates and is in the position to decide how to match them. Adopting this point of view, we identify the optimum matches pending on the constraints set by the distribution of groups in the selected sample. Since partners choose each other on the basis of multiple social characteristics with unequal effects on break-ups, we consider simultaneously various criteria rather than one at a time.

As presented in section 3, in this paper, we consider four criteria which proved to have a significant impact on the likelihood of divorce: differences of age, cultural origin, educational level and prior experience of divorces. We subdivide each criterion into several categories and assign to each category a weight given by the log odds indicated in Table 1.

Figure 1 shows the weight assigned to the five categories of the difference of age: woman much older than man (5 years older and more), woman older than man (from 2 to 4 years

older), both have the same age (from 1 year older to 1 year younger), man older than woman (from 2 to 4 years older), man much older than woman (5 years older and more).

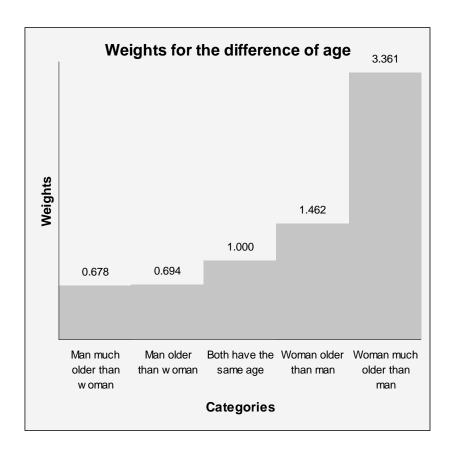


Figure 1: Weight attribution for the difference of age.

The weights assigned to the difference of age are not linear but rather in a "staircase" form. Furthermore, the weight attribution is also asymmetric in the sense that for the same difference of age, couples in which women are older than men receive a higher weight than couples in which men are older than women.

For each couple, we estimate the value for the age criterion by subtracting the age of the man from the age of the woman. This value allows us to classify each couple in one of five categories and assign the weight of this category to the couple.

For the differences of cultural origin, educational level and prior experience of divorces we use the same weight attribution process that can be formulized as follows.

Given:

W a set of women

M a set of men

$$|W| = |M|$$

We choose C a set of criteria influencing the stability of couples (i.e. age, cultural origin, education, divorce, experience in our case)

All the values that can take a criterion $c \in C$ define its domain D_c

We divide each criterion $c \in C$ into a set of categories K_c

All the values that can take a category $k_c \in K_c$ define its domain D_{k_c}

Within a criterion, the domains of all categories must be exclusive

$$D_{k_c} \cap D_{k'_c} = \varnothing, \forall k_c, k'_c \in K_c$$

The domain of a criterion is defined by the domains of all its categories

$$\bigcup_{k_c} D_{k_c} = D_c, \forall k_c, k_c' \in K_c$$

We assign to each category $k_c \in K_c$ a cost p_{k_c} given by the logistic regression (see Table 1).

We compute v_{wm}^c , the value of criterion $c \in C$ for each couple formed by woman $w \in W$ and by man $m \in M$

$$v_{wm}^c = f_c(v_w^c, v_m^c)$$

 f_c is a specific function for criterion $c \in C$

 v_w^c is the value of woman $w \in W$ for criterion $c \in C$

 v_m^c is the value of man $m \in M$ for criterion $c \in C$

Finally, we calculate p_{wm}^c , the cost of criterion $c \in C$ for each couple formed by woman $w \in W$ and by man $m \in M$

so that
$$p_{wm}^c = p_{k_c}$$
 if $v_{wm}^c \in D_{k_c}$

To take into account simultaneously all the criteria, the total cost for each couple formed by woman $w \in W$ and by man $m \in M$ is given by $\sum_{c} p_{w,m}^{c}$

The weight attribution presented above is integrated in our optimization model as follows

$$\min \sum_{c} \sum_{w} \sum_{m} p_{w,m}^{c} x_{w,m}$$
s.t.
$$\sum_{m} x_{w,m} = 1, \forall w \in W$$

$$\sum_{w} x_{w,m} = 1, \forall m \in M$$

$$x_{wm} = \begin{cases} 1 & \text{if woman } w \in W \text{ is assigned to man } m \in M \\ 0 & \text{otherwise} \end{cases}$$

If we replace $\sum_{c} p_{w,m}^{c}$ by $\pi_{w,m}$ (the cost of all criteria for woman $w \in W$ and for man $m \in M$), we obtain the classical *linear assignment problem* (many papers have been published on model applications and algorithm developments, for a general overview see for instance Burkard et al., 2008; Lovász and Plummer, 1986; Schrijver, 2003).

The cost matrix $\pi_{w,m}$ reflects the probability of divorce. If appropriately transformed, it could also be transformed and used to set up a "preference matrix" as defined in the *stable marriage* problem (Gale and Shapley, 1962; Gusfield and Irving, 1989; Lustig and Puget, 2001).

5. Implementation and results

We implement the optimization model presented above into an IP model using the GAMS modeling language. From the 1074 couples of the survey, we eliminate 7 couples that present missing values for a least one of the criteria considered. The model has then 1067 couples, 1067^2 binary variables and 1067! different solutions. We pre-calculated the cost function $\pi_{w,m} = \sum_{c} p_{w,m}^c$ for the 1067^2 possible couples and solve the model using CPLEX.

5.1 Global results

The solution we obtained for 1067 couples is optimal. In this solution, 99.81% couples have been reallocated. Figure 2 compares the objective function of all couples in the initial solution (the clear points) and in the final solution (the dark points). As the objective function is improved in the final solution for about 68% of the couples, most of the dark points are lower than the light points.

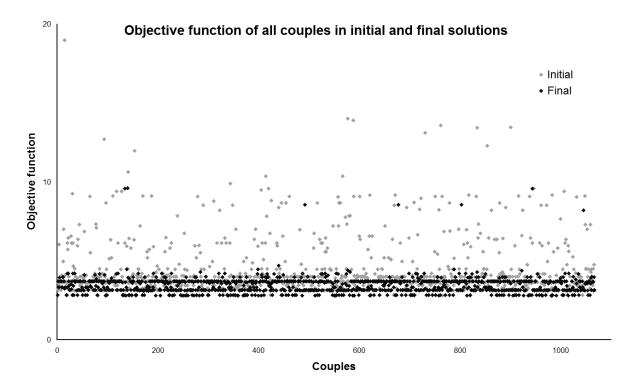


Figure 2: The objective function is improved for 68% of the couples in the final solution.

Figure 3 compares shows that, for all criteria, the objective function in the final solution (the dark columns) is lower than the objective function in the initial solution (the white columns). In other words, no criterion was "worsened" to improve other criteria through the optimization process. Figure 3 also shows that, with all the criteria included, the total objective function is improved about 21% of its initial value.

Cost of each criterion in initial and final solutions

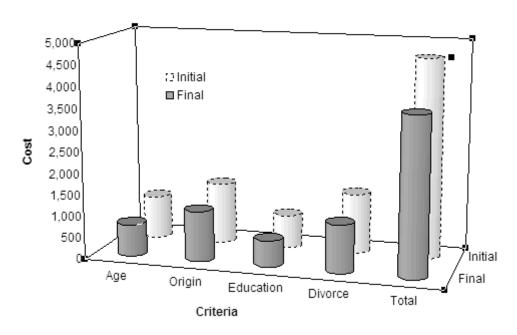


Figure 3: The objective function is improved for all criteria in the final solution.

Figure 4 and 5 show the number of couples in each category of each criterion in the initial solution (represented by the white columns) and in the final solution (represented by the dark columns). The cost function (represented by the growing up dark line) indicates that low cost categories are grouped on the left and high cost categories are on the right of the charts. Both figures show that except for the first category in the age criterion, white columns are lower than dark columns on the left (low cost categories) and white columns are higher than dark columns on the right (high cost categories). That means that in final solutions couples with high cost are replaced by couples with low cost for all criteria, considered together.

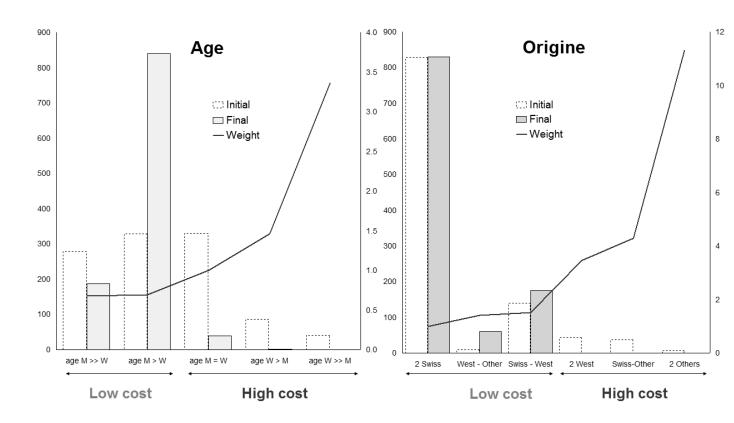


Figure 4: The number of couples in high cost categories decreases in the final solution for criteria concerning age and cultural origin.

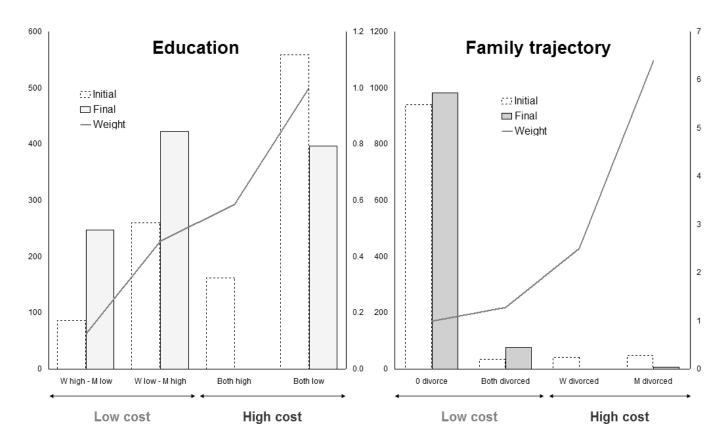


Figure 5: The number of couples in high cost categories decreases in the final solution for criteria concerning educational level and prior experience of divorce or separation.

Overall, 69% of individuals have been attributed a partner in a different age category in the final solution; 53% for education, 21% for citizenship of origin, and 9% for family trajectory.

5.2 Portrait analysis

What does it means at couples level? Table 3 analyzes the profiles of the extreme couples of the final solution compared with the initial situation. The best case in the initial solution is a couple with a total weight equal to 2.803 in which both partners are Swiss and have no prior experience of divorce, the woman has higher educational level than the man and the man is much older than the woman. The best couple arrangement in the final solution shows exactly the same profile. In fact, 2.803 is the sum of the lowest weights of each criterion and it is impossible for the optimization process to improve this optimal profile. However, if the objective function doesn't change best couple's arrangement, the probability of break-up is far more lower when worst allocations are considered. The worst case in the initial solution is a couple with a total weight equal to 18.979 in which both partners have high education level and are nor Swiss neither from another Western country, the man has prior experience of divorce and is much older than the woman. In the final solutions, the worst case is a couple with a total weight equal to 9.600 in which both partners have low educational level, the man has prior experience of divorce and is older than the woman, and the couple associates one Swiss and one foreigner from a Western country. The objective function permits therefore to reduce by half the weight in cases of couples with the highest risk of break-up.

Best couple in initial and final solution		Worst couple in initial solution		Worst couple in final solution		
Weight	Couple portrait	Weight	Couple portrait	Weight	Couple portrait	
2.803	Total weight	18.979	Total weight	9.600	Total weight	
0.678	Wife is 5+ years younger than husband	0.678	Wife is 5+ years younger than husband	0.694	Wife is 5+ years younger than husband	
1	No previous divorce	6.401	Husband has previously divorced	6.401	Husband has previously divorced	
0.125	Wife has a higher level than husband	0.586	Wife & husband have a high level	1.000	Wife & husband have a low level	
1	Wife & husband are Swiss	11.314	Both from non-Western country	1.505	One is Swiss the other from non-Western country	

Table 3: Portraits of best and worst couples in the initial and the final solutions.

By reducing the gap between the best and the worst cases, the optimization process forms more homogeneous couples compared to the initial solutions. This result is confirmed by a lower standard deviation of the objective function in the final solutions (0.59) compared to the initial solution (1.73). This can also be observed in Figure 6. This figure orders all the couple profiles found in the initial and final solutions by its objective function. In the front, the light area represents the number of couples in the initial solution. In the background, the dark area displays the number of couples in the final solution. Clearly, the initial solution has more high cost profiles on the right as the light area hides the dark area. But for low cost profiles grouped on the left, as the light area is exceeded by the dark area, there are much more couples in the final solution than in the initial solution. If we take into account costs lower than 4, the initial situation includes 58.95% of couples, whereas the final solution reaches 94.18%.

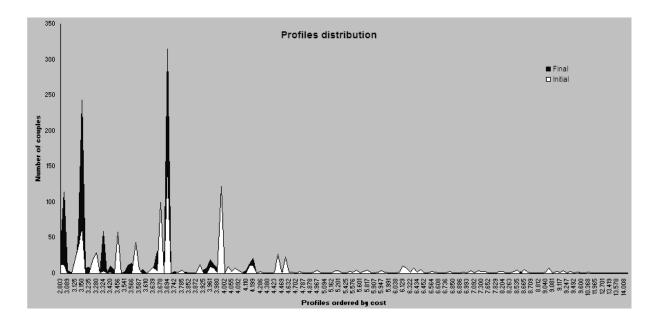


Figure 6: Profiles distribution of couples ordered by cost

6. Alternative choices versus unique solution

The choice of a partner could be subjective and influenced by other criteria than those proposed in this article. For this reason, it is interesting to offer alternative choices to couples instead of a unique optimal solution.

In our case, it is possible to generate other good solutions from the optimal solution we optain with CPLEX (see §5.1). Given

w1 and w2 two women

 $mil\ the\ man\ associated\ to\ woman\ w1$ in the initial solution,

mi2 the man associated to woman 2 in the initial soltion

mf1 the man associated to woman w1 in the optimal solution

mf2 the man associated to woman w2 in the optimal solution

We assume that if w1 and w2, respectively mi1 and mi2, mf1 and mf2 have the same value for all the criteria considered then we can create a new optimal solution in which woman w1 is associated to man mf2 and woman w2 is associated to mf1.

The table below shows 74 groups in which all the women, all the men to whom they are associated in the initial and in the final solution have the same caracteristics. There are 4 couples in group 4. That means that we can generate 4! = 24 optimal solutions by permuting the final men in this groups. The total number of optimal solutions we can generate with 74 is 214 solutions.

		N !			N!			N!
Grou	N	distinct		N	distinct		N	distinct
р	Couples	solutions	Group	Couples	solutions	Group	Couples	solutions
1	2	2	26	2	2	51	2	2
2	2	2	27	2	2	52	2	2
3	3	6	28	2	2	53	2	2
4	3	6	29	2	2	54	2	2
5	2	2	30	2	2	55	2	2
6	2	2	31	2	2	56	2	2
7	2	2	32	2	2	57	2	2
8	2	2	33	3	6	58	2	2
9	2	2	34	2	2	59	2	2
10	2	2	35	2	2	60	2	2
11	3	6	36	2	2	61	2	2
12	2	2	37	2	2	62	2	2
13	4	24	38	3	6	63	2	2
14	2	2	39	2	2	64	2	2
15	2	2	40	3	6	65	2	2
16	2	2	41	2	2	66	3	6
17	2	2	42	2	2	67	2	2
18	2	2	43	2	2	68	2	2
19	2	2	44	2	2	69	2	2
20	2	2	45	3	6	70	2	2
21	2	2	46	2	2	71	3	6
22	3	6	47	2	2	72	2	2
23	2	2	48	3	6	73	2	2
24	2	2	49	2	2	74	2	2
25	2	2	50	2	2			

In other words, couples can interact with the optimal solution. Within a given group, women can of n couples, we can offerta wome women can choose in each group can choose one men within the group.

Imagine we have a matrimonial agency with an optimal assignment. Within a same group of n couples, the agency can make different arrangements to get n! optimal alternatives.

7. Discussion and conclusion

This paper proposes an innovative method for optimizing the allocation of partners by

applying an operation research model on the allocation of partners in couples. In the optimization model we have assigned men to women in a way that minimizes the number of divorces for a given society. This allocation was done according to empirical criteria stemming from straightforward statistical analyses such as logistic regressions. We found that, within the set of demographic constraints characterizing the Swiss marriage market, about 7 individuals over 10 (68%) can be reallocated to a couple with a higher chance of survival. This reallocation leads the initial non optimal situations to the final optimal situations with a reduction of the objective function by 21% of its initial value.

Based on this set of evidence, one can also state that current marriage markets are suboptimally organized and that matching individuals according to a small number of objective criteria such as age, education and cultural origin following well-known algorithms of optimization may help reduce divorce and thus alleviate its economic, social and psychological costs. Marriage was for years under the supervision of the community in the Western world (Goode, 1962). Only recently was marriage considered as a purely private matter serving the psychological needs of individuals, with mate selection left apparently to chance only. The increasing number of couples matched on the Internet by on-line specialized agencies suggests that this time may be soon over. The stiffening of regulations concerning opposite sex relations at work and elsewhere, and the overall complexification of life trajectories (Sapin et al., 2007), among a variety of sociological trends, make more and more individuals ask for such institutional support when searching for a spouse or a partner. Therefore, it is the right time to develop efficient allocation tools for matching individuals, which may grant those agencies and their clients with a sound understanding of the survival chance of their matching. On the long run, such expertise may help societies fight against rising rates of divorce or separation.

The proposed methodology allows also to better understand how structures of the marriage

market impact on conjugal mismatches. Some characteristics, although, strongly associated with divorce cannot lead to large reallocations of partners because their distribution in the sample are highly skewed. This is the case of nationality: although matches between Swiss and non-Western individuals have very high odds of separation or divorce, the number of individuals that can be reallocated is small due to the very low pool of available mates of the right nationality with good matches on other dimensions (age, education, previous divorce). When several parameters are intercorrelated (as in the case of birth cohorts and education) reallocation becomes tricky. The great social diversity of contemporary societies makes this problem extremely relevant: with increasing social diversity and increasing individualism, the likelihood of finding an optimal match on all criteria becomes smaller. In order to solve this and other related problems, we intend in the next step of this research to add more criteria, such as religiosity and personality factors to the experimentation. We also wish to integrate a dynamic structure in the optimization model in order to better take into account the different states of life trajectories. These developments may help to better estimate the impact of mate selection within the highly complex set of demographic constraints characterizing current marriage markets.

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