Student sectioning for minimizing potential conflicts on multi-section courses

David Schindl

Abstract In sufficiently large schools, lessons are given to classes in sections of various sizes, depending on the subject taught. Consequently, classes have to be split into various given numbers of sections. We focus on how to subdivide a class in subgroups, so as to be able to reproduce all required sections by merging subgroups together, while minimizing the number of edges in the resulting course conflict graph. As a main result, we show that subdividing the students set in a regular way is optimal. We then discuss our solution uniqueness and feasibility, as well as practical issues concerning teacher assignments to sections.

Keywords student sectioning · timetabling · conflict graph

1 Introduction

Depending on the subject taught, classroom sizes, teachers preferences or budget, the number of subgroups - or *sections* - created from a given class may vary from one subject to the other. For instance, given a class with 120 students, one may need to create 3 sections of 40 students for economics classes, 5 sections of 24 students for informatics and 6 sections of 20 students for english courses. One of the tasks included in a timetable creation is to actually decide how to dispatch students into these sections. If handled first, this task, called *student sectioning*, has a crucial importance when it comes to timetabling, i.e. assigning timeslots to courses[1].

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[1] Although the word "timetabling" usually refers to the whole process of creating a timetable for each student and each teacher, in this paper we call timetabling the task of assigning timeslots to courses, as a problem separate from student sectioning.

While student sectioning usually refers to assigning students to optional courses (see Section 2), we are interested here in the special but nontrivial case where all courses are mandatory. Such a situation occurs in many, if not most, schools: as soon as there are at least two different section numbers for the same class, the question inevitably arises about how should the students be dispatched into these sections. It is in particular the case each semester on several classes at Haute Ecole de Gestion de Genève, and this is what motivated this topic. We describe in next paragraph a reasonable criterion to evaluate a sectioning and will show later that a natural construction produces an optimal solution.

A timetable should avoid assigning two courses a common timeslot if they have at least one student in common. Taking only this kind of constraints into account, there is a straightforward feasible timetable obtained by assigning all courses of the same subject the same set of timeslots. Indeed, since each student has exactly one course of each subject, two courses of the same subject will never share a student and can therefore be scheduled in parallel. However, there are usually many other constraints avoiding this simple timetable, starting with the conflict between courses sharing the same teacher. Adding, among others, teachers and classes availability constraints, it quickly becomes very difficult (in general NP-complete) to predict what a feasible schedule will be like, or if there is one at all. However, in order to maximize our chances to obtain a good schedule, it is reasonable, while dispatching students into sections, to make the number of pairs of potentially conflicting courses as small as possible. This number is precisely the criterion we aim at minimizing by dispatching students into sections in an appropriate way.

The paper is organized as follows. Section 2 consists of a literature review related to student sectioning and more specifically to our problem. A precise formulation is given in Section 3 and its solution and proof are presented in Section 4. We follow with a discussion in Section 5 and conclude by asking two open questions.

2 Related literature

Our main contribution is based on mathematical considerations and concerns a sectioning strategy to be applied on mandatory courses before timetabling, whereas most literature on student sectioning deal with optional courses and present heuristic approaches which are run before, in parallel of, or after a timetabling algorithm. We refer the interested reader to [5] for a detailed review on these methods, and we mention here only those who are most linked to this work.

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In [2], students have some mandatory courses and some optional ones, to choose from a list. Prior to timetabling, the author proposes to create an initial, called *homogeneous* sectioning by grouping students in clusters according to *similarities* between their optional course selections. The problem is then

decomposed into smaller subproblems and greedy algorithms are run on each of them to assign timeslots to courses. After timetabling, some assignments are modified using an alternating tree approach, in order to decrease conflicts in students schedules. The authors of [6] present and discuss several practical timetabling issues in creating the timetable of a large university. Their initial student sectioning is created in the same homogeneous way as above and once the timetable is settled, the number of conflicts is lowered via a local search over the student to sections assignments. This work is extended in [5], where the same homogeneous sectioning is initially computed, but after the timetable is set,it is improved by a batch sectioning heuristic using several kinds of neighborhoods to diminish the number of conflicts. The obtained sectioning can then be further modified in an online manner since students may make schedule change requests, which are then processed by an algorithm trying to satisfy them without changing previously sectioned students assignments. In [1], an experimental study is reported about the impact on space utilization and timetabling feasibility of varying the maximum size of groups and the number of sections. They show that the latter parameter has the most impact on feasibility and observe that allowing schedule changes while assigning students also has a significant impact on feasibility. A similar issue is handled with a graph theoretical approach in [7], where the author discusses on how many vertex splits permit to decrease the chromatic number, i.e. how many additional sections must be introduced in order to decrease the number of necessary timeslots to ensure feasibility. A lower bound is given in function of the total number of available subjects and the number of subjects to be chosen by each student. The author of [4] proposes a heuristic consisting of two tabu searches over the set of timetables allowing conflicts in student schedules. In a first search, courses are only assigned to timeslots, and in the second search, only students exchanges are allowed between different sections of a course, to try to diminish the number of conflicts in the student's schedules. In [3], the authors investigate the problem of sectioning with a given timetable in terms of complexity. They show that a basic version of sectioning where each course uses exactly one timeslot is solvable with a network flow algorithm. On the other hand, they prove NP-completeness of three generalizations, one of them they call "BSS + (E)", where courses may have several timeslots. Our problem can be viewed as a special case of this last version, since we additionally require sections of a same subject to be balanced. An even more special case with practical interest could be obtained by requiring timeslots associated to courses to be consecutive. To our knowledge the complexity statuses of these two problems are not settled yet.

3 Problem formulation

Consider a class of students, each one following mandatory courses on k subjects. Each subject i, with $1 \leq i \leq k$, is taught in n_i sections of equal sizes (plus or minus 1). Consequently, for each i, our class has to be split into n_i

sections, each containing the fraction $\frac{1}{n_i}$ of the students. Courses on a same subject i will be denoted $c_{i,1}, \dots c_{i,n_i}$ and they have the same duration d_i. Duration may however vary from one subject to the other. Each course has to be assigned a set of students and a set of timeslots.

For the student assignment, we define for each subject $i \in \{1, \dots, k\}$ a function

$$s_i : [0, 1] \to \{1, \dots, n_i\}$$

Each function s_i represents the students assignment to one of the n_i sections of subject i and will be called *sectioning* i. For instance, if $s_3(x) = 4 \ \forall \ x \in [0.4, 0.45[$, it means that each student ranked (for instance according to alphabetical order) between 40% and 45% belongs to the 4^{th} section in the 3^{rd} sectioning. The set of sectionings together will be simply called a *sectioning set* and denoted by S. Figure 1 is a schematic view of the following sectioning set $S = \{s_1, s_2, s_3\}$ with $k = 3$ subjects, $n_1 = 2$, $n_2 = 3$ and $n_3 = 5$.

$$s_1(x) = \begin{cases} 1 & \text{if } x \in [0, 0.3[\cup [0.6, 0.8[\\ 2 & \text{if } x \in [0.3, 0.6[\cup [0.8, 1] \end{cases}$$

$$s_2(x) = \begin{cases} 1 & \text{if } x \in [0, 0.2[\cup [0.867, 1[\\ 2 & \text{if } x \in [0.2, 0.533[\\ 3 & \text{if } x \in [0.533, 0.867] \end{cases}$$

$$s_3(x) = \begin{cases} 1 & \text{if } x \in [0, 0.1[\cup [0.45, 0.55[\\ 2 & \text{if } x \in [0.1, 0.2[\cup [0.3, 0.4[\\ 3 & \text{if } x \in [0.2, 0.3] \ \cup \ [0.9, 1] \\ 4 & \text{if } x \in [0.4, 0.45] \ \cup \ [0.55, 0.7] \\ 5 & \text{if } x \in [0.7, 0.9] \end{cases}$$

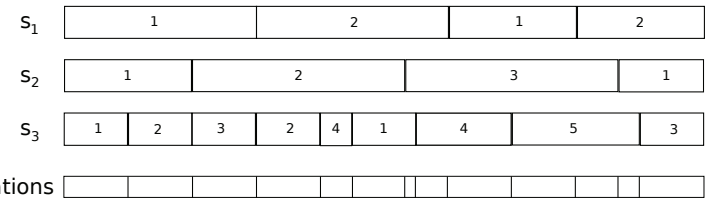

Fig. 1 Sectioning example with $k = 3$ subjects, $n_1 = 2$, $n_2 = 3$ and $n_3 = 5$.

For the timeslots assignment, we define the following function which we will call a *timetable* :

$$T : C \to \mathcal{P}(P)$$

where C is the set of courses and P is the set of time periods or timeslots. For instance, if $T(c_{3,4}) = \{5, 8, 9\}$, the fourth section of subject 3 is scheduled on timeslots 5, 8 and 9.

In a feasible timetable, two courses sharing a timeslot should not share a student. We consequently define potential conflict graphs of the two following kinds on the set of courses C. Given a sectioning set $S = \{s_1, \ldots, s_n\}$, two vertices $c_{i,j}$ and $c_{i',j'}$ are adjacent in the *students conflict graph* $G_S = (C, E_S)$, if and only if $s_i^{-1}(j) \cap s_{i'}^{-1}(j') \neq \emptyset$. Notice that for each subject i, the vertex set $C_i = \{c_{i,j} : j = 1 \ldots n_i\}$ is a stable set in G_S. Given a timetable T, two vertices $c_{i,j}$ and $c_{i',j'}$ are adjacent in the *timeslots conflict graph* $G_T = (C, E_T)$, if and only if $T(c_{i,j}) \cap T(c_{i',j'}) \neq \emptyset$.

4 Optimal sectioning

Our main result, which is presented in this section, shows how to choose a sectioning set S so as to minimize the number of edges of G_S, regardless of the timetable. By $\mathrm{GCD}(a, b)$ we denote the greatest common divisor of positive integers a and b and by $\mathrm{LCM}(a, b)$ their least common multiple. These are linked by the formula $\mathrm{GCD}(a, b)\mathrm{LCM}(a, b) = ab$.

Consider subjects i and i' and a sectioning set S. Since the vertex sets C_i and $C_{i'}$ are stable in G_S, we have

$$\begin{aligned}\min_{s_1,\ldots,s_k} |E(G_S)| &= \min_{s_1,\ldots,s_k} \sum_{i,i' \in \{1,\ldots,k\}, i<i'} |E(G_S[C_i \cup C_{i'}])| \\ &\geq \sum_{i,i' \in \{1,\ldots,k\}, i<i'} \min_{s_i, s_{i'}} |E(G_S[C_i \cup C_{i'}])|\end{aligned}$$

Consequently, if we are able to find a sectioning set $S = \{s_1, \ldots, s_k\}$ such that

$$|E(G_S[C_i \cup C_{i'}])| = \min_{s_i, s_{i'}} |E(G_S[C_i \cup C_{i'}])| \quad \forall \quad 1 \leq i < i' \leq k$$

we are done.

Next Proposition shows that it is actually the case, and the corresponding sectionings are the ones where all preimages $s_i^{-1}(j)$ are intervals. Figure 2 displays the optimal sectioning set corresponding to our example above.

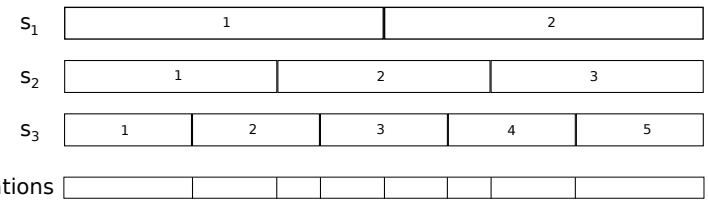

Fig. 2 Optimal sectioning with $k = 3$ subjects, $n_1 = 2$, $n_2 = 3$ and $n_3 = 5$.

Proposition 1 *If $s_i(x) = \lceil n_i x \rceil \ \forall\, i \in \{1, \dots, k\}$, then for each $i < i'$ we have*

$$\begin{aligned} |E(G_S[C_i \cup C_{i'}])| &= \min_{s_i, s_{i'}} |E(G_S[C_i \cup C_{i'}])| \\ &= n_i + n_{i'} - \mathrm{GCD}(n_i, n_{i'}) \end{aligned}$$

Proof We begin our proof with the following observations on properties we may assume on optimal sectionings s_i and $s_{i'}$.

Observation 1. There are optimal sectionings s_i and $s_{i'}$, with $s_i(x) = \lceil n_i x \rceil$.

Indeed, let $\tilde{s}_i$ and $\tilde{s}_{i'}$ be optimal. Since $|\tilde{s}_i^{-1}(j)| = |s_i^{-1}(j)| = \frac{1}{n_i} \quad \forall j \in \{1, \dots, n_i\}$, there is a bijection σ from the $[0, 1]$ interval into itself, such that $\tilde{s}_i(\sigma(x)) = \lceil n_i x \rceil = s_i(x)$. Then choosing $s_{i'}(x) = \tilde{s}_{i'}(\sigma(x))$, s_i and $s_{i'}$ induce exactly the same conflict graph as $\tilde{s}_i$ and $\tilde{s}_{i'}$, since $x \in s_i^{-1}(j) \cap s_{i'}^{-1}(j')$ if and only if $\sigma(x) \in \tilde{s}_i^{-1}(j) \cap \tilde{s}_{i'}^{-1}(j')$. In particular, both conflict graphs have the same, minimal, number of edges.

From now on, we assume $s_i(x) = \lceil n_i x \rceil$.

Observation 2. There are optimal sectioning s_i and $s_{i'}$ such that the sets $s_i^{-1}(j) \cap s_{i'}^{-1}(j')$ are intervals.

For each $1 \leq j \leq n_i$, an argument similar to the above, with σ being a bijection from the interval $[\frac{j-1}{n_i}, \frac{j}{n_i}[$ into itself, permits to assume that the sets

$$s_i^{-1}(j) \cap s_{i'}^{-1}(j') = \left[\frac{j-1}{n_i}, \frac{j}{n_i}\right[\bigcap s_{i'}^{-1}(j'),\ 1 \leq j' \leq n_{i'}$$

are connected, and hence are intervals.
We may thus assume that each set $s_i^{-1}(j) \cap s_{i'}^{-1}(j')$ is an interval. Recalling that $|E(G[C_i \cup C_{i'}])|$ is equal to the number of sets $s_i^{-1}(j) \cap s_{i'}^{-1}(j')$ that are non empty, $|E(G[C_i \cup C_{i'}])|$ is equal to the number of intervals $s_i^{-1}(j) \cap s_{i'}^{-1}(j')$.

We now show that $|E(G[C_i \cup C_{i'}])| \geq n_i + n_{i'} - \mathrm{GCD}(n_i, n_{i'})$. Assume by contradiction that the number of intervals $s_i^{-1}(j) \cap s_{i'}^{-1}(j')$ is at most $n_i + n_{i'} - \mathrm{GCD}(n_i, n_{i'}) - 1$. Of course, if there are, say p intervals, there are $p-1$ *separations*, i.e. pairs of adjacent (in $[0, 1]$) intervals (see Figures 1 and 2). This means that there are at most $n_i + n_{i'} - \mathrm{GCD}(n_i, n_{i'}) - 2$ such separations. Further, exactly $n_i - 1$ of them are induced by the intervals $s_i^{-1}(j)$. We call them *original*. Hence there are at most $n_{i'} - \mathrm{GCD}(n_i, n_{i'}) - 1$ separations that are non original and exclusively induced by the sets $s_{i'}^{-1}(j')$. Let us call them *additional*.
Construct an auxiliary graph $H = (V_h, E_h)$ with $V_h = \{h_1, \dots, h_{n_{i'}}\} = \{s_{i'}^{-1}(j') : 1 \leq j' \leq n_{i'}\}$ and two vertices $h_{j'_1}$ and $h_{j'_2}$ being adjacent in H if and only if there is at least one pair of intervals $s_i^{-1}(j_1) \cap s_{i'}^{-1}(j'_1)$ and $s_i^{-1}(j_1) \cap s_{i'}^{-1}(j'_2)$ sharing an additional separation. The number of connected components of H is at least $|V_h| - |E_h| = n_{i'} - (n_{i'} - \mathrm{GCD}(n_i, n_{i'}) - 1) = \mathrm{GCD}(n_i, n_{i'}) + 1$. Consider the smallest one. It has at most $\left\lfloor \frac{n_{i'}}{\mathrm{GCD}(n_i, n_{i'})+1} \right\rfloor$

vertices. Observe that each connected component of the corresponding union of intervals in $[0,1]$ is only delimited by original separations. As a consequence, the sum of its interval lengths must be a multiple of $\frac{1}{n_i}$, say $\frac{a}{n_i}$, $a \in \mathbb{N}^*$:

$$\begin{aligned} \frac{a}{n_i} &= \left\lfloor \frac{n_{i'}}{\mathrm{GCD}(n_i, n_{i'}) + 1} \right\rfloor \cdot \frac{1}{n_{i'}} \\ \Leftrightarrow a n_{i'} &= \left\lfloor \frac{n_{i'}}{\mathrm{GCD}(n_i, n_{i'}) + 1} \right\rfloor n_i \end{aligned}$$

In particular $an_{i'} \geq \mathrm{LCM}(n_i, n_{i'})$. On the other hand,

$$\begin{aligned} \frac{a}{n_i} &= \left\lfloor \frac{n_{i'}}{\mathrm{GCD}(n_i, n_{i'}) + 1} \right\rfloor \cdot \frac{1}{n_{i'}} \\ &< \frac{n_{i'}}{\mathrm{GCD}(n_i, n_{i'})} \cdot \frac{1}{n_{i'}} = \frac{1}{\mathrm{GCD}(n_i, n_{i'})} \\ &= \frac{\mathrm{LCM}(n_i, n_{i'})}{n_i n_{i'}} \end{aligned}$$

Hence $an_{i'} < \mathrm{LCM}(n_i, n_{i'})$, a contradiction.

To show that for $s_i(x) = \lceil n_i x \rceil$ and $s_{i'}(x) = \lceil n_{i'} x \rceil$ we have $|E(G[C_i \cup C_{i'}])| = n_i + n_{i'} - \mathrm{GCD}(n_i, n_{i'})$, we only need to observe that the points in $[0,1]$ that are both original and additional separations are

$$a \frac{\mathrm{LCM}(n_i, n_{i'})}{n_i n_{i'}}, \quad a \in \{1, \ldots, \mathrm{GCD}(n_i, n_{i'}) - 1\}$$

Indeed, separations that are common to both s_i and $s_{i'}$ are common multiples of $\frac{n_i}{n_i n_{i'}}$ and $\frac{n_{i'}}{n_i n_{i'}}$.
So the total number of separations is

$$(n_i - 1) + (n_{i'} - 1) - (\mathrm{GCD}(n_i, n_{i'}) - 1) = n_i + n_{i'} - \mathrm{GCD}(n_i, n_{i'}) - 1$$

and the number of edges of the conflict graph, which is equal to the number of intervals, is $n_i + n_{i'} - \mathrm{GCD}(n_i, n_{i'})$.

In the sequel, we call *regular* the sectioning set of Proposition 1, i.e. $S = \{s_1, \ldots s_k\}$, with $s_i(x) = \lceil n_i x \rceil, i = 1, \ldots, k$. Notice that unsurprisingly, regular sectioning follows the same principle as homogeneous sectioning proposed in [2], in the sense that if two students are involved in the same sections for subjects 1 to $k-1$, it means that they are ranked closely and are therefore likely to be also involved in the same section for subject k.

5 Discussion

5.1 Uniqueness

Observe that if there are only two different section numbers n_1 and n_2, other sectioning sets may also minimize the edges in the conflict graph. For instance,

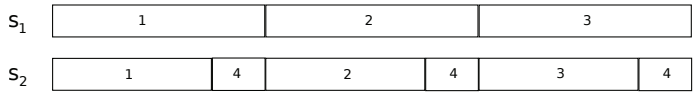

Fig. 3 Sectioning set $\tilde{S}$.

if $n_1 = 3$ and $n_2 = 4$, the sectioning set $\tilde{S}$ depicted in Figure 3 also produces $n_1 + n_2 - \mathrm{GCD}(n_1, n_2)$ edges in $G_{\tilde{S}}$. However, this sectioning set is specific to the section numbers $n_1 = 3$ and $n_2 = 4$ and finding an optimal sectioning for a third subject with a larger number of sections may become tedious, if not impossible. In contrast, our regular sectioning function $s_i = \lceil n_i x \rceil$ does not depend on other values $n_j,\ j \neq i$, permitting to add any number of sections with guaranteed optimality as long as regular sectioning is used.

5.2 Timetabling feasibility

Given any sectioning set, it is not clear whether a conflict free timetable exists or not, since there may be several additional constraints preventing it, like restrictive teachers availabilities. However, permuting sections (i.e. set of students) of the same subject may be allowed as long as teacher assignments remain unchanged. Given a timetable T, we call a sectioning set S *feasible* with respect to T if there exists a set of section permutations $\sigma_1, \ldots, \sigma_k$, with σ_i over the set $\{1, \ldots, n_i\}$, such that the resulting timetable is conflict free, i.e. such that $T(c_{i,j}) \cap T(c_{i,j'}) = \emptyset$ whenever $s_i^{-1}(\sigma_i(j)) \cap s_i^{-1}(\sigma_i(j')) \neq \emptyset$. Obviously, regular sectioning may not be feasible for any timetable, the simplest example being where all courses are timetabled in parallel. A less trivial and more useful question is the following: if there exists a feasible sectioning set for some timetable T, is the regular sectioning set also feasible? In the general case the answer is negative, and we illustrate it with a small example of 12 students, 2 subjects of 3 and 4 sections, durations $d_1 = 1$ and $d_2 = 2$, and the timetable depicted in Table 1. Indeed, Figure 4 displays the corresponding

Course $c_{i,j}$	$c_{1,1}$	$c_{1,2}$	$c_{1,3}$	$c_{2,1}$	$c_{2,2}$	$c_{2,3}$	$c_{2,4}$
$T(c_{i,j})$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{4,5\}$

Table 1 Timetable incompatible with regular sectioning.

timeslots conflict graph[2]. If we apply the regular sectioning approach, the resulting sectioning conflict graph is, up to section permutations within the same subject, depicted in Figure 5. Clearly, S is feasible with respect to T if and only if there is a permutation of $\{c_{1,1}, c_{1,2}, c_{1,3}\}$ (or $\{c_{2,1}, c_{2,2}, c_{2,3}, c_{2,4}\}$) such that no pair of vertices is an edge in both conflict graphs. It is not the case,

[2] For the sake of simplicity, we do not display edges between vertices of the same subject, since these potential conflicts can never be realized.

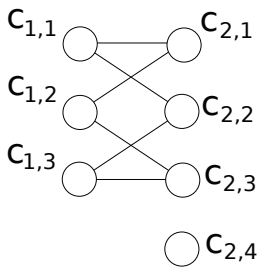

Fig. 4 Timeslots conflict graph G_T.

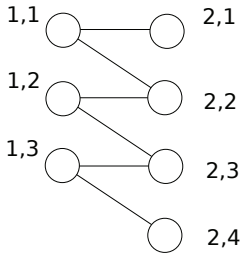

Fig. 5 Sectioning conflict graph G_S obtained with regular sectioning.

since there are 3 vertices of degree 2 among the $c_{2,j}$ vertices in G_T, whereas there are only 2 vertices of degree one among them in G_S. Now consider the same sectioning set $\tilde{S}$ as in Figure 3, and described in Table 2 for our 12 students. The corresponding sectioning conflict graph is displayed in Figure

Student	1	2	3	4	5	6	7	8	9	10	11	12
Section for subject 1	1	1	1	1	2	2	2	2	3	3	3	3
Section for subject 2	3	3	3	4	2	2	2	4	1	1	1	4

Table 2 Sectioning set $\tilde{S}$.

6. This graph is the bipartite complement of the timeslots conflict graph G_T, thus $\tilde{S}$ is feasible with respect to T.

This example shows that if regular sectioning permits to minimize the number of edges, it may not be the only optimal solution in that sense, and there may exist timetables that are not feasible for S, but feasible for another sectioning set. However, in this example, the course $c_{2,2}$ is assigned timeslots 1 and 3, and we did not find such an example where all course are assigned only consecutive timeslots, which we may expect in practice. The existence or not of such an example remains an open question.

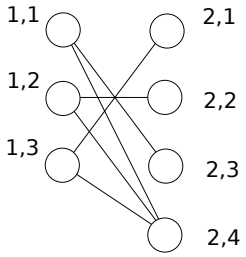

Fig. 6 Sectioning conflict graph $G_{\tilde{S}}$ obtained with sectioning set $\tilde{S}$.

5.3 Teacher assignments

In most cases, teachers are indifferent to which sections they are assigned. The section assignments among teachers within a same subject can therefore be freely permuted. We provide here some hints on how to exploit this flexibility prior to timetabling. We again base our discussion on the number of potential conflicts, and consider teachers like students, in the sense that they contribute to edges in the student conflict graph.

If we restrict attention to only one subject, we observe that changing teacher assignments to sections will not change the conflict subgraph (up to isomorphism) induced by sections of this subject, hence the number of edges.

If a teacher gives courses in two subjects, it may be preferable to assign him or her sections with similar sets of students. Indeed, these sections are likely to be already adjacent in the student conflict graph, so adding the teacher to them will create only few new edges, if any. Observe that this similarity argument is consistent with regular sectioning and homogeneous sectioning ([2]).

Finally, for the vast majority of cases where two teachers give courses in two different subjects, a criterion could be based on their availabilities: it may be suitable to assign similar sections to teachers having different availabilities. This criterion suggests a preprocessing consisting in permuting teachers among sections in order to maximize the probability of existence of a feasible timetable. However, this task does not promise to be trivial: if two teachers have complementary availabilities and are assigned similar sections, what about a third teacher having availabilities similar to the first one?

6 Conclusion

In this paper, we address the problem of student sectioning on mandatory courses with various numbers of sections, before knowing the timetable. We prove that sectioning students in a regular way permits to minimize the number of potential conflicts between courses. We then discuss uniqueness of this

optimal solution, the existence of a feasible timetable and some issues about teachers assignments. We leave the reader with the following open questions:

- Are there section numbers $n_1, \dots n_k$ on k subjects, with durations $d_1, \dots, d_k$, a non-regular balanced sectioning $\tilde{S}$ and a timetable T where all courses are assigned consecutive timeslots, such that $\tilde{S}$ is feasible with respect to T, but the regular sectioning is not?
- Is the problem "BSS + (E)" defined in [3] still NP-complete, when restricted to the case where sections of a same subject have to be balanced and courses are assigned only consecutive timeslots?

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