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Optimal Monetary Policy when Information is Market-Generated*

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Abstract

In this paper we show that endogenous - i.e. market-generated - signals observed by the private sector have crucial implications for monetary policy. When information is endogenous, achieving the optimum through price stabilization is elusive. The optimal policy then consists, on the contrary, in exacerbating the natural response of prices to shocks. In our framework, where supply shocks are naturally deflationary, optimal policy is then countercyclical, whereas the standard price-stabilizing policy would have been procyclical. The role of endogenous signals is independent of the possibility of the central bank to directly communicate its private information through public announcements.

Keywords: Optimal monetary policy, information frictions, expectations.

JEL codes: D83, E32, E52.

1 Introduction

The state of the fundamentals cannot be perfectly observed by the private agents that populate the economy. They have to rely on signals that help them formulate their expec-

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tations. We define as exogenous information the one that comes from sources that are not market-determined, like exogenous signals or news. By endogenous information instead we mean the one that can be inferred from variables that are market-determined, like prices or quantities. Obviously, agents can learn from both sources of information. In this paper we show that the endogenous nature of signals affects optimal monetary policy.

Using a model with imperfectly competitive price-setters, flexible prices and a central bank that maximizes social welfare and receives a - possibly noisy - private signal on the state of fundamentals, we show that the nature of the signals received by price-setters dramatically affects the way in which monetary policy should be conducted. When information is endogenous, prices should signal the policymaker's information, especially when shocks drive efficient fluctuations. This contrasts with the case with exogenous information, where price stabilization has been found to be optimal. Our results remain true even if we assume that the central bank can directly communicate its private information to the agents.

To understand, suppose signals are purely exogenous. In this case, it is optimal for the central bank to implement a "demand management' type of policy aimed at directly steering the economy towards the efficient allocation. Consider for example a labor supply shock that decreases the marginal cost. Price-setters, who are imperfectly informed on this shock, would decrease their price to stimulate demand, but less than what would be optimal, and production would increase less than in a perfect information world, generating a negative output gap. The optimal policy of an informed central bank would then be to close the output gap by implementing an expansionary policy. In fact, optimal policy is such that the central bank "does the job", so that price-setters do not need to change their price. This policy therefore amounts to stabilizing prices. This policy also implies that individual prices do not need to reflect the individual information of price-setters, which contains price dispersion.

We assume instead that information is endogenous, by allowing price-setters to use the observed demand for their individual good as a signal. This has two crucial consequences. First, the central bank cannot close the output gap through price stabilization. In our example, an expansionary policy would partially reveal itself to the agents through higher individual demand and would then be offset through higher prices, so this policy cannot effectively stimulate the economy nor stabilize prices. The central bank cannot rely on traditional demand management. Second, this does not imply that the central bank is powerless. In fact, as markets do reveal some information to the agents, this provides a new channel for policy: the central bank can transmit its knowledge to the public by affecting the endogenous signals that agents observe through monetary policy. Indeed, for a given individual price, the individual demand comoves positively with the aggregate price, which reflects to a certain extent the fundamental shock. With its action the central bank can then emphasize the natural movement of prices so that individual demand becomes a powerful signal to which agents could react. A restrictive monetary policy reinforces the reduction in prices due to the shock, making prices more informative about the underlying productivity shock. The stronger reduction in prices generates a more accurate response of price-setters to the positive shock, which helps close the output gap. Because price-setters are more informed about the underlying fundamentals, this policy also reduces price dispersion.

Our paper confers a crucial informational role to equilibrium variables. This approach dates back to Phelps (1969) and Lucas (1972). Among recent papers that analyze how endogenous information affects economic outcomes, without looking at the monetary policy implications, are Angeletos and Werning, (2006), Amador and Weill (2010, 2012), Benhima (2014), Hellwig and Venkateswaran (2009) and Gaballo (2015). An earlier literature, including King (1982), Dotsey and King (1986) and Weiss (1980), examines how monetary policy affects the information content of prices. However, they focus on optimal feedback rules, as they assume that the central bank has no private information on the current state, contrary to us.[1] The problem of the central bank in our context is how to optimally use its private information on the shocks hitting the economy. In a standard framework, monetary policy should accommodate these shocks and aim for price stability. But we show that the traditional results in terms of optimal policy are reversed when information is endogenous.

Indeed, the analysis of optimal monetary policy has been done mostly in the context of exogenous imperfect information. In that context, the general result is that optimal monetary policy targets price stability, at least for shocks driving efficient fluctuations, like productivity shocks. This extends the results of the sticky-price New-Keynesian literature such as Gali (2008) and Woodford (2003), which has also established the benefits of price stability. Ball et al. (2005) find this in a sticky-information à la Mankiw and Reis (2002), while Adam (2007) finds this in a rational inattention model à la Sims (2003). Paciello and Wiederholt (2014) show that, when inattention is endogenous, price stabilization also generalizes to shocks that drive inefficient fluctuations, like mark-up shocks. Lorenzoni (2010) studies optimal policy in the case of dispersed information and the central bank

[1] Besides, in King (1982) and Dotsey and King (1986), dispersed information play a key role, which is not the case in our context.

has no informational advantage over private agents. In these papers, all the signals received by decision-makers are exogenous, contrary to our approach, where some signals are endogenous.[2]

An exception is Angeletos and La'o (2013) who find, like us, that price stabilization is not optimal, and that policy should target a negative correlation between prices and economic activity. It is important to note, however, that our result depends on the presence of endogenous information, while theirs originates from the assumption, in their model, of both nominal and real frictions. In an earlier version of their paper (Angeletos and La'o, 2008), endogenous information accentuate their finding, but the role it plays, as opposed to the role played by real frictions, is not obvious.

Our approach is also linked to the literature on how the central bank should communicate on its private information. In our paper, indeed, we assume that the central bank receives signals that are distinct from the private agents'. This is backed by the empirical literature on the signalling channel of monetary policy, which suggests that the central bank might have some information that is not common knowledge, and that can therefore be communicated through policy.[3] Some papers have explored theoretically the consequences of the signaling channel of monetary policy through monetary instruments, as Tang (2015), Baeryswil and Cornand (2010) or Berkelmans (2011). Our approach differs from theirs as we assume the agents' source of information is their local market. As a result, the information conveyed by monetary policy is mediated by the structure of the economy and blurred by the shocks hitting the economy. This has crucial implications on the optimal elasticity of the policy instrument to the central bank's information. The central bank can also choose to communicate directly its information to the public, with more or less precision.[4] In an extension, we consider the case where the central bank additionally communicates on its information (or equivalently, on its instrument) and our results are unchanged, whatever the transparency of central bank communication.

Finally, our main focus is on shocks that drive efficient fluctuations (labor supply shocks). In that case, the central bank objective is to maximize the information content of endogenous variables. However, as Angeletos and Pavan (2007) stress, more information can be detrimental in the case of shocks that drive inefficient fluctuations, like mark-up

[2]Paciello and Wiederholt (2014) study an extension where decision-makers choose the optimal linear combination of shocks to observe. In our case, the signals observed by agents come from the market they are involved in, it is therefore not necessarily optimal, and this is what drives our results.

[3]See Romer and Romer (2010), Justiniano et al. (2012), Nakamura and Steinsson (2015), Sims (2002) and Melosi (2013).

[4]See for example Morris and Shin (2002), Hellwig (2005), Woodford (2005), Angeletos and Pavan (2007), Amador and Weill (2010), who study the optimal level of central bank "transparency".

shocks. We therefore introduce these shocks in an extension and show that, contrary to labor supply shocks, the central bank wants to minimize the the information content of endogenous variables. However, this does not mean that the central bank is passive. On the opposite, the inflationary tendency induced by mark-up shocks has to be counteracted by a contractionary monetary policy. All in all, the price reaction of "efficient shocks" has to be accentuated while the price reaction of "inefficient shocks" has to be counterbalanced.

The structure of the paper is the following. Section 2 presents our baseline model with price-setters and productivity shocks. In section 3 we study the equilibrium of the model. Section 4 studies optimal policy. Section 5 presents extensions with alternative information sets. Section 6 presents an extension with mark-up shocks, which drive inefficient fluctuations in output. Section 7 concludes.

2 The model

We consider a one-period model with flexible prices. There is a representative household composed of a consumer and a continuum of workers distributed over the unit interval, indexed by $i \in [0,1]$, and a central bank. The consumer consumes a bundle of goods C and each worker i supplies her labor N_i in order to produce a differentiated good, also indexed by i, in the quantity Y_i. The central bank conducts monetary policy with the goal of maximizing the utility of the representative agent.

2.1 The household

The utility function of the representative household depends on the consumption of the final good C, on labor N and on a preference shock Z:

$$u(Y,N,Z) = Z\frac{Y^{1-\gamma}}{1-\gamma} - \frac{N^{1+\eta}}{1+\eta}, \tag{1}$$

Z acts as a labour supply shifter so we refer to it as a "supply shock". Y is the consumption bundle, defined as $Y = \left(\int_0^1 C_i^{\frac{\varrho-1}{\varrho}} di\right)^{\frac{\varrho}{\varrho-1}}$, where C_i is the consumption of good i and $\varrho > 1$ is the elasticity of substitution between goods, and total labor is $N = \int_0^1 N_i di$, where N_i is the amount of labor used to produce good i. The aggregate supply shifter $z = \log(Z)$ is a gaussian iid shock with mean zero and variance σ_z^2. The budget constraint of the representative household is

$$\int_0^1 P_i C_i di = \int_0^1 P_i Y_i di + T \tag{2}$$

where Y_i is the quantity produced of the individual good i, and P_i is the price of good i. T are the monetary transfers from the central bank. The income generated by production plus the monetary transfers are used for consumption.

The consumer shops the differentiated goods. He observes the prices and the quantities purchased. The individual good demand equation is then given by:

$$C_i = C\left(\frac{P_i}{P}\right)^{-\varrho}, \tag{3}$$

where he consumption price index,P, is defined as

$$P = \left(\int_0^1 (P_i)^{1-\varrho} di\right)^{\frac{1}{1-\varrho}}, \tag{4}$$

Each good i is produced and sold by worker i. More precisely, worker i produces the good using the following linear technology:

$$Y_i = N_i, \tag{5}$$

2.2 The Price-Setting Equation

Worker i is a price-setter. She chooses N_i and P_i monopolistically in order to maximize the expected household utility, (1), subject to the individual good demand equation, (3), the budget constraint, (2), the production technology, (5), and equilibrium in the good market $C_i = Y_i$, $i \in [0,1]$. Denote by I_i the information set of worker i when she decides the price. We denote by $E_i(.) = E(.|I_i)$ the individual expectations and with $\bar{E}(.) = \int_0^1 E_i(.)di$ their cross-individual average. We denote variables in logs by lower-case letters.

Using the model specified in the previous section it is possible to show that the optimal price set by the individual price-setter i is [5]

$$p_i = \chi E_i p + (1-\chi)[E_i q - \delta_z E_i z], \tag{6}$$

[5] The details can be found in the Appendix.

where z is the labor supply shifter, p is the consumer price index for this economy and q stands for nominal aggregate demand, defined as

$$q = y + p. \tag{7}$$

Equation (6) states that the optimal price of each good depends on the aggregate price and on the nominal marginal cost, which depends positively on the nominal aggregate demand q and negatively on the supply shock z. $0 < \chi < 1$ is the degree of strategic complementarities in price-setting: it describes by how much optimal individual prices increase with the average price in the economy for a given level of nominal aggregate demand. δ_z and χ are functions of the parameters of the model. Here we have

$$\begin{aligned} \chi = & \tfrac{1-\gamma+(\varrho-1)\eta}{1+\varrho\eta}; \\ \delta_z = & \tfrac{1}{\eta+\gamma}. \end{aligned}$$

Given the presence of imperfect information, the optimal price defined in equation (6) depends on the expectations formulated by the price-setter i on the basis of her information set I_i.

The model is closed simply with a quantity equation. That is, we assume a cash-in-advance constraint that implies that nominal spending depends on money supply:

$$q = m + v, \tag{8}$$

where m is the log of money supply set by the central bank and v is money velocity. Velocity v is a gaussian iid shock with mean zero and variance σ_v^2. Nominal aggregate demand is thus partially controlled by the central bank, up to the velocity shock v. This assumption captures the facts that there are exogenous shifts in aggregate demand that policy cannot control.

The demand y_i for the individual good i can also be written in log-linear form:

$$y_i = y - \varrho(p_i - p), \tag{9}$$

Equations (6)-(9), along with the approximations $\int_0^1 p_i di = p$ and $\int_0^1 y_i di = y$, compose the reduced-form log-linear model. However, the model is not quite closed. We still have to specify the information set of workers I_i, $i \in [0, 1]$, as well as monetary policy m.

2.3 Information Structure

The consumer and the workers have different information sets. We assume, without loss of generality, that the consumer knows all the shocks.[6] Workers - price-setters - do not observe z and participate only to the market for good i, so they have a more limited information set.

Price-setter i does not observe z and v. Instead, she observes an exogenous private signal z_i:

$$z_i = z + \varepsilon_i \tag{10}$$

that describes with an idiosyncratic error the supply shock hitting the economy. ε_i is a gaussian iid shock with mean zero and variance σ_ε^2. It averages out in the aggregate so $\int_0^1 \varepsilon_i di = 0$.

Besides, because she sells good i, price-setter i observes her individual demand y_i. Our assumption is then that each price-setter can observe its own demand schedule, described in Equation (9) without knowing what are exactly the forces that move it. By combining their individual demand and their price p_i, price-setters can easily construct a variable $\tilde{y}$ that is independent of idiosyncratic shocks:

$$\tilde{y} = y_i + \varrho p_i = y + \varrho p,$$

where we used Equation (9). $\tilde{y}$ can be interpreted as an adjusted measure of the demand for goods that reflects only aggregate shocks. For a given individual price p_i, an increase in individual demand y_i can reflect either an increase in aggregate demand y or an increase in the aggregate price p, which makes the individual good more attractive. In this sense, the signal $\tilde{y}$ is an imperfect description of the state of the economy. Importantly, $\tilde{y}$ is an endogenous variable: it depends on the structure of the economy and in particular on monetary policy. Monetary policy can then affect the information set of agents through that channel.

The monetary authority does not observe z and v. Similarly to price-setters, it receives a noisy signal z^{cb}:

$$z^{cb} = z + \xi \tag{11}$$

where ξ is a gaussian iid shock with mean zero and variance σ_ξ^2. The central bank does not observe the z_i signals as those are private information to price-setters.

[6]Even if the consumer would not directly observe all shocks, he would have all the relevant information. Indeed, the consumer perfectly observes the supply shock z, as well as the set of prices and quantities, because he participates to all markets.

Importantly, we assume that neither z^{cb}, nor m, nor q are part of the price-setters' information set. This assumption hinges on the idea that aggregate information is not always available contemporaneously to private agents (this is typically the case for q), and when it is, private agents do not necessarily pay attention to it (this is typically the case for m, the monetary instrument and for z^{cb}, the central bank's information). This assumption is relaxed in Section 5 by allowing the central bank to communicate its information, and we show that it is harmless.

2.4 Monetary Policy

The goal of the central bank (CB) is to choose money supply m in order to maximize the welfare of the representative agent. In the Appendix, we show that this is akin to minimizing the loss function L:

$$L = V(y - y^*) + \Phi V(p_i - p) \tag{12}$$

with

$$y^* = \delta_z z \tag{13}$$

whose arguments are the volatility of the output gap and the dispersion of individual prices. The parameter Φ is a function of the deep parameters of the model: $\Phi = \varrho/(1-\chi)$. It is useful to recognize that the output gap is tightly linked to the price gap $p - p^*$, with $p^* = q - y^*$, as $y - y^* = -(p - p^*)$. Therefore, $V(y - y^*) = V(p - p^*)$. As a consequence, the goal of the central bank amounts to helping price-setters to set their price at the right level.

Money supply is assumed to react to the central bank information in the following way: $m = \beta_z z^{cb}$. The nominal aggregate demand defined in equation (8) therefore follows:

$$q = \beta_z(z + \xi) + v = \beta_z z + \nu, \tag{14}$$

where $\nu = \beta_z \xi + v$ is the total demand shock, which is composed of the monetary noise $\beta_z \xi$ and the velocity shock v. We assume that the central bank commits to β_z before the realization of the shocks.

3 Equilibrium

In this section, we study the equilibrium for a given policy parameter β_z. In particular, we examine how policy shapes the information of price-setters. We show that, to improve welfare, the central bank has two options: either stabilize prices, or improve the price-setters' information. We show that, because monetary policy is always reflected, to a certain extent, in the endogenous signals received by price-setters, it is in fact impossible to reach price stability.

An equilibrium is a set of quantities $\{y_i\}_{i\in[0,1]}$, and prices $\{p_i\}_{i\in[0,1]}$ such that price-setting follows (6), aggregate demand follows (7), monetary policy follows (14), $p = \int_0^1 p_i di$ and $y = \int_0^1 p_i di$, and the information set I_i of price setter i includes z_i and y_i, for all $i \in [0,1]$.

Endogenous signal and signal extraction Following the literature on noisy rational expectations, we restrict ourselves to analyze linear equilibria. We guess that, by combining their individual signal z_i and their individual demand y_i (or, equivalently, the adjusted demand $\tilde{y}$), price-setters can extract an endogenous signal of z of the following form:

$$\tilde{z} = z + \kappa_z^{-1}\nu,$$

where κ_z is a combination of the parameters of the model. We will show that this is indeed the case and we will characterize the solution for κ_z. Note that κ_z is an essential determinant of the precision of the endogenous signal. This signal accounts for the fact that, similarly to what happens in a Lucas' economy, price-setters cannot precisely understand whether changes in their individual demand are due to demand factors subsumed in ν, or to a change in labor supply z.

Agents use their exogenous signal z_i and the endogenous one $\tilde{z}$ in order to formulate their expectations:

$$E_i[z|\tilde{z}, z_i] \quad = \quad \gamma_z z_i + \tilde{\gamma}_z \tilde{z}, \tag{15}$$

where γ_z and $\tilde{\gamma}_z$ are defined as a function of the precisions of the signals:

$$\begin{aligned} \gamma_z &= \tfrac{\sigma_\varepsilon^{-2}}{\sigma_\varepsilon^{-2}+\sigma_z^{-2}+P_z} \\ \tilde{\gamma}_z &= \tfrac{P_z}{\sigma_\varepsilon^{-2}+\sigma_z^{-2}+P_z}. \end{aligned} \tag{16}$$

where $P_z = \kappa_z^2(\sigma_v^2 + \beta_z^2\sigma_\xi^2)^{-1}$ is the precision of the endogenous signal $\tilde{z}$.

Policy, strategic and information wedges The equilibrium of the economy should satisfy the following equations (the proof can be found in the Appendix):

$$\begin{array}{rcllll} p\_p^* & = & (\beta_z - \delta_z - \kappa_z) & (1+\gamma_z\tilde{\chi}_z) & \left[\bar{E}(z) - z\right] \\ p_i - p & = & (\beta_z - \delta_z - \kappa_z) & [1-(1-\gamma_z)\tilde{\chi}_z] & \left[E(z) - \bar{E}(z)\right] \end{array} \quad (17)$$

where $\tilde{\chi}_z = \chi/(1-\chi\gamma_z)$, γ_z and $\tilde{\gamma}_z$ are defined in (16) and

$$\begin{array}{rcl} \bar{E}(z) - z & = & -(1-\gamma_z-\tilde{\gamma}_z)z + \tilde{\gamma}_z\kappa_z^{-1}\nu \\ E_i(z) - \bar{E}(z) & = & \gamma_z\varepsilon_i. \end{array} \quad (18)$$

The price gap, $p - p^*$, and the individual deviations of prices from their average, $p_i - p$, depend on three wedges: the "policy" wedge $\beta_z - \delta_z - \kappa_z$ that is function of the policy parameter β_z, the strategic wedge that depends on the strategic complementarities parameter $\tilde{\chi}_z$, and the information wedges represented by the average expectation error $(\bar{E}(z) - z)$ and by the dispersion in expectations $(E_i(z) - \bar{E}(z))$.

Consider the policy wedge first. The policy wedge reflects the price-setters' incentive to react to their forecasts of z when setting their price. To understand, consider the price-setting equation in the absence of strategic complementarities ($\chi = \tilde{\chi}_z = 0$), written so that only the endogenous signal and the expectation of z appear:

$$p_i = (\beta_z - \delta_z - \kappa_z)E_i(z) + \kappa_z\tilde{z} \quad (19)$$

The endogenous signal $\tilde{z}$, in the price-setting equation, is used to account for the demand disturbance ν, as $\tilde{z} = z + \kappa_z^{-1}\nu$. This signal, because it is common knowledge, affects neither the price gap nor the price dispersion. Now consider the effect of expectations. If price-setters expect z to increase, they decrease their price ($-\delta_z$ term). Additionally, they expect the central bank to receive a positive signal on z as well and to adjust the money supply as a reaction to that signal. If they expect the money supply to respond positively ($\beta_z > 0$), then they should set a higher price as a response to an expected higher nominal demand. Crucially, the price should also respond negatively to expectations of z if $\kappa_z > 0$. Indeed, for a given endogenous signal $\tilde{z} = z + \kappa_z^{-1}\nu$, higher expectations of the supply shock z are consistent with lower expectations of the demand shock ν, which induce price-setters to set lower prices.

The information wedge reflects the price-setters forecasting errors. If the central bank is not too reactive and κ_z is small ($\beta_z < \delta_z + \kappa_z$), then the policy wedge is negative and the optimal response to a supply shock is negative. But if price-setters collectively overestimate the supply shock ($\bar{E}(z) > z$), then the price gap is negative, because they set too low prices. This implies a positive output gap, as it overly stimulates demand. Similarly, if a price-setter is relatively more optimistic on supply than average ($E_i(z) > \bar{E}(z)$), then it set lower prices than average.

With strategic complementarities ($\chi > 0$), there is an additional wedge that reflects the well-known lower incentives to react to private signals, which increases the volatility of the price gap and reduces the cross-sectional dispersion.

Notice that, if κ_z were exogenous, it would be easy to close the policy wedge by setting $\beta_z - \kappa_z = \delta_z$. This means that the incentives to set a lower price as a response to the expectation of a supply shock (δ_z) are compensated by the incentives to set a higher price as a response to the *implied* expected increase in demand ($\beta_z - \kappa_z$). In that case, there is neither an output gap nor any cross-sectional dispersion, and prices are stabilized since prices need to respond only to the signal $\tilde{z}$: $p_i = p = \kappa_z \tilde{z}$. Concretely, the central bank "does to job", by appropriately stimulating the economy when a positive shock occurs. The inference of z becomes therefore irrelevant for price-setters. This is reminiscent of the "divine coincidence", where stabilizing prices coincides with output gap stabilization.

However, recall that the parameter κ_z is not exogenously given, but is a function of the parameters of the model and hence of the policy parameter β_z. This has two implications that may shape the effect of monetary policy in equilibrium. First, the policy wedge cannot necessarily be closed. This will depend on how β_z affects κ_z. Second, monetary policy potentially affects the output gap and the cross-sectional dispersion not only through the policy wedge, but also through the informational wedge, as the precision of the endogenous signal depends on κ_z, which may be affected by monetary policy in equilibrium.

Equilibrium precision of the endogenous signal The precision of the endogenous signal $P_z = \kappa_z^2(\sigma_v^2 + \beta_z^2\sigma_\xi^2)^{-1}$ depends both on β_z and κ_z. κ_z is a function of the parameters of the model and, importantly, of the policy parameter β_z. We thus denote the precision $P_z(\beta_z)$.

In equilibrium, the resulting κ_z is described in the following lemma (see proof in the Appendix):

Lemma 1 *For a given policy parameter β_z, κ_z is characterized in equilibrium by*

$$\kappa_z = \beta_z - \frac{(\varrho - 1)(1-\chi)\gamma_z\delta_z}{1 + [\varrho(1-\chi) - 1]\gamma_z}. \tag{20}$$

where γ_z is defined in equation (16). A solution for $\kappa_z(\beta_z)$ always exists and, when $\beta_z < 0$, *it is unique.*

Equation (20) shows that κ_z is closely related to β_z. The more the monetary policy reacts to the economic shock, the more precise price-setters' information about the shock. It also shows that κ_z is always lower than the policy parameter, $\kappa_z < \beta_z$, since $\varrho > 1$ and $\delta_z > 0$. This can be understood by considering the adjusted demand $\tilde{y}$ from which the agents derive the endogenous signal $\tilde{z}$:

$$\tilde{y} = y_i + \varrho p_i = y + \varrho p = q + (\varrho - 1)p \tag{21}$$

This adjusted demand index is affected positively by nominal demand q. It is also affected by p through two channels. An aggregate demand channel $(q - p)$ and a relative price channel $(p_i - p)$. The higher the price level, the lower aggregate demand and hence the lower the demand for each individual good. At the same time, the higher the price level, the lower the relative price of good i, and the higher the demand for good i. As ϱ, the elasticity of substitution between goods, increases, the relative price channel becomes stronger in comparison to the aggregate demand channel and p has a more positive effect on the adjusted demand index.

In the limit case where $\varrho = 1$, these two channels cancel out and the adjusted demand index depends only on aggregate demand. In this case, agents perfectly observe aggregate demand, which yields the endogenous signal $\tilde{z} = z + \beta_z^{-1}\nu$. Therefore, $\beta_z = \kappa_z$, as implied by Lemma 1.

When instead $\varrho > 1$, the adjusted demand $\tilde{y}$ reflects not only movements in money supply but also changes in the price level. In particular, a supply shock generates a negative response in prices. This introduces a negative bias in the elasticity of $\tilde{y}$ to z. The adjusted demand reflects both the reaction, measured by β_z, of money supply to the increase in z, and the reduction in prices due to the same shock. The resulting equilibrium value for κ_z must then be lower than β_z, as suggested by Lemma 1.

The elusive price stabilization We now consider the equilibrium policy wedge and ask whether achieving the optimum through a zero policy wedge, that is, through price stabilization, is possible. We show that it is not, because the policy wedge is incompressible in equilibrium. Price stabilization is therefore an elusive objective.

Notice that Lemma 1 implies an equilibrium policy wedge given by

$$\beta_z - \delta_z - \kappa_z = -\frac{\delta_z(1 - \gamma_z \chi)}{1 + [\varrho(1 - \chi) - 1]\gamma_z}, \tag{22}$$

As $\gamma_z \in [0,1]$, the absolute value of the policy wedge $|\beta_z - \delta_z - \kappa_z|$ is larger than $\max\{\delta_z/\varrho, \delta_z(1-\chi)\}$, which is strictly positive. This means that the policy wedge cannot approach zero in equilibrium, it is incompressible. As explained earlier, a positive supply shock generates a need to adjust prices downward, to accommodate both for a higher supply shock (δ_z) and for a lower expected demand (κ_z). To close the policy gap, the central bank has to increase money supply and set $\beta_z = \delta_z + \kappa_z$ so as to offset this need by creating an incentive to set higher prices. However, in equilibrium, a higher β_z generates a higher κ_z. A higher κ_z in turn generates a need for an even higher β_z to close the policy gap,.. etc. As a result, it is never possible for the monetary authority to completely close the policy gap.

To understand better, take the limit case where $\varrho = 1$. In that case, $\kappa_z = \beta_z$ and the policy wedge is $-\delta_z$. As discussed above, this case is equivalent to observing the nominal demand directly. Therefore, any change in nominal demand, including a change in β_z, will be perfectly offset by a price adjustment. As a result, the expectation error affects the price gap only through the optimal response to the supply shock, which is $-\delta_z$. In the more general case where $\varrho > 1$, agents do not observe the aggregate demand but a closely related variable, which still gives them enough information to partially respond to nominal demand. The bottom line is that the endogenous signal gives the agents the relevant information to offset the effect of policy, which does not leave room for policy to offset the actions of the agents. Therefore, minimizing the output gap necessarily entails minimizing the informational wedges as well.

4 Optimal monetary policy

We consider now a central bank that would set β_z in order to minimize the loss function (12). Contrary to common wisdom, optimal policy does not entail price stabilization. On the opposite, the purpose of monetary policy is to accentuate the natural movement of prices on order to maximize the informational content of endogenous signals.[7]

The following Lemma establishes that the optimal policy maximizes the information content of the endogenous signal:

Lemma 2 *Denote by β_z^* the β_z that minimizes L under the constraint (17) with $\kappa_z = \kappa_z(\beta_z)$. β_z^* is also the value that maximizes the precision of the endogenous signal $P_z(\beta_z)$.*

[7] In the Appendix, we consider an alternative policy rule that would be specified in terms of output gap and price targets, and show that our results still hold.

This Lemma implies that optimal policy has to make the endogenous signal respond as much as possible to the supply shock, and as little as possible to the demand disturbance. Interestingly, it shows that there is no trade-off between the different wedges. It is enough for the central bank to focus on the information wedge only.

In what follows, we use this to characterize the optimal β_z, first in the special case where the central bank perfectly observes the supply shock, then in the general case where it observes it with noise.

4.1 Perfect information of the central bank

We consider first the case where the monetary authority perfectly observes the supply shock. Therefore we assume that the error term ξ equals zero and therefore that the total demand disturbance in equation (14) is $\nu = v$. The equilibrium precision of the endogenous signal is then $P_z(\beta_z) = \kappa_z(\beta_z)^2\sigma_v^{-2}$, where $\kappa_z(\beta_z)$ is the solution to equation (20). The following Proposition shows how policy can maximize that precision (see proof in the Appendix):

Proposition 1 *The equilibrium precision of the endogenous signal $P_z(\beta_z)$ is maximised for $\beta_z^* \to \pm\infty$. Optimal policy is therefore achieved for $\beta_z^* \to \pm\infty$.*

The CB can make the monetary signal infinitely informative on z by making the money supply respond hyperelastically to z. Whether the CB responds positively or negatively to z does not matter. The optimal policy in the presence of endogenous information can be either procyclical or countercyclical, as long as the monetary signal is infinitely elastic to z.

With endogenous information, this infinitely cyclical monetary policy generates a zero price gap $p - p^* = 0$ (and hence zero output gap $y - y^* = 0$) and no price dispersion, while prices would have infinitely negative or positive movements. It might be surprising that an infinitely volatile money supply does not produce disruptive consequences. But, in this environment, agents are able to recognize both the supply and the demand shocks, so they can adjust their price accordingly. As a result, the volatility in money supply translates exclusively into price volatility.

4.2 Imperfect information of the central bank

In what follows we show that the introduction of noise in the information available to the CB allows us to pin down a finite optimal monetary policy. In order to do that we go back to the assumption that the CB observes the shock with an error: We reintroduce the error ξ and we assume that $\sigma_\xi > 0$. Now the monetary noise is $\nu = v + \beta_z \xi$ and the equilibrium precision of the endogenous signal is $P_z(\beta_z) = \kappa_z(\beta_z)^2(\sigma_v^2 + \beta_z^2 \sigma_\xi^2)^{-1}$.

The following Proposition then characterizes the optimal policy:

Proposition 2 *The equilibrium precision of the endogenous signal $P_z(\beta_z)$ is maximized for β_z^*, where β_z^* is the unique solution to*

$$\frac{1}{\beta_z^*}\left(\frac{\sigma_v}{\sigma_\xi}\right)^2 = \frac{-(\varrho-1)(1-\chi)\sigma_\varepsilon^{-2}\delta_z}{\sigma_z^{-2} + \varrho(1-\chi)\sigma_\varepsilon^{-2} + \sigma_\xi^{-2}\left(1 + \frac{\sigma_v^2}{\sigma_\xi^2}\left(\frac{1}{\beta_z^*}\right)^2\right)}. \tag{23}$$

Optimal policy is therefore achieved for $\beta_z = \beta_z^*$.

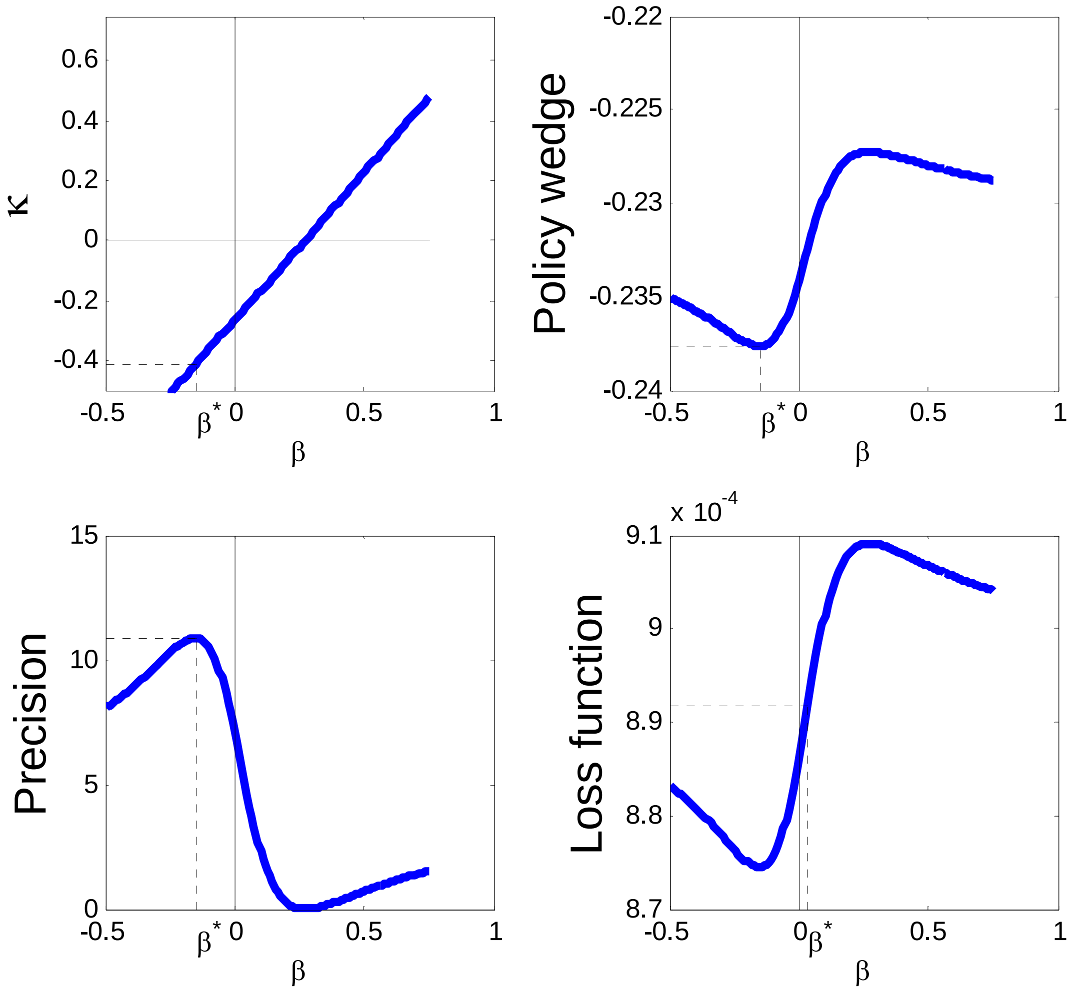
The value of the policy parameter that maximizes the precision of the endogenous signal, β_z^*, in this case becomes finite and negative. Indeed, in this new context the policy parameter β_z affects the precision of the endogenous signal in different ways. As suggested by equation (20), β_z and κ_z are positively related: an increase in β_z increases κ_z and has a positive effect on the precision of $\tilde{z}$. At the same time, however, the same increase in β_z inflates the term ν and, with it, the noise of the signal. Intuitively this trade-off implies that the optimal β_z needs to be anchored to a finite value.

The value of β_z^* actually turns out to be negative. This derives from the fact that $\kappa_z < \beta_z$. For $\tilde{z}$ to be a good signal, κ_z needs to be large in absolute value. As compared to setting a positive β_z, setting a negative β_z generates a larger κ_z in absolute value, while adding the same amount of noise. Comparative statics on Equation (23) in fact show that the absolute value of β_z^* is decreasing in the variance of the central bank noise σ_ξ and increasing in the "natural" elasticity of the adjusted demand signal $\tilde{y}$ to z, which depends on $(\varrho - 1)(1-\chi)\delta_z$.

To understand, consider the adjusted demand $\tilde{y}$, which yields the endogenous signal $\tilde{z}$. Equation (21) shows that aggregate prices p affect positively the signal. The CB then uses monetary policy to emphasize the natural effect of a shock on the signal,through nominal demand q. Given that an increase in z would naturally reduce prices, the monetary authority implements a countercyclical policy to emphasize that reduction in prices.

In Figure 1, we show how monetary policy affects the variables of the model when supply is hit by a unitary (positive) shock. We take $\varrho = 7$, $\gamma = \eta = 1$, which yields $\delta_z = 0.5$ and $\chi = 0.75$. We set $\sigma_z = \sigma_v = \sigma_\epsilon = 0.1$ and $\sigma_\xi = 0.5$. In the case with the endogenous signal, the precision of the monetary signal $\tilde{z}$ is maximized at $\beta_z^* < 0$.

Figure 1: Optimal policy - With endogenous signal

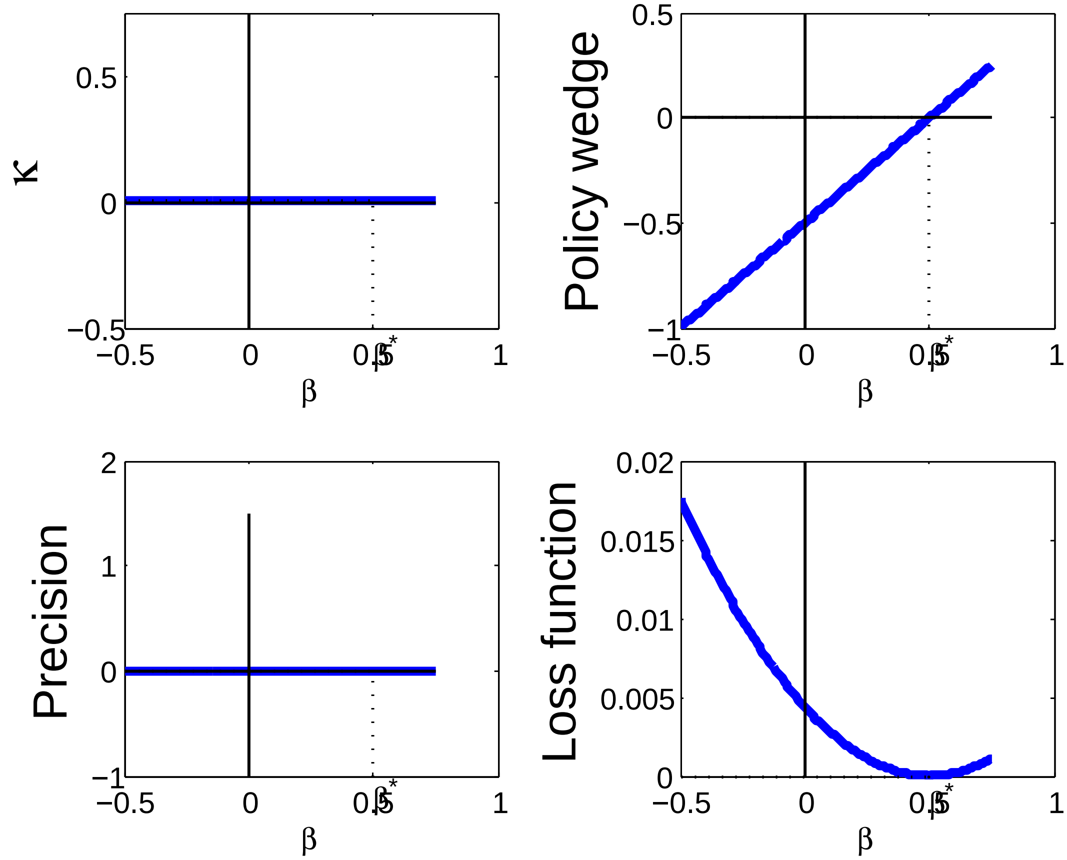


Note: We set $\varrho = 7$, $\gamma = \eta = 1$, which yields $\delta_z = 0.5$ and $\chi = 0.75$. We set $\sigma_z = \sigma_v = \sigma_\epsilon = 0.1$ and $\sigma_\xi = 0.5$.

Indeed, the absolute value of κ_z is larger than the one that would emerge with $\beta_z = |\beta_z^*|$. A negative β_z^* thus reduces the information wedge through a stronger reaction of the endogenous signal to z while limiting the effect of the central bank noise.

It is important to emphasise how our result differs from the findings of the literature that studies optimal monetary policy with exogenous information. As already explained in the previous section, if the information were exogenous, monetary policy would perfectly counterbalance the natural reduction in prices due to the supply shock. To illustrate this, we represent in Figure 2 the case with an exogenous signal where $\kappa_z = 0$, which reflects

Figure 2: Optimal policy - With exogenous signal



Note: We set $\varrho = 7$, $\gamma = \eta = 1$, which yields $\delta_z = 0.5$ and $\chi = 0.75$. We set $\sigma_z = \sigma_v = \sigma_\epsilon = 0.1$ and $\sigma_\xi = 0.5$.

the case where the agents would observe the demand disturbance ν but not the supply shock, using the same parameters. In the case with exogenous signal, the loss function of the central bank is minimized for a positive policy parameter β_z, more precisely for $\beta_z = \delta_z$, the value that shuts down the policy wedge. Optimal policy is procyclical so that prices do not need to respond to the supply shock. As you can note, in that case the precision of the exogenous signal is constant and does not vary with β_z. Note that, in the exogenous information case, optimality is reached by emptying prices from their informational content. In our endogenous information setup, the information content of prices is maximized.

5 Alternative information sets

As a robustness analysis, we consider two alternative information sets. First, in the baseline model, we assumed that the observed individual demand could be perfectly backed out to an endogenous signal that depends only on aggregate shocks. However, it is reasonable to assume that either cognitive limits, or some individual demand shocks could blur the endogenous signal. We therefore allow the individual demand y_i to be observed with an idiosyncratic noise. Second, we assumed that price-setters did not have access to any source of aggregate information. This assumption is challenged by the fact that some aggregate information is usually available. In particular, central banks typically communicate on their assessment on the economic situation, and their instrument is transparent. We therefore allow for other sources of information through central bank communication. We show that our results hold as long as the endogenous signal is not too noisy. As for the information communicated by the central bank (and in general, aggregate information), it does not affect the results as long as it is perceived with some individual noise, which can be arbitrarily low.

5.1 Private endogenous signal

Departing from our previous assumptions, we assume here that the level of sales y_i cannot be perfectly observed because, say, agents are inattentive. Agents instead observe $y_i + x_i$, where x_i is a gaussian iid shock with mean zero and variance σ_x^2 that averages out in the aggregate: $\int_0^1 x_i di = 0$. Price-setter i thus observes her demand with an error along with the private signal z_i, as before. The endogenous signal extracted from this observation is therefore perturbed both by the demand shock ν and by the idiosyncratic noise x_i: $\tilde{z}_i = z + \kappa_z^{-1}\nu + \lambda_z^{-1}x_i$, where κ_z and λ_z are endogenously determined. Both κ_z and λ_z can depend in equilibrium on β_z. Denote by $\tau = \kappa_z^{-2}\sigma_\nu^2/(\kappa_z^{-2}\sigma_\nu^2 + \lambda_z^{-2}\sigma_x^2)$ the share of the public noise in the total noise of the endogenous signal. The parameter τ is now lower than 1, while in the previous version of the model it was exactly equal to 1.

The results here are simulated. However, it is useful, before considering the results, to look at the equilibrium under the simplifying assumption of no complementarities (χ=0).

$$\begin{aligned} p - p^* &= (\beta_z - \delta_z - \tau\kappa_z)\left[\bar{E}(z) - z\right] - (1-\tau)\nu \\ p_i - p &= (\beta_z - \delta_z - \tau\kappa_z)\left[E_i(z) - \bar{E}(z)\right] + \tau\kappa\lambda^{-1}x_i \end{aligned} \quad (24)$$

where in equilibrium we have

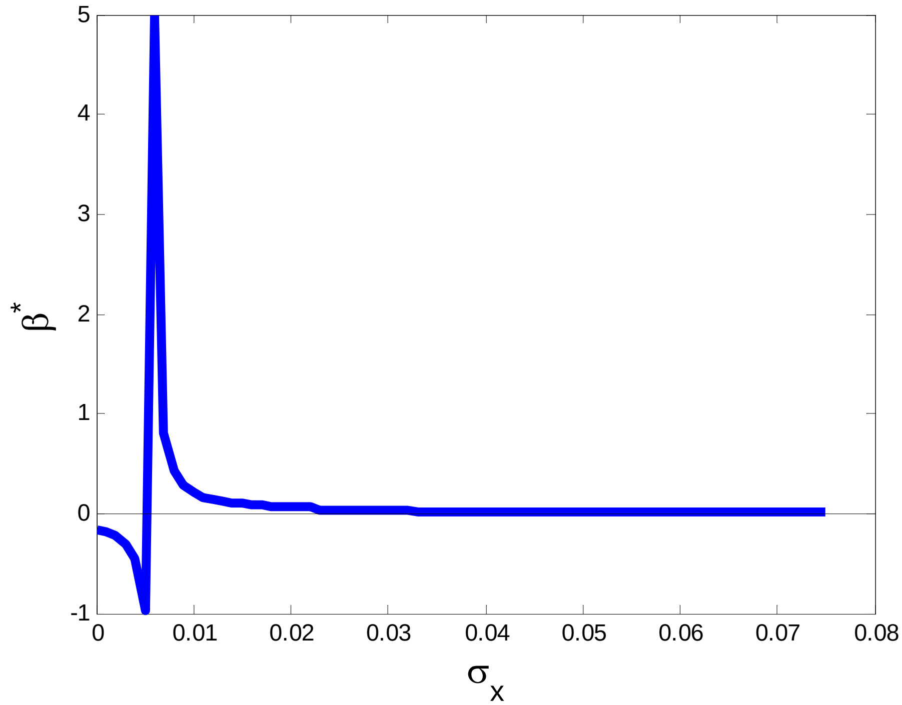
$$\beta_z - \delta_z - \tau\kappa_z = \frac{\beta_z(1-\tau) - \delta_z}{1 + (\varrho - 1)\gamma\tau} \tag{25}$$

The first terms in the price gap and in the individual price dispersion are familiar. They reacts to the aggregate and individual expectational errors $\bar{E}(z) - z$ and $E_i(z) - \bar{E}(z)$ with the policy wedge $\beta_z - \delta_z - \tau\kappa_z$. As you can note, the policy wedge and its interpretation is still very similar to the one that we gave in section 3. However, it is important to underline that, as long as the demand is observed with idiosyncratic noise (i.e., if $\tau < 1$),the policy wedge can be closed by setting $\beta_z(1-\tau) = \delta_z$. Because of idiosyncratic noise, price-setters are less prone to let their price react to the endogenous signal. The monetary policy reaction to the central bank's signal is not fully offset by price-setting and, as a consequence, the policy gap can be perfectly closed.

Notice also that the price gap and the price dispersion include additional terms ($-(1-\tau)\nu$ and τx_i). In the absence of idiosyncratic noise in the endogenous signal, there is a perfect mapping between the expectation error on the total demand disturbance, ν, and on the supply shock, z: if a price-setter underestimates z, then she also overestimates ν. In the absence of the idiosyncratic noise then the price gap and price dispersion can be fully related to agents' errors on z and ν would not matter *per se*. Assuming the presence of the idiosyncratic noise x_i, instead, this perfect mapping disappears: for a given error on z, the price-setters can either underestimate or overestimate ν, depending on whether the error in the endogenous signal is driven by ν or by x_i. When there is a positive demand shock ν, price-setters underestimate nominal demand, and they all set relatively low prices. That's why ν affects negatively the price gap, (see equation (24)). When there is a positive idiosyncratic noise shock instead, the price-setter overestimates nominal demand and sets a relatively high price as compared to the average, as shown in equation (24). As a result, ν and x_i play an additional role in the deviations of prices from their optimum.

Consider now Figure 3, which represents the effect of σ_x on the optimal β_z, letting $\chi > 0$. We let σ_x vary between 0 and 0.08, and use the same parameters as before. Note that in the absence of idiosyncratic noise ($\sigma_x = 0$), the parameter τ is equal to 1 and we are back to the situation described in section 4.2. As already explained, the policy wedge is incompressible and the optimum can be reached by closing the information wedge. The optimal β_z then is negative, consistently with the findings of the previous section. As σ_x increases, β_z becomes more negative. This is because a larger β_z reduces the relative contribution of the private noise to the endogenous signal. As a consequence,

Figure 3: Optimal policy with noisy endogenous signal - Effect of σ_x



Note: We set $\varrho = 7$, $\gamma = \eta = 1$, which yields $\delta_z = 0.5$ and $\chi = 0.75$. We set $\sigma_z = \sigma_v = \sigma_\epsilon = 0.1$ and $\sigma_\xi = 0.5$.

as σ_x increases, the policy-maker has the incentive to set a β_z that is, in absolute value, even higher than before. After a certain threshold value of σ_x, the policy wedge become more relevant than the information wedge, and it can be closed by setting a positive β. As σ_x goes to infinity, the optimal β_z converges to the one that would hold in the absence of endogenous signal, which is positive.

This numerical exercise suggests that, for β_z^* to be negative, the idiosyncratic noise associated to the endogenous signal must be relatively small. This is backed by the literature on rational inattention, and especially Mackowiak and Wiederholt (2009), who show that price-setters must pay almost all their attention to idiosyncratic shocks, as opposed to aggregate shocks, because they estimate that idiosyncratic volatility is almost one order of magnitude larger than aggregate volatility. This implies that agents must be particularly attentive to their local source of information, represented here by y_i.

5.2 Central bank communication

We now assume that the central bank can communicate its assessment of the state of the economy $z^{cb} = z + \xi$. Notice that this is equivalent to communicating on the policy instrument $m = \beta_z z^{cb}$. However, to make the problem non-trivial, we assume that the central bank signal is processed with cost by the agents, so that agents receive the communication signal $\bar{z}_i = z + \xi + u_i$ where u_i is gaussian iid idiosyncratic noise with mean zero and variance σ_u^2. It averages out in the aggregate so $\int_0^1 u_i di = 0$. For simplicity, we come back to the baseline assumption according to which agents observe y_i without additional noise.

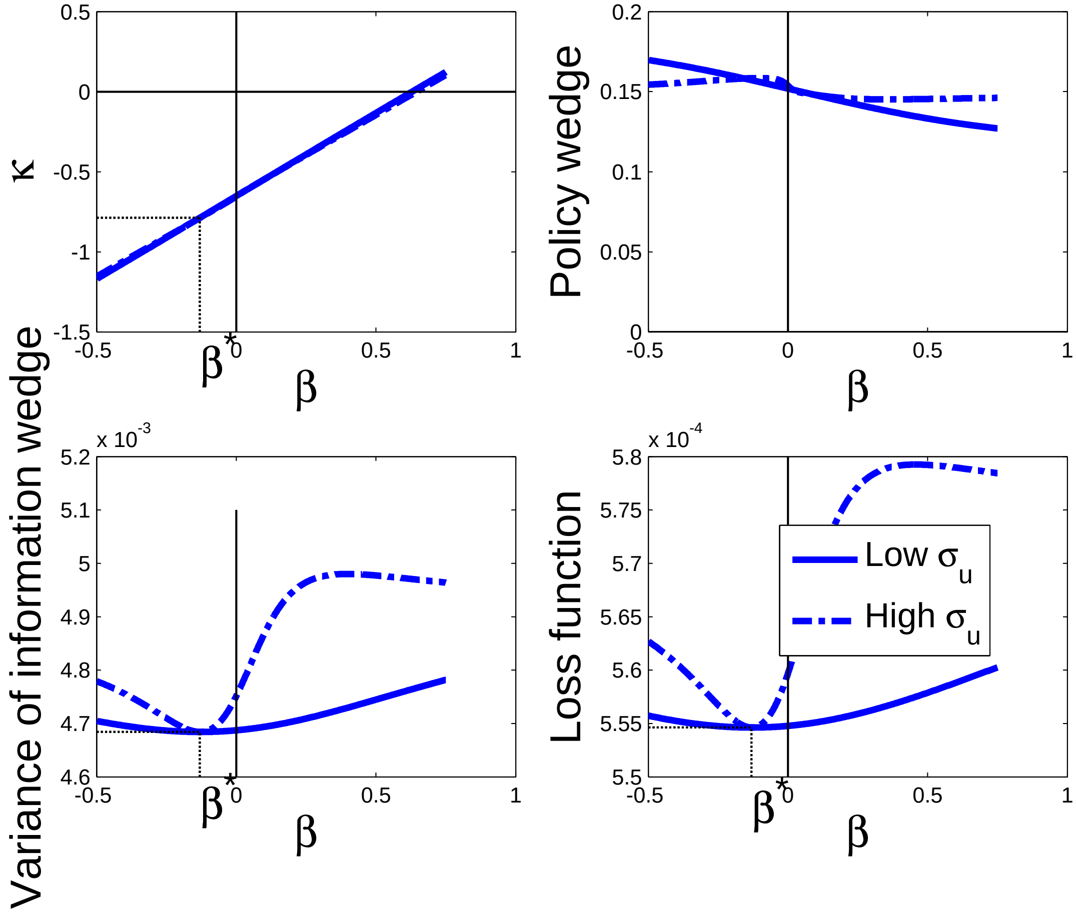
The results are simulated here as well. But before analyzing the results of the simulation, we consider the simple case with no complementarities ($\chi = 0$). Since the price-setters now have an additional source of information on ξ, the endogenous signal might not depend on the total demand disturbance $\nu = \beta_z \xi + v$ only, but might also react independently to ξ. That's why we conjecture that agents extract an endogenous signal of the form $\tilde{z} = z + \kappa_z^{-1} v + \lambda_z^{-1} \beta \xi$. In this case, the equilibrium price gap and the price deviations are:

$$\begin{array}{rcll} p - p^* & = & (\beta_z - \delta_z - \kappa_z)\left[\bar{E}(z) - z\right] & +\beta(\lambda_z - \kappa_z)\left[\bar{E}(\xi) - \xi\right] \\ p_i - p & = & (\beta_z - \delta_z - \kappa_z)\left[E_i(z) - \bar{E}(z)\right] & +\beta(\lambda_z - \kappa_z)\left[E_i(\xi) - \bar{E}(\xi)\right] \end{array} \quad (26)$$

The first terms in the price gap and price deviations are composed of the familiar policy and information wedges. However, now they cannot be summarized to the effect of the error on z. Because the agents have additional information on ξ, the errors on ξ affect the gaps independently from the errors on z. Note that their effect depends on $\lambda_z - \kappa_z$. Indeed, overestimating the central bank noise leads on the one hand price setters to set excessively high prices. On the other hand, overestimating this noise leads them to underestimate the velocity shock v, which drives them to set excessively low prices. The first effect dominates if the endogenous signal is a relatively poorer signal of ξ, that is if $\lambda_z > \kappa_z$. Notice that in the absence of communication (σ_u goes to infinity), we would go back to the original case analysed in section 4. In fact, the endogenous signal would become $\tilde{z} = z + \kappa_z^{-1} v + \lambda_z^{-1} \beta_z \xi = z + \kappa_z^{-1}(v + \beta_z \xi)$, which implies that $\kappa_z = \lambda_z$. The two opposite effects would perfectly balance each other and the last term in equation (26) would disappear.

The simulations show that the optimal β_z is independent of σ_u. In fact β_z^* is always equal to the value that holds in the absence of communication that is, it is equal to the value found in Section 4. Communication by the central bank adds additional information, but does not fundamentally change the way the endogenous signal affects the equilibrium outcome and optimal policy.

Figure 4: Optimal policy with noisy communication

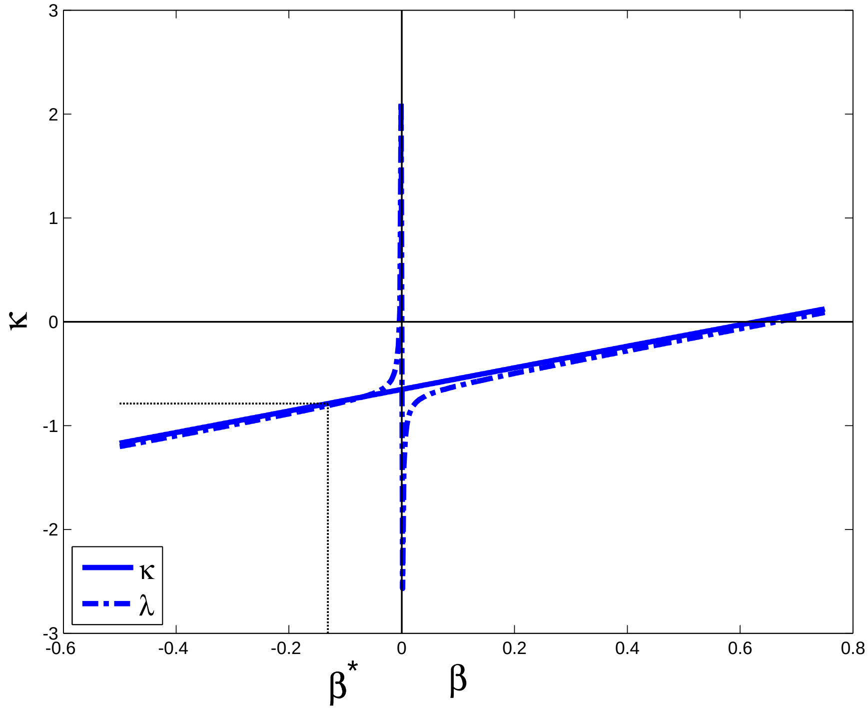


Note: We set $\varrho = 7$, $\gamma = \eta = 1$, which yields $\delta_z = 0.5$ and $\chi = 0.75$. We set $\sigma_z = \sigma_v = \sigma_\epsilon = 0.1$ and $\sigma_\xi = 0.5$. Low σ_u corresponds to $\sigma_u = 0.1$ and high σ_u corresponds to $\sigma_u = 1$.

To understand, consider Figure 4 which represents the equilibrium outcome as a function of β_z, for low and high values of σ_u ($\sigma_u = 0.1$ and $\sigma_u = 1$). Otherwise, we use the same parameters as before. Notice that the policy wedge cannot be closed whatever the value of β, and that the optimal β_z, the one that minimizes the loss function, coincides with the one that minimizes the variance of the information wedge, just like in the baseline case. Notice also that the optimal β_z is the same for both values of σ_u. The precision of the communication signal has therefore no effect on optimal policy. In fact, it seems that the second terms of (26) do not matter for optimal policy.

This can be understood further by considering the equilibrium λ_z as represented in

Figure 5: Noisy communication - κ_z and λ_z as a function of β_z



Note: We set $\varrho = 7$, $\gamma = \eta = 1$, which yields $\delta_z = 0.5$ and $\chi = 0.75$. We set $\sigma_z = \sigma_v = \sigma_\epsilon = \sigma_u = 0.1$ and $\sigma_\xi = 0.5$.

Figure 5. Up to the parameter β_z, the distance between κ_z and λ_z measures the contribution of the second term to the price gap and price dispersion. It appears, from the figure, that λ_z is very close to κ_z, except when β_z is close to zero, in which case the contribution of the second term is small too. This means that the relative response of the endogenous signal to v and ξ does not change when agents have more information. With a communication signal, price-setters are better informed about ξ and v individually, contrary to the benchmark case, where they are informed about $\nu = v + \beta_z\xi$ as a whole. However, the optimal response still depends on their overall assessment of ν, so a better knowledge on its components is not relevant for decision-making. Since prices still respond to the agents' expectations of ν, the endogenous signal still responds to a combination of v and ξ that is close to ν. All in all, the outcome is similar to the benchmark case.

6 Mark-up shocks

We introduce shocks to the elasticity of substitution between goods ϱ, which is now time-varying. Mark-ups are inversely related to ϱ, so we denote ρ the opposite of the log-deviation of ϱ from its steady state, and interpret ρ as shocks to the mark-up. By introducing mark-up shocks, we introduce shocks that drive inefficient fluctuations. In that case, the central bank does not necessarily want to improve the information of price-setters.

We assume that ρ is a gaussian shock with mean zero and variance σ_ρ^2. The optimal price-setting equation now becomes

$$p_i = \chi E_i p + (1-\chi)[E_i q - \delta_z E_i z + \delta_\rho E_i \rho], \tag{27}$$

with

$$\delta_\rho = \frac{1}{(\eta+\gamma)(\varrho-1)}$$

The price responds to the price-setter's expectation of the mark-up shock. Notice that δ_ρ is positive, which means that a mark-up shock tends to be inflationary, contrary to the supply shock, which is deflationary.

Price-setters have the same information set as before, but they also receive an individual signal ρ_i on ρ:

$$\rho_i = \rho + \omega_i \tag{28}$$

where ω_i is a gaussian iid shock with mean zero and variance σ_ω^2.

The central bank observes the signal z^{cb} on the supply shock z (defined above) and and a signal ρ^{cb} on the mark-up shock ρ:

$$\rho^{cb} = \rho + \varphi \tag{29}$$

where φ is a gaussian iid shock with mean zero and variance σ_φ^2.

It then sets money supply as $m = \beta_z z^{cb} + \beta_\rho \rho^{cb}$, so that the nominal demand becomes

$$q = \beta_z z + \beta_\rho \rho + \nu,$$

with $\nu = v + \beta_z \xi + \beta_\rho \varphi$.

We first consider the simple case with only mark-up shocks ($\sigma_z = 0$), then we turn to the general case with two shocks ($\sigma_z > 0$).

Only mark-up shocks Consider first the case with only mark-up shocks and no supply shocks so $\sigma_z = 0$. In a first step, we derive the equilibrium for a given policy parameter β_ρ. Then we derive the optimal β_ρ.

Following the literature on noisy rational expectations, we restrict ourselves to analyze linear equilibria. We guess that, by combining their individual signal ρ_i and their individual demand y_i, price-setters can extract an endogenous signal of the following form:

$$\tilde{\rho} = \rho + \kappa_\rho^{-1}\nu,$$

where κ_ρ is a combination of the parameters of the model. We check later that this is indeed the case and characterize the solution for κ_ρ.

Denote by γ_ρ and $\tilde{\gamma}_\rho$ the weights of respectively the exogenous and the endogenous signal in the expectations of ρ:

$$\begin{aligned} \gamma_\rho &= \tfrac{\sigma_\omega^{-2}}{\sigma_\omega^{-2}+\sigma_\rho^{-2}+P_\rho} \\ \tilde{\gamma}_\rho &= \tfrac{P_\rho}{\sigma_\omega^{-2}+\sigma_\rho^{-2}+P_\rho}. \end{aligned} \tag{30}$$

where $P_\rho = \kappa_\rho^2(\beta_\rho^2\sigma_\varphi^2 + \sigma_v^2)^{-1}$ is the precision of the endogenous signal $\tilde{\rho}$.

All the analysis of Section 3 holds, except for the definition of the price gap $p - p^*$ and for Proposition 2 which defined optimal policy. Indeed, since mark-up shocks drive inefficient output fluctuations, the optimal individual output is $y^* = 0$. The spread between the average realized and optimal price, and between the individual and average price is then

$$\begin{aligned} p - p^* &= \delta_\rho\rho + (\beta_\rho + \delta_\rho - \kappa_\rho) \quad (1 + \gamma\tilde{\chi}_\rho) \quad [\bar{E}(\rho) - \rho] \\ p_i - p &= (\beta_\rho - \delta_\rho - \kappa_\rho) \quad [1 - (1 - \gamma_\rho)\tilde{\chi}_\rho] \quad \left[E_i(\rho) - \bar{E}(\rho)\right]. \end{aligned} \tag{31}$$

where $\tilde{\chi}_\rho = \chi/[1 - \chi\gamma_\rho]$ and

$$\begin{aligned} \bar{E}(\rho) - \rho &= -(1 - \gamma_\rho - \tilde{\gamma}_\rho)\rho + \tilde{\gamma}_\rho\kappa_\rho^{-1}\nu \\ E_i(\rho) - \bar{E}(\rho) &= \gamma_\rho\omega_i. \end{aligned} \tag{32}$$

As in the case with supply shocks, the demand disturbance (ν) affect the information wedge, which drive deviations from the optimum through a policy wedge and a strategic wedge. As before, these deviations can be shut down only by minimizing the informational wedge and therefore by making the endogenous signal as informative as possible. However, this would not be necessarily optimal here because this would make agents respond better to mark-up shocks, which would drive inefficient fluctuations.

Indeed, in the case of mark-up shocks the private optimum does not coincide with the public optimum. The deviation of the private optimum from the social one is described by the additional term $\delta_\rho \rho$, which we can interpret as a "social wedge". As a result, the deviation driven by the mark-up shock ρ would still be equal to $\delta_\rho \rho$ if the informational wedge were equal to zero. It could then be optimal for the central bank to maintain or even maximize the informational wedge by making the endogenous signal less precise.

As in the case with supply shocks, the resulting κ_ρ is still described by Lemma 1, where δ_z, β_z, γ_z and κ_z are replaced respectively by $-\delta_\rho$, β_ρ, γ_ρ and κ_ρ. Note that Proposition 2 is still partly valid: the precision of $\tilde{\rho}$, $P_\rho(\beta_\rho)$, is still maximized for a β_ρ that is defined by analogy with β_z^*. However, this time, the optimal monetary policy does not maximize the precision as mark-up shocks drive inefficient fluctuations. The optimal policy is summarized in the following proposition (the proof is available in the Appendix):

Proposition 3 *Denote by β_ρ^* the β_ρ that minimizes L under the constraint* (31) *with $\kappa_\rho = \kappa_\rho(\beta_\rho)$. β_ρ^* is given by the value that minimizes the precision of the endogenous signal, so that*

$$\beta_\rho^* = \frac{-(\varrho - 1)(1-\chi)\delta_\rho \sigma_\omega^{-2}}{\sigma_\rho^{-2} + \varrho(1-\chi)\sigma_\omega^{-2}}$$

and $\kappa_\rho(\beta_\rho^*) = P_\rho(\beta_\rho^*) = 0$. Optimal policy is therefore achieved for $\beta_\rho = \beta_\rho^*$.

Now, the purpose of the central bank is to reduce the information content of the endogenous signal by counteracting the effect of mark-up shocks on prices. Since a mark-up shock is inflationary, monetary policy is restrictive following a mark-up shock, so that the endogenous signal does not reveal anything about that shock to the agents. This contributes to lower the responsiveness of output to the inefficient mark-up shocks.

Two types of shocks Combining the two types of shocks is quite straightforward. The central bank gets two signals, one for each shock: $z^{cb} = z + \xi$ and $\rho^{cb} = \rho + \varphi$ and sets the two policy parameters, β_z and β_ρ, optimally. The price-setters get the endogenous signal

$$s = \kappa_z z + \kappa_\rho \rho + \nu$$

where $\nu = \upsilon + \beta_z \xi + \beta_\rho \varphi$. Consistently with our previous findings, the central bank would set $\beta_z = \beta_z^*$ and $\beta_\rho = \beta_\rho^*$. In this way the endogenous signal would optimally signal only the supply shock.

7 Conclusion

In this paper, we have shown that, when information is endogenous, monetary policy, because it affects the way the economy reacts to shocks, also shapes the information received by agents. This has crucial implications for monetary policy. The bad news is that endogenous variables partly reveal monetary policy. As a result, price-setters can offset the effects of policy, thus annihilating the central bank's ability to perform a "nominal accommodation" of shocks. But the good news is that endogenous information gives rise to another, potentially powerful channel for monetary policy. The central bank can now shape monetary policy in order to make endogenous variables as revealing as possible of the fundamental shocks hitting the economy. Put differently, in this environment the central bank cannot achieve output gap stabilization through price stabilization. Nevertheless it can achieve it by exacerbating the response of prices to fundamental shocks.

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A The Price-setting Equation

Baseline model Substituting Y and N_i into household period utility, (1), using the individual good demand equation, (3), the budget constraint, (2), the production technology, (5), and $C_i = Y_i$, we obtain the consolidated utility

$$u(Y, N, Z) = Z \frac{\left(Y \int_0^1 \left(\frac{P_i}{P}\right)^{1-\varrho} di \right)^{(1-\gamma)}}{1-\gamma} - \frac{\left(Y \int_0^1 \left(\frac{P_i}{P}\right)^{-\varrho} di \right)^{(1+\eta)}}{1+\eta},$$

The optimality condition with regard to price-setting for worker i resulting from the model specified in section 2 is then

$$E_i \left\{ (\varrho - 1) \frac{ZY^{-\gamma}}{P} \left(\frac{P_i}{P} \right)^{-\varrho} \right\} = E_i \left\{ \varrho \frac{Y^{\eta}}{P_i} \left(\frac{P_i}{P} \right)^{-\varrho(\eta+1)} \right\}. \tag{33}$$

The log-linear version of equation (33) is:

$$p_i = E_i \left\{ p + \frac{1}{\varrho\eta} \left[(\eta + \gamma) y - z \right] \right\}.$$

Now, we substitute for y using (7), which yields

$$\begin{aligned} p_i &= E_i \left\{ p + \frac{1}{1+\varrho\eta} \left[(\eta + \gamma)(q - p) - z \right] \right\} \\ &= E_i \left\{ \frac{1}{1+\varrho\eta} \left[(\eta + \gamma) q + (1 - \gamma + \eta[\varrho - 1]) p - z \right] \right\} \end{aligned}$$

This gives (6).

To derive (9), we use the log-linear version of individual demand for good i (3).

Model with mark-up shocks When ϱ is time-varying, (33) yields the following log-linear approximation

$$p_i = E_i \left\{ p + \frac{1}{\varrho\eta} \left[(\eta+\gamma)y - z + \frac{1}{\varrho - 1}\rho \right] \right\}.$$

where $\rho = -[\log(\varrho) - \log(\bar{\varrho})]$. Now, we substitute for y using (7), which yields

$$\begin{aligned} p_i &= E_i \left\{ p + \frac{1}{1+\varrho\eta} \left[(\eta+\gamma)(m-p) - z + \frac{1}{\varrho-1}\rho \right] \right\} \\ &= E_i \left\{ \frac{1}{1+\varrho\eta} \left[(\eta+\gamma)m + (1-\gamma+\eta[\varrho-1])p - z + \frac{1}{\varrho-1}\rho \right] \right\} \end{aligned}$$

This gives (27).

B The Welfare Approximation

The efficient level of output is defined as the one that emerges under a social planner in the symmetric equilibrium, where $P_i = P$ for all i. In that case, the marginal rate of substitution between labor and consumption is equal to the marginal product of labor. With $N = Y$, this gives

$$Y^{\gamma+\eta} = Z$$

that in logs becomes

$$y = \frac{1}{\gamma+\eta} z. \tag{34}$$

Efficient output is therefore $y^* = \delta_z z$. Besides, (34) should be valid in the steady state as well so $\bar{y} = \frac{1}{\gamma+\eta}\bar{z}$.

Using the full employment condition and the production function for each individual good i, we can rewrite the utility function as

$$u(Y, N, Z) = e^z \frac{e^{(1-\gamma)y}}{1-\gamma} - \frac{\left(\int_0^1 e^{y_i} di\right)^{1+\eta}}{1+\eta}. \tag{35}$$

The second-order approximation of the utility function is

$$u(Y, N, Z) \approx e^{\bar{z}+(1-\gamma)\bar{y}} \left(y + \frac{1-\gamma}{2} y^2 + zy \right) - e^{(1+\eta)\bar{y}} \int_0^1 \left(y_i + \frac{(1+\eta)}{2} y_i^2 \right) di + t.i.p.,, \tag{36}$$

where $t.i.p.$ stands for "terms that are independent of policy" and includes the terms that are completely exogenous, like productivity.

Given that $(1-\gamma)\bar{y} = (1+\eta)(\bar{y} - \bar{a})$ and $E_i(y_i^2) = Var_i(y_i) + E_i(y_i)^2)$, equation (36) becomes

$$u(Y,N,Z) \approx e^{\bar{z}+(1-\gamma)\bar{y}} \left(y + \frac{1-\gamma}{2} y^2 + zy - \int_0^1 y_i di - \frac{(1+\eta)}{2} \int_0^1 y_i^2 di \right) + t.i.p.,, \quad (37)$$

Defining the cross sectional mean as $E_i(y_i) = \int_0^1 y_i di$, the cross sectional variance as $Var_i(y_i) = E_i(y_i^2) - E_i(y_i)^2$, and using the second-order approximation of the consumption bundle

$$y = E_i(y_i) + \frac{1-\varrho^{-1}}{2} Var_i(y_i) \quad (38)$$

we can rewrite the approximation in (37) as

$$\begin{aligned} u(Y,N,Z) &\approx e^{\bar{z}+(1-\gamma)\bar{y}} \left(E_i(y_i) + \frac{1-\varrho^{-1}}{2} Var_i(y_i) + \frac{1-\gamma}{2} y^2 + zy - E_i(y_i) \right. \\ & \left. -\frac{(1+\eta)}{2} \left[Var_i(y_i) + y^2 \right] \right) + t.i.p., \\ &\approx -e^{\bar{z}+(1-\gamma)\bar{y}} \left(\frac{\eta\varrho+1}{2\varrho} Var_i(y_i) + \frac{\gamma+\eta}{2} y^2 - zy \right) + t.i.p. \\ &\approx -e^{\bar{z}+(1-\gamma)\bar{y}} \left(\frac{\eta\varrho+1}{2\varrho} Var_i(y_i) + \frac{\gamma+\eta}{2} \left[y^2 - 2\frac{1}{\gamma+\eta} zy + \left(\frac{1}{\gamma+\eta} \right)^2 z^2 \right] \right) + t.i.p. \\ &\approx -e^{\bar{z}+(1-\gamma)\bar{y}} \left(\frac{\eta\varrho+1}{2\varrho} Var_i(y_i) + \frac{\gamma+\eta}{2} \left[y - \frac{1}{\gamma+\eta} z \right]^2 \right) + t.i.p. \end{aligned}$$

Given that $Var_i(y_i) = Var_i(y_i - y)$, and $\delta_z = 1/(\gamma+\eta)$, this becomes

$$u(Y,N,Z) \approx -e^{\bar{z}+(1-\gamma)\bar{y}} \left(\frac{\eta\varrho+1}{2\varrho} Var_i(y_i - y) + \frac{\gamma+\eta}{2} [y - \delta_z z]^2 \right) + t.i.p., \quad (39)$$

Dropping the proportionality factor and substituting in the second term the full information production $y^* = \delta_z z$, gives:

$$\begin{aligned} -Eu(Y,N,Z) &\approx E(y-y^*)^2 + \tfrac{\eta+\varrho^{-1}}{\gamma+\eta} Var_i(y_i - y) + t.i.p. \\ &\approx Var(y-y^*) + \tfrac{\eta+\varrho^{-1}}{\gamma+\eta} Var_i(y_i - y) + t.i.p., \end{aligned} \quad (40)$$

Finally, note that the individual demand equation (9) yields $y_i - y = -\varrho(p_i - p)$, so (40) rewrites as

$$-Eu(Y,N,Z) \approx Var(y-y^*) + \frac{\varrho(\varrho\eta+1)}{\gamma+\eta} Var_i(p_i - p) + t.i.p.$$

Then simply note that $\Phi = \varrho(\varrho\eta+1)/(\gamma+\eta) = \varrho/(1-\chi)$.

Note that this approximation is still valid with mark-up shocks.

C Output gap and price targets

Here we examine how optimal policy would translate into a standard policy rule. We consider a standard Taylor-type of rule, where money supply reacts to the output gap and the price level. We consider two cases. First, we assume that the output gap and the aggregate price level are perfectly observed. In this case, it is optimal for the central bank to react infinitely to the output gap and to prices. Second, we assume that the central bank observes the output gap and the price level with noise. This second case is a natural way to introduce central bank noise that pins down the Taylor rule coefficients to finite values. We find that the optimal policy described in Section 4 can be replicated by letting the money supply respond positively to the output gap and prices.

Output gap and prices perfectly observed The monetary policy rule is $m = -\beta_y(y-y^*) - \beta_p p$, so that nominal demand follows:

$$q = -\beta_y(y-y^*) - \beta_p p + v \tag{41}$$

When combined with (7), this yields

$$q = \frac{\beta_y}{1+\beta_y} y^* - \frac{\beta_p - \beta_y}{1+\beta_y} p + \frac{1}{1+\beta_y} v$$

As $y^* = \delta_z z$, a policy targeting the supply shock is mapped with $\beta_y = \beta_p$. In that case we would have

$$q = \frac{\beta_y}{1+\beta_y}\delta_z z + \frac{1}{1+\beta_y} v \tag{42}$$

In this context, letting β_y go to $+\infty$ or $-\infty$ would reduce the contribution of the velocity component v relative to the fundamental component z in the endogenous signal, achieving optimality.

To see this, denote $\tilde{z} = z + \kappa_y^{-1} v = z + [(1+\beta_y)\kappa_y^{-1}]v/(1+\beta_y)$. Transposing our previous analysis by just replacing κ_z with $\kappa_y/(1+\beta_y)$ and β_z with $\delta_z\beta_y/(1+\beta_y)$, we get

$$\frac{\kappa_y}{1+\beta_y} = \delta_z \left[\frac{\beta_y}{1+\beta_y} - \frac{(\varrho-1)(1-\chi)\gamma_z}{1+[\varrho(1-\chi)-1]\gamma_z}\right]$$

where γ_z follows (16). The policy wedge is now denoted $\frac{\beta_y}{1+\beta_y} - \delta_z - \frac{\kappa_y}{1+\beta_y}$, but its equilibrium value remains the same as expressed in (22), so it is still incompressible.

The optimum is then reached by maximizing the precision of the endogenous signal P_z, which is a function of β_y. We denote $P_z = P_y(\beta_y) = \kappa_y(\beta_y)^2\sigma_v^{-2}$. We have:

$$\kappa_y = \delta_z \left[\frac{\beta_y(1-\chi\gamma_z) - (\varrho-1)(1-\chi)\gamma_z}{1+[\varrho(1-\chi)-1]\gamma_z}\right] \tag{43}$$

β_y should then go to $+\infty$ or $-\infty$ in order to maximize $|\kappa_y|$ and hence the precision of the endogenous signal.

Output gap and prices observed with noise Suppose now that the output gap and the price level are observed with noise by the central bank. Denote by ξ_y and ξ_p these respective noises, so that the central bank observes $y - y^* + \xi_y$ and $p + \xi_p$. Applying a monetary rule similar to (41) with the noisy measure of the output gap would yield, when we set $\beta_p = \beta_y$:

$$q = \frac{\beta_y}{1+\beta_y}\delta_z z + \frac{\beta_y}{1+\beta_y}\xi + \frac{1}{1+\beta_y}v \tag{44}$$

with $\xi = \xi_y + \xi_p$. Now denote $\tilde{z} = z + \kappa_y^{-1}\nu = z + [(1+\beta_y)\kappa_y^{-1}]\nu/(1+\beta_y)$, with $\nu = v + \beta_y\xi$. Transposing our previous analysis by just replacing κ_z with $\kappa_y/(1+\beta_y)$ and β_z with $\delta_y\beta_y/(1+\beta_y)$, we would get the same κ_y as described in (43). Now, however, the precision of the signal is given by $P_y(\beta_y) = [\kappa_y(\beta_y)]^2(\sigma_v^2 + \beta_y^2\sigma_\xi^2)^{-1}$.

The previous analysis can be transposed, as the policy wedge is identical. The following Proposition then characterizes the optimal policy:

Proposition 4 *The equilibrium precision of the endogenous signal $P_y(\beta_y)$ is maximized for β_y^*, where β_y^* is strictly negative.*

Proof. See proof below. ■

The optimal response of the money supply to both the output gap and the price level is therefore positive. The purpose of monetary policy is indeed to emphasize the response of prices. The response of both prices and the output gap to a supply shock tends to be negative, so the optimal rule makes the money supply respond negatively to a supply shock, which achieves the goal of accentuating the negative price movement. However, the fact that the central bank can make mistakes in its assessment of the output gap and the price puts a limit to the response of money supply.

D Proofs

D.1 Proof of Equations (17)

Replacing the monetary policy rule (14) in the price-setting equation (6), we obtain

$$p_i = \chi E_i(p) + (1-\chi)E_i[(\beta_z - \delta_z)z + v] \tag{45}$$

We make the following guess: $p = \alpha z + \tilde{\alpha}\kappa_z^{-1}v$. Taking expectations of p, z and v and replacing in (45), we get

$$\begin{aligned} p_i \;=\; & \Big\{\chi\Big[\alpha(\gamma_z+\tilde{\gamma}_z)+\tilde{\alpha}(1-\gamma_z-\tilde{\gamma}_z)\Big]+(1-\chi)\Big[(\beta_z-\delta_z)(\gamma_z+\tilde{\gamma}_z)+\kappa_z(1-\gamma_z-\tilde{\gamma}_z)\Big]\Big\}z \\ & +\Big\{\chi\Big[\alpha\tilde{\gamma}_z+\tilde{\alpha}(1-\tilde{\gamma}_z)\Big]+(1-\chi)\Big[(\beta_z-\delta_z)\tilde{\gamma}_z+\kappa_z(1-\tilde{\gamma}_z)\Big]\Big\}\kappa_z^{-1}v \\ & +\Big\{\chi(\alpha-\tilde{\alpha})\gamma_z+(1-\chi)(\beta_z-\delta_z-\kappa_z)\gamma_z\Big\}\varepsilon_i \end{aligned}$$

Averaging across price-setters to obtain p and identifying the coefficients, we find after arranging

$$\alpha = -\delta_z + \beta_z + \frac{(\delta_z+\kappa_z-\beta_z)(1-\gamma_z-\tilde{\gamma}_z)}{1-\chi\gamma_z}$$

$$\tilde{\alpha} = \kappa_z - \frac{(\delta_z+\kappa_z-\beta_z)\tilde{\gamma}_z}{1-\chi\gamma_z}$$

Since $p^* = (\beta_z - \delta_z)z + v$, then we have

$$p - p^* = -\frac{(\delta_z+\kappa_z-\beta_z)}{1-\chi\gamma_z}[(1-\gamma_z-\tilde{\gamma}_z)z - \tilde{\gamma}_z\kappa_z^{-1}v]$$

Using the above expression for p_i, we can also write

$$p_i - p = -\frac{(1-\chi)(\delta_z+\kappa_z-\beta_z)}{1-\chi\gamma_z}\gamma_z\varepsilon_i$$

Finally, note that

$$\begin{aligned} \bar{E}z - z &= -(1-\gamma_z-\tilde{\gamma}_z)z + \tilde{\gamma}_z\kappa_z^{-1}v \\ E_iz - \bar{E}z &= \gamma_z\varepsilon_i \end{aligned}$$

which yields (17).

D.2 Proof of Lemma 1

Characterization Price-setters observe the adjusted demand $\tilde{y} = y + \varrho p = q + (\varrho - 1)p = \beta_z z + v + (\varrho - 1)p$. Using our guess-and-verify solution for p, we can write:

$$p = \left[-\delta_z + \beta_z + \frac{(\delta_z + \kappa_z - \beta_z)(1 - \gamma_z - \tilde{\gamma}_z)}{1 - \chi\gamma_z} \right] z + \left[\kappa_z - \frac{(\delta_z + \kappa_z - \beta_z)\tilde{\gamma}_z}{1 - \chi\gamma_z} \right] \kappa_z^{-1} v$$

As a result, we have

$$\begin{aligned} \tilde{y} &= \left\{ \beta_z + (\varrho - 1) \left(\kappa_z - \tfrac{\delta_z + \kappa_z - \beta_z}{1 - \gamma_z \chi} [(1 - \chi)\gamma_z + \tilde{\gamma}_z] \right) \right\} z \\ &+ \left\{ \kappa_z + (\varrho - 1) \left(\kappa_z - \tfrac{\delta_z + \kappa_z - \beta_z}{1 - \gamma_z \chi} \tilde{\gamma}_z \right) \right\} \kappa_z^{-1} v \end{aligned}$$

$\tilde{y}$ depends linearly on z and v. For our guess to be true, we need that $\tilde{y} = f\tilde{z}$ for some constant f. Identifying the coefficients, we get

$$\beta_z + (\varrho - 1) \left(\kappa_z - \frac{\delta_z + \kappa_z - \beta_z}{1 - \gamma_z \chi} [(1 - \chi)\gamma_z + \tilde{\gamma}_z] \right) = \kappa_z + (\varrho - 1) \left(\kappa_z - \frac{\delta_z + \kappa_z - \beta_z}{1 - \gamma_z \chi} \tilde{\gamma}_z \right)$$

which yields (20).

Existence and unicity Equation (20) can be rewritten as $X(\kappa_z) = \beta_z$ with

$$X(\kappa_z) = \kappa_z + \frac{(\varrho - 1)(1 - \chi)\sigma_\varepsilon^{-2}\delta_z}{\sigma_z^{-2} + \kappa_z^2 \sigma_v^{-2} + \varrho(1 - \chi)\sigma_\varepsilon^{-2}} \tag{46}$$

X is continuous on $\mathbb{R}$ with values between $-\infty$ and $+\infty$, so a solution $\kappa_z(\beta_z)$ to (20) exists.

Suppose $\beta_z < 0$. According to (46), $\kappa_z < X(\kappa_z)$ as $\delta_z > 0$, so necessarily a solution $\kappa_z(\beta_z) < \beta_z < 0$. For the solution to be unique, it is then sufficient that X is strictly increasing for all $\kappa_z < 0$. X is continuously differentiable with

$$X'(\kappa_z) \quad = \quad 1 - \tfrac{2\kappa_z(\varrho - 1)(1 - \chi)\sigma_\varepsilon^{-2}\sigma_v^{-2}\delta_z}{[\sigma_z^{-2} + \kappa_z^2\sigma_v^{-2} + \varrho(1 - \chi)\sigma_\varepsilon^{-2}]^2}$$

This is positive if $\kappa_z < 0$, as $\delta_z > 0$. So X is strictly increasing in κ_z for $\kappa_z \in \mathbb{R}-$. Therefore, for $\beta_z < 0$, the solution $\kappa_z(\beta_z)$ of $X(\kappa_z) = \beta_z$ is unique.

When $\beta_z > 0$, there could be multiple solutions.

D.3 Proof of Lemma 2

We first write more explicitly the loss function:

$$L = \delta_z^2 \frac{\sigma_z^{-2} + P_z(\beta_z) + \Phi(1-\chi)^2\sigma_\varepsilon^{-2}}{[\sigma_z^{-2} + P_z(\beta_z) + \varrho(1-\chi)\sigma_\varepsilon^{-2}]^2} \tag{47}$$

where $P_z(\beta_z) = \kappa_z(\beta_z)^2\sigma_v^{-2}$. This expression is obtained by replacing the output gap and the individual deviations using (17), then replacing κ_z in the policy wedge using (20), and finally by replacing γ_z, $\tilde{\gamma}_z$ and $\tilde{\chi}_z$ by their expressions.

We then replace $\Phi = \varrho/(1-\chi)$ and obtain

$$L = \frac{\delta_z^2}{\sigma_z^{-2} + P_z(\beta_z) + \varrho(1-\chi)\sigma_\varepsilon^{-2}} \tag{48}$$

L depends on β_z only through the precision $P_z(\beta_z)$. L decreases monotonously with the precision $P_z(\beta_z)$. Therefore, the level of β_z that maximizes the precision also minimizes L.

D.4 Proof of Proposition 1

If we replace γ_z in the equilibrium value of κ_z (20), κ_z can be characterized as follows

$$\kappa_z = \beta_z - \frac{(\varrho - 1)(1-\chi)\sigma_\varepsilon^{-2}\delta_z}{\sigma_z^{-2} + \kappa_z^2\sigma_v^{-2} + \varrho(1-\chi)\sigma_\varepsilon^{-2}}$$

As the second term in the right-hand-side is bounded, then the solution(s) of $X(\kappa_z) = \beta_z$ go(es) to $+\infty$ ($-\infty$) if and only if β_z goes to $+\infty$ ($-\infty$). As a consequence, $P_z(\beta_z) = \kappa_z(\beta_z)^2\sigma_v^{-2}$ goes to $+\infty$ if and only if β_z goes to $+\infty$ or $-\infty$. Both $\beta_z = +\infty$ and $\beta_z = -\infty$ thus maximize the precision $P_z(\beta_z)$. Applying Lemma 2, we then derive Proposition 1.

D.5 Proof of Proposition 2

We have

$$P_z'(\beta_z) = 2\left(\sigma_v^2 + \beta_z^2\sigma_\xi^2\right)^{-1}\kappa_z(\beta_z)Q(\beta_z). \tag{49}$$

where

$$Q(\beta_z) = \kappa_z'(\beta_z) - \kappa_z(\beta_z)\beta_z\sigma_\xi^2\left(\sigma_v^2 + \beta_z^2\sigma_x i^2\right)^{-1}$$

$P_z'(\beta_z) = 0$ either for $\kappa_z(\beta_z) = 0$ or for $Q(\beta_z) = 0$. Since $\kappa_z(\beta_z) = 0$ characterizes the minimum level of precision ($P_z(\beta_z) = 0$), $Q(\beta_z) = 0$ characterizes a local maximum for the precision.

Suppose there exists β_z^* such that $Q(\beta_z^*) = 0$. This implies $P_z'(\beta_z^*) = 0$. As a result, $\kappa_z'(\beta_z^*) = 1$, since, according to (20),

$$\kappa_z'(\beta_z) = 1 + \frac{(\varrho-1)(1-\chi)\delta_z(\sigma_\varepsilon^2)^{-1}P_z'(\beta_z)}{[\sigma_z^{-2} + P_z(\beta_z) + \varrho(1-\chi)\sigma_\varepsilon^{-2}]^2}$$

Replacing in $Q(\beta_z^*)$ and rearranging, we find that β_z^* must satisfy

$$\kappa_z(\beta_z^*) = \beta_z^* + \frac{\sigma_v^2}{\beta_z^*\sigma_\xi^2} \tag{50}$$

Replacing $\kappa_z(\beta_z^*)$ in Equation (20) using (50), we obtain Equation (23) which characterizes β_z^* uniquely.

Finally, notice that $\beta_z - (\varrho-1)(1-\chi)\sigma_\varepsilon^{-2}\delta_z/[\sigma_z^{-2} + \varrho(1-\chi)\sigma_\varepsilon^{-2}] < \kappa_z < \beta_z$, so κ_z goes to $+\infty$ if β_z goes to $+\infty$ and κ_z goes to $-\infty$ if β_z goes to $-\infty$. Besides, κ_z/β_z is bounded. As a result, $P_z(\beta_z) = \kappa_z^2/(\sigma_v^2 + \beta_z^2\sigma_\xi^2)$ goes to σ_ξ^{-2} when β_z goes to $-\infty$ or $+\infty$. This value lies strictly below $P_z(\beta_z^*)$, so $P_z(\beta_z^*)$ is the global maximum of P_z.

D.6 Proof of Proposition 3

We proceed as in the proof of Lemma 2 and write more explicitly the loss function:

$$L = \delta_\rho^2 \frac{[P_\rho(\beta_\rho) + \varrho(1-\chi)\sigma_u^{-2}]^2\sigma_\rho^2 + P_\rho(\beta_\rho) + \Phi(1-\chi)^2\sigma_u^{-2}}{\left[\sigma_\rho^{-2} + P_\rho(\beta_\rho) + \rho(1-\chi)\sigma_u^{-2}\right]^2} \tag{51}$$

where $P_\rho(\beta_\rho) = \kappa_\rho(\beta_\rho)^2(\sigma_v^2 + \beta_\rho^2\sigma_\varphi^2)^{-1}$. This expression is obtained by replacing the output gap and the individual deviations using (31), then replacing κ_ρ in the policy wedge using (20) (where δ_z, γ_z, β_z and κ_z are replaced respectively by $-\delta_\rho$, γ_ρ, β_ρ and κ_ρ), and finally by replacing γ_ρ, $\tilde{\gamma}_\rho$ and $\tilde{\chi}_\rho$ by their expressions.

Replacing $\Phi = \varrho/(1-\chi)$, we obtain

$$\begin{aligned} L &= \delta_\rho^2 \tfrac{[P_\rho(\beta_\rho)+\varrho(1-\chi)\sigma_u^{-2}]^2\sigma_\rho^2 + P_\rho(\beta_\rho)+\varrho(1-\chi)\sigma_u^{-2}}{\left[\sigma_\rho^{-2}+P_\rho(\beta_\rho)+\varrho(1-\chi)\sigma_u^{-2}\right]^2} \\ &= \delta_\rho^2\sigma_\rho^2[P_\rho(\beta_\rho) + \varrho(1-\chi)\sigma_u^{-2}]\tfrac{\sigma_\rho^{-2}+P_\rho(\beta_\rho)+\varrho(1-\chi)\sigma_u^{-2}}{\left[\sigma_\rho^{-2}+P_\rho(\beta_\rho)+\varrho(1-\chi)\sigma_u^{-2}\right]^2} \\ &= \delta_\rho^2\sigma_\rho^2 \tfrac{P_\rho(\beta_\rho)+\varrho(1-\chi)\sigma_u^{-2}}{\sigma_\rho^{-2}+P_\rho(\beta_\rho)+\varrho(1-\chi)\sigma_u^{-2}} \end{aligned} \tag{52}$$

L depends on β_ρ only through the precision $P_\rho(\beta_\rho)$. L increases monotonously with the precision $P_\rho(\beta_\rho)$. Therefore, the level of β_ρ that minimizes the precision also minimizes L.

D.7 Proof of Proposition 4

The expression for L (47) is still valid, with $P_z(\beta_z)$ replaced with $P_y(\beta_y)$, with $P_y(\beta_y) = \kappa_y(\beta_y)^2(\sigma_v^2 + \beta_y^2\sigma_\xi^2)^{-1}$. Therefore, we still have that L varies in an opposite way to P_y. Consequently, the minima of L are the maxima of P_y.

We have

$$P_y'(\beta_y) = 2\left(\sigma_v^2 + \beta_y^2\sigma_\xi^2\right)^{-1}\kappa_y(\beta_y)Q(\beta_y). \tag{53}$$

where

$$Q(\beta_y) = \kappa_y'(\beta_y) - \kappa_y(\beta_y)\beta_y\sigma_\xi^2\left(\sigma_v^2 + \beta_y^2\sigma_x i^2\right)^{-1}$$

$P_y'(\beta_y) = 0$ either for $\kappa_y(\beta_y) = 0$ or for $Q(\beta_y) = 0$. Since $\kappa_y(\beta_y) = 0$ characterizes the minimum level of precision ($P_y(\beta_y) = 0$), $Q(\beta_y) = 0$ characterizes a local maximum for the precision.

Suppose there exists β_y^* such that $Q(\beta_y^*) = 0$. This implies $P_y'(\beta_y^*) = 0$. As a result, according to (43),

$$\kappa_y'(\beta_y^*) = \delta_z\frac{(1-\chi\gamma_z)}{1+[\varrho(1-\chi)-1]\gamma_z}$$

Replacing in $Q(\beta_y^*)$ and rearranging, we find that β_y^* must satisfy

$$(\beta_y^*)^2 - \frac{(\varrho-1)(1-\chi)\gamma_z}{1-\chi\gamma_z}\beta_y^* - \sigma_\xi^{-2}[\sigma_v^2 + (\beta_y^*)^2\sigma_\xi^2] = 0 \tag{54}$$

This second-order polynomial admits 2 roots:

$$\begin{aligned}\beta_1 &= \tfrac{1}{2}\left[\tfrac{(\varrho-1)(1-\chi)\gamma_z}{1-\chi\gamma_z} + \sqrt{\left(\tfrac{(\varrho-1)(1-\chi)\gamma_z}{1-\chi\gamma_z}\right)^2 + 4\sigma_\xi^{-2}[\sigma_v^2+\beta_y^2\sigma_\xi^2]}\right]\\ \beta_2 &= \tfrac{1}{2}\left[\tfrac{(\varrho-1)(1-\chi)\gamma_z}{1-\chi\gamma_z} - \sqrt{\left(\tfrac{(\varrho-1)(1-\chi)\gamma_z}{1-\chi\gamma_z}\right)^2 + 4\sigma_\xi^{-2}[\sigma_v^2+\beta_y^2\sigma_\xi^2]}\right]\end{aligned}$$

Replacing in (43), it appears that $(\kappa_y(\beta_1))^2 < (\kappa_y(\beta_2))^2$ and that $(\beta_1)^2 > (\beta_2)^2$. Therefore, $P_y(\beta_1) < P_y(\beta_2)$. β_2 is therefore the value of β_y that maximizes the precision of the endogenous signal. The optimal β_y is thus $\beta_y^* = \beta_2$. We have also $\beta_2 < 0$.