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Value of Statistical Life and Changes in Ambiguity

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Abstract: This paper investigates the effect of changes in ambiguity on the value of statistical life (VSL) under the smooth ambiguity model developed by Klibanoff, Marinacci and Mukerji (2005). Changes in ambiguity over the mortality risk are expressed through the specific concept of stochastic dominance of order n defined by Ekern (1980). We provide sufficient conditions on the individual attitudes towards ambiguity such that both an ambiguity-averse and an ambiguity-seeking individual modify their VSL in the face of changes in ambiguity. These results have important implications for cost-benefit applications on the risk of human life.

JEL classification: D81, Q51, I18

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1 Introduction

Because society has limited resources that it can spend on health and safety improvements, it is crucial for policymakers to assign an appropriate economic value to change in the risk of human life. The value of statistical life (VSL) is a measure widely used for the evaluation of changes in mortality risk for public policies in medicine, the environment, transportation safety, among others (Drèze, 1962; Jones-Lee, 1974). The dominant approach to calculate the VSL is based on individuals' willingness to pay. Under this approach, the VSL estimates correspond to the willingness of people to trade off wealth for a reduction in the probability of death.

As the VSL is used to assess the benefit of many public policies, changes in the economic value of the variation in risks can significantly affect the extent to which alternative policies appear more beneficial. Hence, knowing which factors influence the VSL allows to examine how economic values may differ between individuals or groups, but also makes it possible to investigate and identify the presence of bias in the estimation of VSL. This explains why the literature on VSL has concentrated on how different attributes affect the VSL, whether it be wealth (Hammitt, 2000), baseline risk (Pratt and Zeckhauser, 1996), background risk (Eeckhoudt and Hammitt, 2001), age (Viscusi and Aldy, 2003), health status (Hammitt, 2002), or altruism (Andersson and Lindberg, 2007). These studies show that the VSL is highly dependent on the environment individuals face.

When evaluating the willingness to pay to trade off wealth for a reduction in the probability of death, it also happens that the baseline risk the individual faces is ambiguous. It could be the case, as stressed by Treich (2010), because various different estimates on this baseline probability are available and the individual is thus uncertain about the true probability, or simply because individuals have an imprecise information about the mortality risk they face. If the individual faces ambiguity on his probability of death, it is likely that this ambiguity may impact his willingness to trade off wealth for a reduction in the probability of death. The impact of ambiguity on the VSL may also depend on the attitude of the individual towards ambiguity, and in particular the aversion towards ambiguity. In that context, Treich (2010) investigated under the recent theory of ambiguity of Klibanoff, Marinacci and Mukerji (2005) (hereafter KMM) how ambiguity and ambiguity-aversion influence the VSL. He showed that the introduction of ambiguity raises the VSL of an ambiguity-averse individual compared to the case of no ambiguity.

However a more general question arises when the individual faces two different situations of ambiguity. Indeed if the level of ambiguity on the baseline probability is different, will the VSL increase? This question is important in cost-benefit studies since not considering the presence of changes in ambiguity could provide bias in VSL estimates. For instance, if changes in ambiguity increased the willingness-to-pay to reduce mortality risk, then estimates of VSL obtained from compensating-wage-differential studies would underestimate the VSL of people facing different levels of ambiguity on their probability of death.

Intuition would suggest that an ambiguity-averse decision-maker (DM) would have a different VSL in the face of a change in ambiguity on the baseline probability, since Treich (2010) showed that the introduction of ambiguity raises the VSL of an ambiguity-averse DM compared to the case of no ambiguity. Yet, extension of Treich's (2010) results to the case of changes in ambiguity is not straightforward and depends on the way change in ambiguity is characterized and on other attitudes towards ambiguity than ambiguity-aversion.

In this paper, we use the particular concept of stochastic dominance defined by Ekern (1980) to define changes in ambiguity in the model of ambiguity of KMM (2005). It makes it possible to link the notion of changes in ambiguity to the properties of the function capturing the individual attitudes towards ambiguity. We provide sufficient conditions on this function such as changes in ambiguity modify the VSL of an ambiguity-averse DM. According the definition of Courbage and Rey (2016), we provide sufficient conditions on this function such as an ambiguity-averse DM increases his VSL when the context becomes more ambiguous.

If ambiguity aversion is usually assumed, recent empirical works have shown that actually DMs are often ambiguity seeking (see e.g. Kocher, Lahno, Trautmann, 2015; Ivanov, 2011). We then extend our results to ambiguity-seeking DM, and provide sufficient conditions on the function capturing ambiguity attitudes such as changes in ambiguity modify the VSL of an ambiguity-seeking DM.

The paper is organised as follow. In the next section, we present the benchmark model of VSL under ambiguity and ambiguity-aversion. In section 3, we characterise changes in ambiguity and their impact on the VSL. In section 4, we provide sufficient conditions on the function capturing individual attitudes towards ambiguity under which a change in ambiguity modifies the VSL of an ambiguity-averse DM. Section 5 provides an illustration of the results of section 4. In section 6, we consider the case of an ambiguity-seeking DM. Finally, a short conclusion is provided in the last section.

2 The benchmark model

The standard VSL model assumes that an individual's utility is given by the expected value of his state-dependent utility of wealth

$$V_0 = (1-p)u_a(w) + pu_d(w) \tag{1}$$

where p is the probability that he dies during the current time period, $u_a(w)$ is the utility of wealth conditional on surviving the period, and $u_d(w)$ is the utility of wealth if he dies (i.e. the utility of bequest).

It is conventionally assumed that $u_a(.)$ and $u_d(.)$ are twice differentiable and such that

$$u_a(w) > u_d(w),\ u_a'(w) > u_d'(w),\ u_a^{''}(w) \leq 0,\ u_d^{''}(w) \leq 0. \tag{2}$$

That is at any level of wealth, both utility and marginal utilities are higher conditional on survival than on death, and that the individual is weakly risk-averse with respect to wealth in both states of the world.

The VSL is defined as the marginal rate of substitution between wealth w and the mortality risk p. It is obtained by totally differentiating Eq. (1), holding expected utility constant:

$$VSL_0 = \frac{dw}{dp} = \frac{u_a(w) - u_d(w)}{(1-p)u_a'(w) + pu_d'(w)} \tag{3}$$

The VSL measures the willingness to trade off wealth for an infinitesimal reduction in the probability of death. Under assumptions (2), the VSL is greater than zero, increases with wealth, and increases with the mortality risk p (effect coined "dead-anyway" effect by Pratt and Zeckhauser (1996)).

So as to introduce ambiguity, we consider the model of ambiguity axiomatized by KMM (2005). Let consider the random variable $\widetilde{\epsilon}$, and add it to the probability of dying p so that $\widetilde{p} = p + \widetilde{\epsilon}$. For the sake of comparison, let's assume that the DM's (subjective) beliefs are such that $E(\tilde{\epsilon}) = 0$ which allows us to consider a mortality risk of the same magnitude as in the case without ambiguity ($E(\tilde{p}) = p$)[1]. According to KMM (2005), the DM's welfare writes as

$$W_{\tilde{\epsilon}} = \Phi^{-1}\left(E[\Phi\{(1-\widetilde{p})u_a(w) + \widetilde{p}u_d(w)\}]\right), \tag{4}$$

where E denotes the expectation operator over the random variable $\widetilde{\epsilon}$. The function Φ captures the attitude towards ambiguity and is supposed to be increasing (assuming differentiability). The DM is considered as strictly ambiguity-averse (seeking) if Φ is strictly concave (convex), as shown by KMM (2005). For a DM with a function Φ such that $\Phi'' < 0$, the introduction of $\widetilde{\epsilon}$ on the probability p reduces his welfare, compared to the case where the probability does not face any ambiguity, i.e. $W_{\tilde{\epsilon}} < V_0$. Hence $\Phi'' < 0$ represents ambiguity aversion, while $\Phi(x) = x$ represents ambiguity neutrality. In the later, the introduction of ambiguity does not modify the DM's welfare. Indeed, as $\Phi'' = 0$, $E[\Phi\{(1-\widetilde{p})u_a(w) + \widetilde{p}u_d(w)\}] = \Phi(E[\{(1-\widetilde{p})u_a(w) + \widetilde{p}u_d(w)\}])$, and then $W_{\tilde{\epsilon}} = V_0$ as, by assumption, $E(\tilde{p}) = p$. Consequently, for an ambiguity-neutral DM, the VSL with ambiguity is equal to the one without ambiguity. In the following, we consider an ambiguity-averse DM (i.e. a DM with a function Φ such that $\Phi'' < 0$).

By totally differentiating Eq. (4) holding $W_{\tilde{\epsilon}}$ constant, we can define the VSL under ambiguity as

$$VSL_{\widetilde{\epsilon}} = \frac{dw}{dp} = \frac{(u_a(w) - u_d(w))E[\Phi'\{(1-\widetilde{p})u_a(w) + \widetilde{p}u_d(w)\}]}{E[((1-\widetilde{p})u_a'(w) + \widetilde{p}u_d'(w))\Phi'\{(1-\widetilde{p})u_a(w) + \widetilde{p}u_d(w)\}]} \tag{5}$$

which we rewrite as (in the goal to simplify notations)

$$VSL_{\widetilde{\epsilon}} = \frac{(u_a(w) - u_d(w))E[\Phi'(\widetilde{V}_0)]}{E[\widetilde{V}_0'\Phi'(\widetilde{V}_0)]} \tag{6}$$

[1] We assume that the realizations of $\tilde{p}$ belong to $[0,1]$.

where $\widetilde{V}_0 = (1-\widetilde{p})u_a(w) + \widetilde{p}u_d(w)$ and $\widetilde{V}_0' = (1-\widetilde{p})u_a'(w) + \widetilde{p}u_d'(w)$.

Treich (2010) shows that the VSL of an ambiguity-averse DM under ambiguity is always higher than the one without ambiguity. In this article, we go one step further and investigate how a change in the level of ambiguity modifies the VSL of an ambiguity-averse individual. To that aim, let's consider two situations where the levels of ambiguity are respectively captured by $\widetilde{\epsilon}_1$ and $\widetilde{\epsilon}_2$ with $E(\tilde{\epsilon}_1) = E(\tilde{\epsilon}_2)$ which allows us to consider a mortality risk of the same magnitude in the two situations i.e. such that $E(p+\tilde{\epsilon}_1) = E(p+\tilde{\epsilon}_2)$.

The analytical expression of the VSL of an ambiguity-averse DM facing the level of ambiguity captured by $\tilde{\epsilon}_i$ $(i=1,2)$ is

$$VSL_{\widetilde{\epsilon_i}} = \frac{(u_a(w)-u_d(w))E[\Phi'(\widetilde{V}_{0i})]}{E[\widetilde{V}_{0_i}'\Phi'(\widetilde{V}_{0i})]}, \tag{7}$$

where $\widetilde{V}_{0i} = (1-\widetilde{p_i})u_a(w) + \widetilde{p_i}u_d(w)$ and $\widetilde{V}_{0i}' = (1-\widetilde{p_i})u_a'(w) + \widetilde{p_i}u_d'(w)$.

Our objective is then to compare $VSL_{\widetilde{\epsilon}_1}$ and $VSL_{\widetilde{\epsilon}_2}$.

3 Changes in the level of ambiguity and VSL

In order to model changes in ambiguity captured by $\widetilde{\epsilon}_1$ and $\widetilde{\epsilon}_2$, we first consider that $\widetilde{\epsilon}_2$ is a "mean-preserving spread" (MPS) over $\widetilde{\epsilon}_1$, i.e. $\tilde{\epsilon}_2 = \tilde{\epsilon}_1 + \tilde{\theta}$ with $E(\tilde{\theta}/\tilde{\epsilon}_1 = \epsilon_1) = 0$ for all[2] ϵ_1. The concept of MPS is widely used to establish a partial ordering of probability distributions (see e.g. Rothshlid and Stiglitz, 1970).

According to Eq. (7), $VSL_{\widetilde{\epsilon}_2} \geq VSL_{\widetilde{\epsilon}_1}$ is equivalent to

$$\frac{E[\Phi'(\widetilde{V}_{02})]}{E[\widetilde{V}_{02}'\Phi'(\widetilde{V}_{02})]} \geq \frac{E[\Phi'(\widetilde{V}_{01})]}{E[\widetilde{V}_{01}'\Phi'(\widetilde{V}_{01})]}, \tag{8}$$

which is equivalent to (since all terms are positive)

$$\frac{E[\Phi'(\widetilde{V}_{02})]}{E[\Phi'(\widetilde{V}_{01})]} \geq \frac{E[\widetilde{V}_{02}'\Phi'(\widetilde{V}_{02})]}{E[\widetilde{V}_{01}'\Phi'(\widetilde{V}_{01})]}. \tag{9}$$

Intuition would suggest that an ambiguity-averse DM would have a higher VSL in the case where the level of ambiguity is captured by a variable being a MPS of another, i.e intuition would suggest that $VSL_{\widetilde{\epsilon}_2} \geq VSL_{\widetilde{\epsilon}_1}$. Indeed, Treich (2010) showed that the introduction of ambiguity raises the VSL of an ambiguity-averse DM compared to the case of no ambiguity; the introduction of ambiguity corresponding to a special case of a

[2]By definition of a MPS, $E_\theta(\epsilon_1 + \tilde{\theta}/\tilde{\epsilon}_1 = \epsilon_1) = \epsilon_1$ for all ϵ_1 that implies $E_{\epsilon_1}[E(\tilde{\epsilon}_1 + \tilde{\theta}/\tilde{\epsilon}_1)] = E_{\epsilon_1}(\tilde{\epsilon}_1)$ and therefore $E(\tilde{\epsilon}_2) = E(\tilde{\epsilon}_1)$.

MPS with $\tilde{\epsilon}_1 = 0$ and $\tilde{\epsilon}_2 = \tilde{\epsilon}$ such that $E(\tilde{\epsilon}) = 0$. However, when we consider a more general form of MPS, other conditions on ambiguity attitudes are required.

To see this, note that since $E(\widetilde{\epsilon}_1) = E(\widetilde{\epsilon}_2)$, then $E(\tilde{V_{01}'}) = E(\tilde{V_{02}'})$. In the case where $\widetilde{\epsilon}_1 = 0$ and $\widetilde{\epsilon}_2 = \widetilde{\epsilon}$ with $E(\widetilde{\epsilon}) = 0$, the term $\widetilde{V_{01}'}$ is constant. Consequently, $E[\tilde{V}_{01}'\Phi'(\tilde{V}_{01})] = E(\tilde{V}_{01}')E[\Phi'(\tilde{V}_{01})] = E(\tilde{V}_{02}')E[\Phi'(\tilde{V}_{01})]$, and then Eq. (9) rewrites

$$E(\tilde{V}_{02}')E[\Phi'(\tilde{V}_{02})] \geq E[\tilde{V}_{02}'\Phi'(\tilde{V}_{02})], \tag{10}$$

or, equivalently

$$cov(\widetilde{V_{02}'}, \Phi'(\widetilde{V}_{02})) \leq 0. \tag{11}$$

Hence, provided death reduces the marginal utility of wealth compared to life, the VSL under ambiguity is always higher than the one without ambiguity for an ambiguity-averse individual as shown by Treich (2010).

Unfortunately, this result can not be extended to a more general form of change in ambiguity than no ambiguity versus ambiguity, even if the two levels of ambiguity are captured by a MPS with no degenerated random variables since, in this case, the term $\widetilde{V_{01}'}$ is not a constant. Consequently, ambiguity-aversion is no longer a sufficient condition to obtain $VSL_{\tilde{\epsilon}_2} \geq VSL_{\tilde{\epsilon}_1}$ without additional conditions on the function Φ capturing the attitudes of the DM towards ambiguity. In the following, we characterise conditons on Φ which allow such comparison in the case of $\widetilde{\epsilon}_2$ being a MPS over $\widetilde{\epsilon}_1$.

A sufficient condition to obtain $VSL_{\tilde{\epsilon}_2} \geq VSL_{\tilde{\epsilon}_1}$, or equivalently, Eq. (9), is:

$$(CC): \quad \frac{E[\Phi'(\widetilde{V}_{02})]}{E[\Phi'(\widetilde{V}_{01})]} \geq 1 \geq \frac{E[\widetilde{V_{02}'}\Phi'(\widetilde{V}_{02})]}{E[\widetilde{V_{01}'}\Phi'(\widetilde{V}_{01})]}. \tag{12}$$

In the goal to simplify notations, we introduce three functions g, z anf H as follows: $g(\epsilon) = \Phi'(V_0 + \epsilon\Delta v)\ \forall\epsilon$ with $\Delta v = u_d(w) - u_a(w)$, $z(\epsilon) = V_0' + \epsilon\Delta v'\ \forall\epsilon$ with $\Delta v' = u_d'(w) - u_a'(w)$ and $V_0' = (1-p)u_a'(w) + pu_d'(w)$, and $H(\epsilon) = z(\epsilon)g(\epsilon)\ \forall\epsilon$. Hence, $E[g(\tilde{\epsilon}_i)] = E[\Phi'(\widetilde{V_{0i}})]$ and $E[H(\tilde{\epsilon}_i)] = E[z(\tilde{\epsilon}_i)g(\tilde{\epsilon}_i)] = E[\tilde{V_{0i}}\Phi'(\tilde{V_{0i}})]$. Condition (CC) rewrites then as

$$(CC): \quad \frac{E[g(\widetilde{\epsilon}_2)]}{E[g(\widetilde{\epsilon}_1)]} \geq 1 \geq \frac{E[H(\widetilde{\epsilon}_2)]}{E[H(\widetilde{\epsilon}_1)]}. \tag{13}$$

According to Rothshild and Stiglitz (1970), the comparison to 1 of the left-hand side term of Eq. (13) is driven by the sign of the second derivative of the function g, i.e. $E[g(\widetilde{\epsilon}_2)] \geq E[g(\widetilde{\epsilon}_1)]$ if $g''(\epsilon) > 0\ \forall\epsilon$. By differentiating g twice, we have $g''(\epsilon) = (\Delta v)^2\Phi'''(V_0 + \epsilon\Delta v)$ that is positive if the DM is ambiguity-prudent i.e. $\Phi''' > 0$. As for the right-hand side term of Eq. (13), we have $E[H(\widetilde{\epsilon}_2)] \leq E[H(\widetilde{\epsilon}_1)]$ if $H''(\epsilon) < 0\ \forall\epsilon$, i.e. (after rearranging the terms) if $-\frac{\Phi'''(V_0+\epsilon\Delta v)}{\Phi''(V_0+\epsilon\Delta v)} < 2\frac{\Delta v'}{(\Delta v)(V_0'+\epsilon\Delta v')}\ \forall\epsilon$. We must characterize then the smallest value of the right-hand side term of this last expression. We define the term S^* as $S^* = \frac{\Delta v'}{(\Delta v)(V_0'+\epsilon_i^*\Delta v')}$, with $\epsilon_i^* = Min\{\underline{\epsilon}_1, \underline{\epsilon}_2\}$ where $\underline{\epsilon}_i$ $(i = 1, 2)$ is the lowest realization[3] of

[3]The term $\underline{\epsilon}_i$ $(\forall i = 1, 2)$ denotes the smallest realization of the random variable $\widetilde{\epsilon}_i$ capturing the level of ambiguity, i.e. the realization of $\widetilde{\epsilon}_i$ such that the probability of death is the lowest or equivalently such that $V_{0i}' = V_0' + \epsilon_i\Delta v'$ is the biggest.

$\tilde{\epsilon}_i$. S^* is the smallest value that can be taken by the term $S = \frac{\Delta v'}{(\Delta v)(V_0' + \epsilon_i \Delta v')}$. Note that S^* is always positive since Δv and $\Delta v'$ are both strictly negative.

We can then exhibit the following proposition.

Proposition 1

Let consider an ambiguity-averse DM and two random variables, $\tilde{\epsilon}_1$ and $\tilde{\epsilon}_2$ capturing two levels of ambiguity, such that $\widetilde{\epsilon}_2$ is a mean preserving spread over $\widetilde{\epsilon}_1$. If the individual is ambiguity-prudent ($\Phi^{'''} > 0$) and such that $-\frac{\Phi'''(x)}{\Phi''(x)} < 2S^* \; \forall x$, then $VSL_{\widetilde{\epsilon}_2} \geq VSL_{\widetilde{\epsilon}_1}$.

Note that the assumption of a prudent-ambiguity DM, i.e. $\Phi''' > 0$, is widely used in the literature. For instance, in their their original paper KMM (2005) suggest using the function $\Phi(x) = -\frac{exp^{(-\alpha x)}}{\alpha}$ with $\alpha > 0$ as an illustration of their model that verifies this assumption. This assumption is also made in Baillon (2015) and is used in Berger (2014) to characterize a precautionary saving behaviour in the face of ambiguity.

The condition, $-\frac{\Phi^{'''}(x)}{\Phi''(x)} < 2S^* \; \forall x$, in Proposition 1, means that the absolute ambiguity prudence index should be sufficiently small, i.e. shoud be inferior to a level equal to $2S^*$. The absolute ambiguity prudence index measures the intensity of ambiguity prudence. This index has been recently used in the literature to define the concepts of "constant absolute ambiguity aversion" and "decreasing absolute ambiguity aversion" (see Berger (2011, 2014), Gierlinger and Gollier (2015)).

Hence, according to Proposition 1, an ambiguity-averse DM increases his VSL in the face of a change in ambiguity captured by a MPS, if he is also ambuity-prudent and if his index of absolute ambiguity prudence is not too large. It should be also noted that if $\Phi''' = 0$, we always have $VSL_{\widetilde{\epsilon}_2} \geq VSL_{\widetilde{\epsilon}_1}$. This is quite obvious as $\Phi^{'''}(x) = 0$ implies $-\frac{\Phi^{'''}(x)}{\Phi''(x)} = 0$ which is always strictly inferior to $2S^*$ since this term is stricly positive. Preferences of an ambiguity-averse DM with $\Phi''' = 0$ are formalized by the quadratic function $\Phi(x) = x - \beta x^2$.

To sum up, when the change in the level of ambiguity is represented by $\tilde{\epsilon}_1$ and $\tilde{\epsilon}_2$, where $\widetilde{\epsilon}_2$ is a MPS over $\widetilde{\epsilon}_1$, $VSL_{\widetilde{\epsilon}_2}$ can be either larger or smaller than $VSL_{\widetilde{\epsilon}_1}$ for an ambiguity-averse DM. If the ambiguity-averse DM is ambiguity-prudent and his index of ambiguity prudence is not too large, then $VSL_{\widetilde{\epsilon}_2} \geq VSL_{\widetilde{\epsilon}_1}$. If the DM is ambiguity-prudent and the index of absolute prudence is sufficiently large, we could obtain that $VSL_{\widetilde{\epsilon}_2} \leq VSL_{\widetilde{\epsilon}_1}$.

4 Generalization

In order to model other changes in ambiguity captured by $\widetilde{\epsilon}_1$ and $\widetilde{\epsilon}_2$, we use the concept of Ekern dominance (1980) which makes possible generalizing partial orderings of probability distributions. It is defined as follows.

Consider two random variables $\widetilde{X}$ and $\widetilde{Y}$ with F and G respectively their two cumulative distribution functions defined over a probability support contained within the interval $[a, b]$. Define $F_1 = F$ and $G_1 = G$. Now define $F_{k+1}(z) = \int_a^z F_k(t)dt$ and $G_{k+1}(z) = \int_a^z F_k(t)dt$ for $k \geq 1$. The variable $\widetilde{X}$ is dominated at the order n $(n \geq 2)$ by the variable $\widetilde{Y}$ $(\widetilde{X} \preceq_n \widetilde{Y})$[4] if $F_n(z) \geq G_n(z)$ for all z in $[a, b]$ where the inequality is strict for some z, and $E(\widetilde{X}^k) = E(\widetilde{Y}^k)$ $\forall k = 1, .., n$, i.e. the $(n-1)$ first moments being identical.

Note that Ekern's (1980) definition is well-known in expected utility theory and includes the cases of mean-preserving increase in risk of Rothshild and Stiglitz (1970) as well as of increase in downside risk defined by Menezes et al. (1980) as respectively a 2^{nd}-degree and a 3^{rd}-degree change. Let us recall the known characterizing property of Ekern dominance[5]:

<u>Lemma:</u> For all $\widetilde{X}$ and $\widetilde{Y}$ such that $\widetilde{X} \preceq_n \widetilde{Y}$, then $E[f(\widetilde{X})] < E[f(\widetilde{Y})]$ for any function f such that $(-1)^{n+1} f^{(n)} > 0$.

Using this lemma, we obtain the following proposition (see Appendix 1).

Proposition 2

Let consider an ambiguity-averse decision-maker and two random variables, $\tilde{\epsilon}_1$ and $\tilde{\epsilon}_2$ capturing two levels of ambiguity, such that $\tilde{\epsilon}_2 \preceq_n \tilde{\epsilon}_1$. If the function Φ is such that $(-1)^{k+1}\Phi^{(k)}(x) > 0$ $\forall x$, $\forall k = n, n+1$ and such that $-\frac{\Phi^{(n+1)}(x)}{\Phi^{(n)}(x)} < nS^*$ $\forall x$, then $VSL_{\tilde{\epsilon}_2} \geq VSL_{\tilde{\epsilon}_1}$ for n even, and $VSL_{\tilde{\epsilon}_2} \leq VSL_{\tilde{\epsilon}_1}$ for n odd.

The condition, $-\frac{\Phi^{(n+1)}(x)}{\Phi^{(n)}(x)} < nS^*$ $\forall x$, in Proposition 2, means that the absolute ambiguity perception index should be not too large, i.e. inferior to nS^*. Naturally, $\Phi^{n+1}(x) = 0$ implies $-\frac{\Phi^{n+1}(x)}{\Phi^{n}(x)} = 0$ which is always strictly inferior to nS^* since this term is strictly positive. Consequently, $\Phi^{n+1}(x) = 0$ (jointly with $(-1)^{(n+1)}\Phi^{(n)}(x) > 0$) is a sufficient condition to obtain the result.

It should be stressed that it is usually assumed in the litterature that the function capturing ambiguity attitudes is such that $(-1)^{(n+1)}\Phi^{(n)} > 0$ for all $n \geq 1$, i.e. $\Phi^{(3)} > 0$, $\Phi^{(4)} < 0$, $\Phi^{(5)} > 0$, $\ldots$, i.e. such that $(-1)^{k+1}\Phi^{(k)}(x) > 0$, $\forall k = n, n+1$. The function $\Phi(x) = -\frac{exp^{(-\alpha x)}}{\alpha}$ with $\alpha > 0$ used by KMM (2005) in their original paper shares the properties of $(-1)^{(n+1)}\Phi^{(n)} > 0$ for all $n \geq 1$. These properties are also shared by the function $\Phi(x) = ln(x)$ for $x > 0$, or the function $\Phi(x) = \frac{x^\gamma}{\gamma}$ with $0 < \gamma < 1$ which are used in Ju and Miao (2012) and Gollier (2011) to express various attitudes towards ambiguity.

One can wonder why, under the condition $-\frac{\Phi^{(n+1)}(x)}{\Phi^{(n)}(x)} < nS^*$, the VSL is higher for the dominated random variable $\tilde{\epsilon}_2$ when n is even, and why the VSL is higher for the non-

[4]The passage from $\tilde{Y}$ to $\tilde{X}$ corresponds to a nth-degree change.

[5]A proof can be found in Ingersoll (1987) for example.

dominated variable $\tilde{\epsilon}_1$ (i.e. for the variable that dominates the other) when n is odd for a DM such that $(-1)^{k+1}\Phi^{(k)} > 0$ for all $k = 1, ..., n$. This result can be interpreted in terms of preferences for harms desagregation, or preferences to "combine good with bad" in a similar way as the one developped by Eeckhoudt and Schlesinger (2006) and Eeckhoudt et al. (2009) in expected utility theory. The condition $-\frac{\Phi^{(n+1)}(x)}{\Phi^{(n)}(x)} < nS^*$ guarantees that ambiguity preferences captured by Φ verify preferences for harms desagregation. The intuitive explanation is the following.

Let us first consider the case where $\tilde{\epsilon}_1$ and $\tilde{\epsilon}_2$ are degenerated random variables, $\tilde{\epsilon}_1 = 0$ and $\tilde{\epsilon}_2 = k$ with $k > 0$. The passage from $\tilde{\epsilon}_1$ to $\tilde{\epsilon}_2$ is considered as an adverse outcome (a "harm") for the DM since in our model it corresponds to an increase in the probability of death. We obtain $VSL_{\tilde{\epsilon}_2} \geq VSL_{\tilde{\epsilon}_1}$. This effect is the well known "dead-anyway" effect (Pratt and Zeckhauser, 1996). Intuitively, a DM facing a large probability of death has little incentive to limit his spending on mortality risk reduction since he is unlikely to survive.

If $\tilde{\epsilon}_1 = 0$ and $\tilde{\epsilon}_2 = \tilde{\epsilon}$ with $E(\tilde{\epsilon}) = 0$, the passage from $\tilde{\epsilon}_1$ to $\tilde{\epsilon}_2$ is an adverse outcome for the DM, it is a "harm". We obtain $VSL_{\tilde{\epsilon}_2} \geq VSL_{\tilde{\epsilon}_1}$ as shown by Treich (2010). Treich gives an intuition of this result based on the dead-anyway effect. Ambiguity over baseline mortality risks thus leads the ambiguity-averse DM to behave in a way that is consistent with a perceived increase of the baseline mortality risk (see Treich (2010) for more details). The VSL is higher for the dominated random variable when $n = 2$ i.e. an even order.

Let us now consider the case $n = 3$ with the two following random variables: $\tilde{\epsilon}_1 = [k, \tilde{\epsilon}; \frac{1}{2}, \frac{1}{2}]$ and $\tilde{\epsilon}_2 = [k + \tilde{\epsilon}, 0; \frac{1}{2}, \frac{1}{2}]$. Recall that k and $\tilde{\epsilon}$ represent adverse outcomes for the DM. Consequently, the DM with an ambiguity function Φ such that $\Phi''' > 0$ prefers not to be confronted with these two adverse outcomes together in one state of nature as is the case with the lottery $\tilde{\epsilon}_2$. He rather prefers to disaggregate these two adverse outcomes across states of nature as it is the case with the lottery $\tilde{\epsilon}_1$. Then the "bad" context is captured by $\tilde{\epsilon}_2$ and we obtain that $VSL_{\tilde{\epsilon}_2} \geq VSL_{\tilde{\epsilon}_1}$. Using the property of Ingersoll (1987), it is easy to verify that $\tilde{\epsilon}_2$ dominates $\tilde{\epsilon}_1$ in Ekern's sense to the order of 3. When $n = 3$ i.e. an odd order, the VSL is higher for the non-dominated random variable.

Such reasoning for disaggregation of adverse outcomes also applies for higher orders. Indeed, let us consider now the case $n = 4$ with the following random variables: $\tilde{\epsilon}_1 = [\tilde{\theta}, \tilde{\epsilon}; \frac{1}{2}, \frac{1}{2}]$ and $\tilde{\epsilon}_2 = [\tilde{\theta} + \tilde{\epsilon}, 0; \frac{1}{2}, \frac{1}{2}]$, with $E(\tilde{\theta}) = 0$ and $\tilde{\theta}$ and $\tilde{\epsilon}$ are independant. Consequently, the DM with an ambiguity function Φ verifying $\Phi^{(4)} < 0$ prefers not to be confronted with these two adverse outcomes together in one state of nature as is the case with the lottery $\tilde{\epsilon}_2$. He rather prefers to disaggregate these two adverse outcomes across states of nature as is the case with the lottery $\tilde{\epsilon}_1$. Then the "bad" context is captured by $\tilde{\epsilon}_2$. We obtain then $VSL_{\tilde{\epsilon}_2} \geq VSL_{\tilde{\epsilon}_1}$. Using the property of Ingersoll (1987), it is easy to verify that $\tilde{\epsilon}_2$ is dominated by $\tilde{\epsilon}_1$ in Ekern's sense to the order of 4. The VSL is higher for the dominated random variable when $n = 4$ i.e. an even order.

This interpretation coincides with the concept of ambiguity apportionment proposed by Courbage and Rey (2016). According these authors, Proposition 2 can be rewritten as follows. Let consider an ambiguity-averse decision-maker and two random variables, $\tilde{\epsilon}_1$ and $\tilde{\epsilon}_2$ such that $\tilde{\epsilon}_2$ is more ambiguous than $\tilde{\epsilon}_1$. If the function Φ is such that $(-1)^{k+1}\Phi^{(k)}(x) > 0$ $\forall x$, $\forall k = n, n+1$ and such that $-\frac{\Phi^{(n+1)}(x)}{\Phi^{(n)}(x)} < nS^*$ $\forall x$, then $VSL_{\tilde{\epsilon}_2} \geq VSL_{\tilde{\epsilon}_1}$.

5 Illustration

In order to provide further intuition on the results, let consider the following illustration. Individuals are presented the possibility to live in two areas, 1 and 2 in which they face a mortality risk due to environmental pollution. Suppose that a public prevention programme is expected to reduce the probability of death by the same level in both areas. The level of ambiguity of the baseline probability of death is different from one area to the other.

First consider that in area 1, a study indicates a risk level of 150 cases per 1 million population, while in area 2, another study cannot indicate with precision the risk level but comes to the conclusion that the risk level is 145 cases per 1 million or 155 cases per 1 million with equiprobability (data of US Environmental Protection Agency (EPA), see Viscusi and Hamilon (1999), Sunstein (2000), Adler (2007)). More formally, with our notations, $\tilde{\epsilon}_1 = 0$ and $\tilde{\epsilon}_2 = [-5/1000000, +5/1000000; \frac{1}{2}, \frac{1}{2}]$. In area 1, the individual faces no ambiguity on the mortality risk, while he faces ambiguity in area 2. According to Treich (2010), the VSL individuals are ready to pay for benefiting from the public prevention programme is higher in area 2 than in area 1 for an ambiguity-averse individual.

Consider now the case where the risk level in area 1 and in area 2 are both ambiguous. In area 1, it is 142 cases per 1 million or 150 cases per 1 million with equiprobability. In area 2, it is 145 cases per 1 million or 147 cases per 1 million with equiprobability. With our notations, we have $\tilde{\epsilon}_1 = [-8/1000000, 0; \frac{1}{2}, \frac{1}{2}]$ and $\tilde{\epsilon}_2 = [-5/1000000, -3/1000000; \frac{1}{2}, \frac{1}{2}]$. This implies that changes in ambiguity from area 1 to 2 corresponds to a MPS, i.e. an even-order change. According to propositions 1 and 2, the VSL individuals are ready to pay for benefiting from the public prevention programme is higher in area 2 than in area 1 for an ambiguity-averse and ambiguity-prudent DM such that $\frac{\Phi'''}{\Phi''} < 2S^*$.

Consider now the case where the risk level in area 1 and in area 2 are the followings. In area 1, it is 145 cases per 1 million with probability $\frac{3}{4}$ or 155 cases per 1 million with probability $\frac{1}{4}$. In the area 2, it is 140 cases per 1 million with probability $\frac{1}{4}$ or 150 cases per 1 million with probability $\frac{3}{4}$. With our notations, we have $\tilde{\epsilon}_1 = [-5/1000000, +5/1000000; \frac{3}{4}, \frac{1}{4}]$ and $\tilde{\epsilon}_2 = [-10/1000000, 0; \frac{1}{4}, \frac{3}{4}]$. This implies that changes in ambiguity from the area 1 to area 2 corresponds to an Ekern 3^{nd}-degree change, i.e. an oddorder change. According to proposition, the VSL individuals are ready to pay for benefiting from the public prevention programme is higher in area 1 than in area 2 for an ambiguity-averse, ambiguity-prudent and ambiguity-temperant DM such that $\frac{\Phi^{(4)}}{\Phi^{(3)}} < 3S^{*}$.

Consider now the case where changes in ambiguity from area 1 to area 2 corresponds to an Ekern 4^{nd}-degree change, i.e. an even-order change. Suppose that $\tilde{\epsilon}_1$ and $\tilde{\epsilon}_2$ are such that: $\tilde{\epsilon}_1 = [-5/1000000, +5/1000000; \frac{1}{2}, \frac{1}{2}]$ and $\tilde{\epsilon}_2 = [-10/1000000, 0, +10/1000000; \frac{1}{8}, \frac{3}{4}, \frac{1}{8}]$. According to propositions 1 and 2, the VSL individuals are ready to pay for benefiting from the public prevention programme is higher in area 1 than in area 2 for an ambiguity-averse, ambiguity-temperant DM and such that $\Phi^{(5)} > 0$ with $\frac{\Phi^{(5)}}{\Phi^{(4)}} < 4S*$.

The same reasoning also applies for higher orders.

6 Ambiguity lovers

While it is often assumed that individuals dislike ambiguity, recent empirical works have showed that individuals in many situations are ambiguity-seeking (see e.g. Kocher, Lahno, Trautmann, 2015; Ivanov, 2011). In this section, we consider how different the effect of changes in ambiguity on the VSL of an ambiguity-lover is from an ambiguity-averse DM?

Let consider now an ambiguity-lover DM with Φ such that $\Phi' > 0$ and $\Phi'' > 0$. By definition, for such an individual, the introduction of ambiguity increases his welfare, i.e. $W_{\tilde{\epsilon}} < V_0$. It is straighforward to show that for an ambiguity-seeking DM the VSL under ambiguity is always smaller than the one without ambiguity. Indeed, $VSL_{\tilde{\epsilon}} \leq VSL_0$ is equivalent to $cov(\tilde{V_0'}, \Phi'(\tilde{V_0})) \geq 0$. Provided death reduces the marginal utility of wealth compared to life ($\Delta v'(x) = u_d'(x) - u_a'(x) < 0\ \forall x$), it is easy to show that this last condition is always satisfied for an ambiguity-seeking DM.

When we compare the VSL for two different levels of ambiguity, as we did in the case of an ambiguity-averse DM, results are less obvious, since they depend on the signs of higher derivatives of Φ. For these higher derivatives, one must distinguish different cases as shown by Ebert (2013)[6]. For instance, consider the power function, $\Phi(x) = x^{\gamma}$, for all $x > 0$ with $\gamma > 0$ that implies $\Phi' > 0$. Imposing $\gamma > 2$ implies $\Phi'' > 0$ and $\Phi''' > 0$, while imposing $\gamma \in]1, 2[$ implies $\Phi'' > 0$ and $\Phi''' < 0$. If $\gamma = 2$, $\Phi'' > 0$ and $\Phi''' = 0$. Then the ambiguity-seeking DM can at the same time be ambiguity-prudent, or ambiguity imprudent or neutral to ambiguity-prudence. More generally, up to some order n (which depends on the power γ), all derivatives of Φ are positive, and for higher orders derivatives alternate in signs. Hence, contrary to the case of an ambiguity-averse DM for whom higher derivatives always alternate in signs, in the case of an ambiguity-seeking DM, various scenarios can occur regarding the signs of higher derivatives of the function

[6] Ebert (2013) provides insights on the behaviors of a risk-lover DM in the framework of expected utility theory.

Φ. We therefore need to take into account these various scenarios.

To that end, let's first consider the case where $\widetilde{\epsilon}_2$ is a MPS over $\widetilde{\epsilon}_1$ (i.e. the case where $n = 2$). If the ambiguity-seeking DM is such that $\Phi''' = 0$, we obtain as the intuition suggests the opposite result as the one obtained with an ambiguity-averse DM, i.e. $VSL_{\widetilde{\epsilon}_2} \leq VSL_{\widetilde{\epsilon}_1}$. When $\Phi''' \neq 0$, results are more complicated. If the DM is ambiguity-prudent $\Phi''' > 0$, we cannot conclude on the comparison between $VSL_{\widetilde{\epsilon}_2}$ and $VSL_{\widetilde{\epsilon}_1}$ and all cases can occur i.e. $VSL_{\widetilde{\epsilon}_2} \leq VSL_{\widetilde{\epsilon}_1}$ or $VSL_{\widetilde{\epsilon}_2} \geq VSL_{\widetilde{\epsilon}_1}$. Therefore, an ambiguity-seeking DM can behave as an ambiguity-averse DM in the sense that he can increase his VSL in the face of a MPS.

If the ambiguity-seeking DM is such that $\Phi''' < 0$, we can characterize a sufficient condition under which $VSL_{\widetilde{\epsilon}_2} \leq VSL_{\widetilde{\epsilon}_1}$. We obtain the following proposition.

Proposition 3

Let consider an ambiguity-seeking DM and two random variables, $\tilde{\epsilon}_1$ and $\tilde{\epsilon}_2$ capturing two levels of ambiguity, such that $\widetilde{\epsilon}_2$ is a mean preserving spread over $\widetilde{\epsilon}_1$. If the DM is ambiguity imprudent ($\Phi''' < 0$) and such that $-\frac{\Phi'''(x)}{\Phi''(x)} < 2S^* \; \forall x$, then $VSL_{\widetilde{\epsilon}_2} \leq VSL_{\widetilde{\epsilon}_1}$.

Hence an ambiguity-lover DM decreases his VSL in the face of a change in ambiguity captured by a MPS if he is also ambiguity imprudent and if his index of absolute prudence has the same magnitude as the one of an ambiguity-lover and ambiguity-prudent DM who increases his VSL in the face of the same change in ambiguity (see proposition 2).

For higher orders changes in ambiguity levels, the same logic as the one in the case of $n = 2$ applies. If all derivatives of Φ are strictly positive, i.e. if $\Phi^{(k)}$ and $\Phi^{(k+1)}$ have the same signs, then we cannot conclude and then $VSL_{\widetilde{\epsilon}_2}$ can be smaller or larger than $VSL_{\widetilde{\epsilon}_1}$. If $\Phi^{(k)}$ and $\Phi^{(k+1)}$ have opposite signs then we obtain the following result.

Proposition 4

Let consider an ambiguity-seeking DM and two random variables, $\tilde{\epsilon}_1$ and $\tilde{\epsilon}_2$ capturing two levels of ambiguity, such that $\tilde{\epsilon}_2 \preceq_n \tilde{\epsilon}_1$ for all $n \geq 3$.

(i) If the function Φ is such that $\Phi^{(n)}(x) > 0$, and $\Phi^{(n+1)}(x) < 0$ and such that $-\frac{\Phi^{(n+1)}(x)}{\Phi^{(n)}(x)} < nS^* \; \forall x$, then $VSL_{\widetilde{\epsilon}_2} \leq VSL_{\widetilde{\epsilon}_1}$ for all n (whether n is odd or even).

(ii) If the function Φ is such that $\Phi^{(n)}(x) < 0$, and $\Phi^{(n+1)}(x) > 0$ and such that $-\frac{\Phi^{(n+1)}(x)}{\Phi^{(n)}(x)} < nS^* \; \forall x$, then $VSL_{\widetilde{\epsilon}_2} \geq VSL_{\widetilde{\epsilon}_1}$ for all n (whether n is odd or even).

Hence, interestingly, in the case where n is odd in (i) and n is even in (ii), an ambiguity-averse DM and an ambiguity-seeking DM behave the same way under the same conditions on the signs of higher derivatives. For instance, if $n = 4$ in (ii), then an ambiguity-lover DM that is also ambiguity-prudent and such that $\Phi^{(4)} < 0$, $\Phi^{(5)} > 0$ and $-\frac{\Phi^{(5)}}{\Phi^{(4)}} < 4S^*$ increases his VSL as does an ambiguity-averse DM that is also ambiguity-prudent and such that $\Phi^{(4)} < 0$, $\Phi^{(5)} > 0$ and $-\frac{\Phi^{(5)}}{\Phi^{(4)}} < 4S^*$ (see proposition 2). This means that in the face of the same changes in ambiguity, an ambiguity-seeking DM can behave as an ambiguity-averse DM. This is due to the fact that the results depends on the alternation in the signs of the higher derivatives at the order n and $n+1$.

7 Conclusion

While the VSL is higher under ambiguity than under no ambiguity for an ambiguity-averse DM, ambiguity-aversion is no longer a sufficient condition to obtain a higher VSL in the face of changes in ambiguity. This paper provides further conditions on the individual attitudes towards ambiguity than ambiguity aversion for such a comparison.

To carry-out our analysis, we use the smooth ambiguity model developed by KMM (2005) which allow us to characterise individual attitudes towards ambiguity. We modelize changes in ambiguity over the mortality risk through the specific concept of dominance stochastic of order n defined by Ekern (1980).

We provide sufficient conditions on the individual attitudes towards ambiguity such that an ambiguity-averse DM modifies his VSL in the face of changes in ambiguity. These conditions concern the signs of the higher derivatives of the function capturing the individual attitudes towards ambiguity as well as the magnitude of the ambiguity perception index. These results can be interpreted in terms of preferences to combine good with bad in a similar way as it is done to interpret the signs of the higher derivatives of the utility function of a risk-averse DM in the expected utility theory. We also provide sufficient conditions such that an ambiguity-seeking DM modifies his VSL in the face of changes in ambiguity. In particular, we show that in the face of the same changes in ambiguity, it can happen that an ambiguity-averse DM and an ambiguity-seeking behave the same way with respect to changes in VSL.

The results of this paper therefore suggest that neglecting differences in ambiguity over mortality risks when evaluating the willingness to pay to trade off wealth for a reduction in the probability of death is likely to lead substantial errors in most cost-benefit applications on the risk of human life, whether the DM being ambiguity-averse or ambiguity-lover.

Appendix 1.

As stressed in section 3, a sufficient condition to have $VSL_{\tilde{\epsilon}_2} \geq VSL_{\tilde{\epsilon}_1}$ is condition (CC). Symmetrically, a sufficient condition to have $VSL_{\tilde{\epsilon}_2} \leq VSL_{\tilde{\epsilon}_1}$ is

$$(CC') : \quad \frac{E[g(\widetilde{\epsilon}_2)]}{E[g(\widetilde{\epsilon}_1)]} \leq 1 \leq \frac{E[H(\widetilde{\epsilon}_2)]}{E[H(\widetilde{\epsilon}_1)]}.$$

Using the lemma introduced in section 4, conditions (CC) and (CC') are verified if the derivative of order k of the function g has an opposite sign of the derivative of order k of the function H. These derivatives are given by:

$z'(\epsilon) = \Delta v'$ and thus $z^{(k)}(\epsilon) = 0 \ \forall k \geq 2$,

$g^{(k)}(\epsilon) = (\Delta v)^k \Phi^{(k+1)}(V_0 + \epsilon \Delta v)$,

$H^{(k)}(\epsilon) = kz'(\epsilon)g^{(k-1)}(\epsilon) + z(\epsilon)g^{(k)}(\epsilon) = k(\Delta v')g^{(k-1)}(\epsilon) + z(\epsilon)g^{(k)}(\epsilon)$.

For all amiguity-averse DM, we have $g^{(k)}(\epsilon) > 0 \ \forall k$ since Φ is such that $(-1)^{s+1}\Phi^{(s)} > 0 \forall s = 1, \ldots, k$, as it is assumed in the paper. Otherwise, we obtain that: $H^{(k)}(\epsilon) < 0$ if and only if (because $g^{(k-1)}(\epsilon) > 0 \ \forall k$)

$$-\frac{\Phi^{(k+1)}(\epsilon)}{\Phi^{(k)}(\epsilon)} < k \frac{\Delta v'}{\Delta v(V_0' + \epsilon \Delta v')}. \tag{14}$$

Using the above-mentioned lemma, we obtain thus that: $E[H(\tilde{\epsilon}_2)] \leq E[H(\tilde{\epsilon}_1)]$ if $\tilde{\epsilon}_2 \preceq_k \tilde{\epsilon}_1$ with k even, and $E[H(\tilde{\epsilon}_2)] \geq E[H(\tilde{\epsilon}_1)]$ if $\tilde{\epsilon}_2 \preceq_k \tilde{\epsilon}_1$ with k odd.

To recapitulate, under the condition (14), we obtain the following result:

when k is even, $g^{(k)}(\epsilon) > 0$ and $H^{(k)}(\epsilon) < 0$, then $E[H(\tilde{\epsilon}_2)] \leq E[H(\tilde{\epsilon}_1)]$ and $E[g(\tilde{\epsilon}_2)] \geq E[g(\tilde{\epsilon}_1)]$, that implies condition (CC),

when k is odd, $g^{(k)}(\epsilon) > 0$ and $H^{(k)}(\epsilon) < 0$, then $E[H(\tilde{\epsilon}_2)] \geq E[H(\tilde{\epsilon}_1)]$ and $E[g(\tilde{\epsilon}_2)] \leq E[g(\tilde{\epsilon}_1)]$, that implies condition (CC').

Note that the result cannot be generalized to a relation of stochastic dominance of order k between $\tilde{\epsilon}_1$ and $\tilde{\epsilon}_2$ since $g^{(k)}(\epsilon) > 0 \ \forall k$ for all ambiguity averse DM.

Appendix 2.

The proof is analog to the one of Appendix 1.

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