

An Equivalent Attenuation Function for the Computation of the Lightning Field Radiated by a Tortuous Channel

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Abstract—The electromagnetic lightning field is usually computed by considering a model consisting in a vertical and rectilinear channel, as well as a Transmission-Line (TL) type propagation characterized by an attenuation function, which expresses how the current decreases while it travels upward along the lightning channel. When dealing with realistic channels, affected by tortuosity, the current attenuation is usually neglected and the field computation is cumbersome, since the discretization of the channel in a large number of linear segments has to be performed. This contribution presents an equivalent attenuation function to be used in the computation of vertical channels in order to reproduce the results of a tortuous one. The attenuation function is computed by solving an inverse problem through the Tikhonov regularization technique, and it is found to be reliable for distances larger than 5 km from the lightning channel.

Keywords—lightning, tortuosity, regularization

I. INTRODUCTION

Lightning represents one of the most dangerous weather related phenomena for people safety and infrastructure security [1], [2]. Towers, wind turbines, buildings, and power systems can be severely damaged by the discharge process; for this reason, researchers have focused their efforts on protection schemes, which require an appropriate modeling of the lightning current, the field propagation through the surrounding medium and the lightning channel. The lightning return stroke current can be described according to different models, such as engineering models (in particular, Transmission-Line (TL) type models) [3]–[6], distributed circuit models [7], gas-dynamical models [8]–[10] or electromagnetic (EM) models [11], whereas the description of the field propagation strongly depends on the lightning location with respect to nearby objects [12]–[14] (such as buildings and trees), as well as on the ground characteristics [15]. On the other hand, the channel is usually considered as vertical and rectilinear, even though the frequent practice does not agree with this assumption.

Assuming a TL-type model, the return-stroke current can be described as the (possibly attenuated) propagation of the channel-base current (CBC) upward along the channel itself, and, if the latter is vertical and rectilinear, the radiated EM field can be computed as shown in [16]. This procedure cannot be applied to tortuous channels. Some studies [17]–[20] have

evaluated the EM fields generated by tortuous channels and their coupling with power systems, thus highlighting the possible differences from the vertical model.

When dealing with channel tortuosity, the standard approach consists in considering a current that is not attenuated during its propagation along the tortuous path and then computing the radiated field as the sum of the effects produced by a large amount of linear segments into which the channel can be subdivided [21]. Such an approach obviously requires a huge computational effort in that it is based on the superposition law, where each segment is considered as a short lightning channel producing its own radiation.

In order to reduce this computational burden, the present paper proposes the introduction of an equivalent attenuation function that aims at incorporating the effects of the channel tortuosity into the classical EM field expressions [16] holding for a vertical channel. The equivalent attenuation function is obtained by solving an inverse problem using a Tikhonov regularization technique coupled with the L-curve method [22].

The paper is structured as follows: Section II describes the methodology from a theoretical viewpoint, whereas its application is exemplified in Section III by means of some results; finally, our conclusions are drawn in Section IV.

II. METHODOLOGY

The basic idea inspiring our method is to replace the tortuous channel with a suitable vertical one, in such a way that the attenuation function of the latter, usually denoted by $P(z')$, includes the effect of tortuosity, rather than the current attenuation along the channel. For this reason, from now on the equivalent attenuation function of a tortuous channel will be denoted as $T(z')$ instead of $P(z')$. As a consequence, the problem of computing $T(z')$ is fully analogous to the one presented in [23], with the difference that the regularization algorithm will now be implemented to deal with model errors rather than the measurement noise affecting the input data (i.e., the CBC and the radiated field), as was the main focus of [23]. In particular, in this paper all the input data will not be blurred by noise. However, in view of the satisfactory results obtained in [23], here we shall maintain the same

regularization procedure adopted there, i.e., Tikhonov regularization coupled with the L-curve method.

Now, let us consider the current flowing along the tortuous channel as a two-variable function $I(z', t)$, where z' represents the curvilinear abscissa of the channel and corresponds to the Cartesian coordinate along the vertical axis if and only if the lightning channel is straight and vertical. Then, the equivalent attenuation function $T(z')$ is evaluated according to the following steps:

1. Choice of the preliminary data, i.e., the CBC $I(0, t)$, the return-stroke speed v and the observation point $P(x, y, z)$;
2. numerical generation of the tortuous channel and computation of the EM fields radiated by it, according to [21]; due to the geometrical asymmetry of the channel, both the electric and magnetic fields have non-zero x , y , and z components;
3. reconstruction of an equivalent attenuation function, $T(z')$, by means of the method presented in [23], where the EM fields have been obtained from the tortuous channel as described in step 2 and the channel-base current has been chosen in step 1. Note that the procedure presented in [23] is proposed for a vertical lightning channel, which involves a cylindrical symmetry: then, the method is applied by considering only the vertical electric field (E_z) and the azimuthal magnetic field (H_ϕ), due to the strong dependence of the radial electric field on soil parameters [24], [25]. Moreover, the input data of the procedure in [23] are expressed in the frequency-domain: thus, a preliminary Fourier Transform (FT) has to be applied to the time-domain signals (CBC and radiated fields). According to [23], a proper selection of the number of discretized frequencies has to be performed in order to formulate the inverse problem as a linear system in matrix form;
4. choice of a set of N validation points $\mathcal{Q} = [Q_1(x, y, z), Q_2(x, y, z), \dots, Q_N(x, y, z)]$ in such a way that a) the channel base O and the projections of the observation point P and of the points \mathcal{Q} onto the xy -plane (the ground plane) are aligned; b) the radial distances in the xy -plane of the points \mathcal{Q} from

O are not smaller than the radial distance of P from O ;

5. computation of the EM fields generated by the vertical channel with the equivalent attenuation function $T(z')$, and their comparison with those radiated by the original tortuous channel. If the comparison provides a relative L_1 -mean error (denoted by ϵ) lower than a certain threshold, the procedure is ended; otherwise, the same steps are repeated after having increased the radial distance of the observation point P from the channel base O . In fact, if the observation point is too close to the lightning channel, the equivalent attenuation function cannot absorb and encode all the inhomogeneities or asymmetries of the tortuous channel; accordingly, it is necessary to reconstruct the function $T(z')$ from larger distances. Obviously (and consistently with step 4), the obtained results will be valid only for radial distances of the validation points \mathcal{Q} larger than that of the observation point P .

A graphical overview of steps 1-3 is provided in Figure 1, whereas the validation steps (4-5) are represented in the scheme of Figure 2.

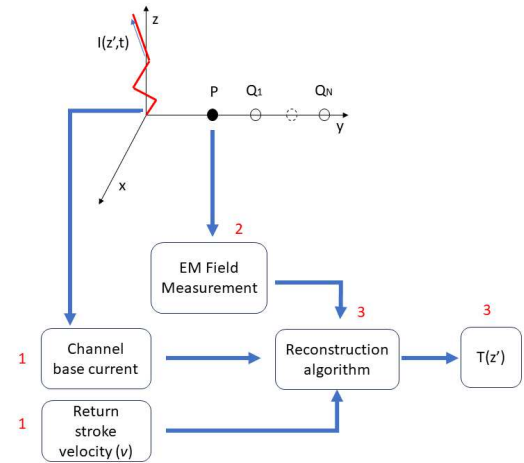


Figure 1. Graphical representation of the tortuous channel and of the steps 1-3 of the proposed procedure.

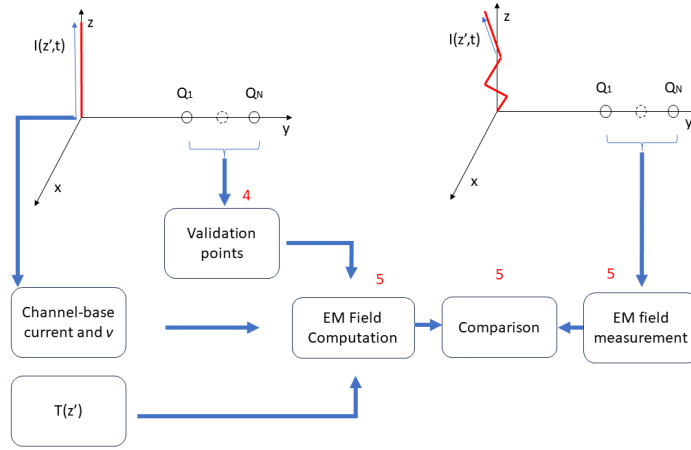


Figure 2, Graphical representation of the steps 4-5 of the proposed procedure.

III. RESULTS

Let us consider a tortuous lightning channel generated according to the algorithm proposed in [21], which is characterized by a first-stroke CBC modeled as the well-known Heidler's function [26], [27] and measured during a time interval of 1 ms. The return-stroke velocity and the total height of the channel are chosen as $c/3$ (i.e., one third of the light speed in vacuum) and 1.95 km, respectively. Moreover, let the observation point P be located at $x=0, y=1$ km, $z=0$ and the set of validation points Q be placed at y -distances ranging from 1 to 100 km along the y -axis, separated from each other by a uniform distance of 1 km.

According to the procedure presented in Section II, the equivalent attenuation function (shown in Figure 3) is obtained from the EM fields measured by the sensor at P and from the knowledge of the CBC. The procedure is applied in the frequency domain by using the first 50 frequencies provided by the fast FT (FFT) of the CBC, which leads to a maximum frequency of 25 kHz [23].

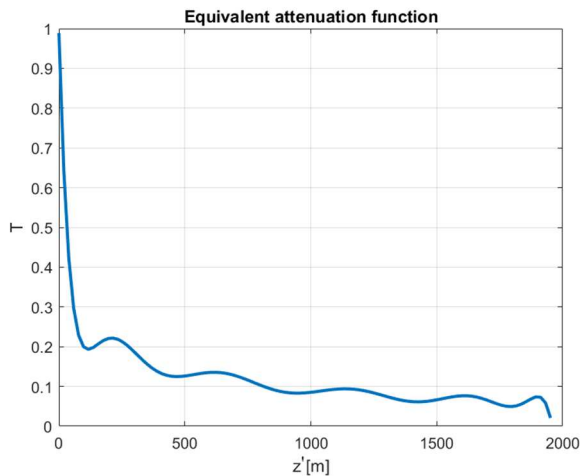


Figure 3. Equivalent attenuation function computed by considering the observation point $P(0, 1 \text{ km}, 0)$.

According to Section II, in order to decide whether the evaluated attenuation function $T(z')$ correctly represents the EM fields generated by the tortuous channel, we should make a comparison between the EM fields (labeled as

'Reconstructed' in the following figures) radiated by the vertical channel with attenuation function $T(z')$ and the EM fields (labeled as 'True' in the following figures) produced by the tortuous channel. The maximum acceptable L_1 -mean error ϵ will be set to the threshold value of 20%. In the same figures, a third graph (labeled as 'Reconstructed with MTLE') will be plotted in order to show the EM fields that a traditional vertical lightning channel with exponential attenuation (with $\lambda=2000$ m) would produce [4]. Moreover, for the sake of readability, in each figure the horizontal axis is limited to an initial time and a final time corresponding respectively to the time necessary for the traveling wave to reach the validation point and to the time for the lightning current to reach the channel top, after which the model loses its validity.

Given the equivalent attenuation function shown in Figure 3, the comparison among the three kinds of EM fields described above has been made by considering a validation point located at $x=0, y=6$ km, $z=0$: the results are displayed in Figure 4 for the azimuthal magnetic field and in Figure 5 for the vertical electric field. As can be observed, the reconstruction is extremely poor, since it drastically underestimates the measured EM fields. This is confirmed by Figure 6, where the relative mean error ϵ is shown as a function of the distance of the validation point in the range from 6 to 100 km.

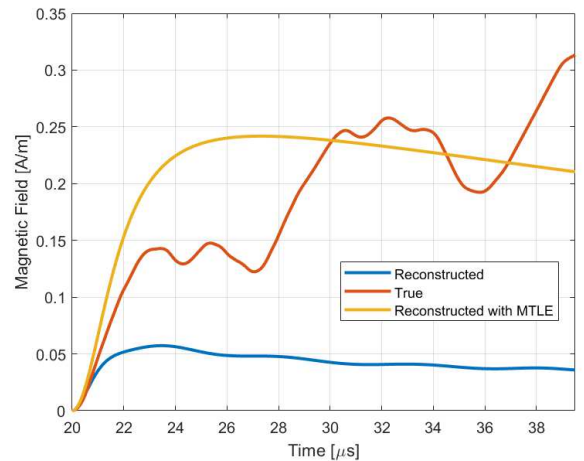


Figure 4. Comparison among the azimuthal magnetic fields obtained by considering either the tortuous channel ('True'), or the vertical channel with

the equivalent attenuation function $T(z')$ ('Reconstructed'), or the vertical channel with a typical exponential decay ('Reconstructed with MTLE'). The validation point is located at $x=0, y=6$ km, $z=0$, whereas the reconstruction has been obtained from an observation point located at $x=0, y=1$ km, $z=0$.

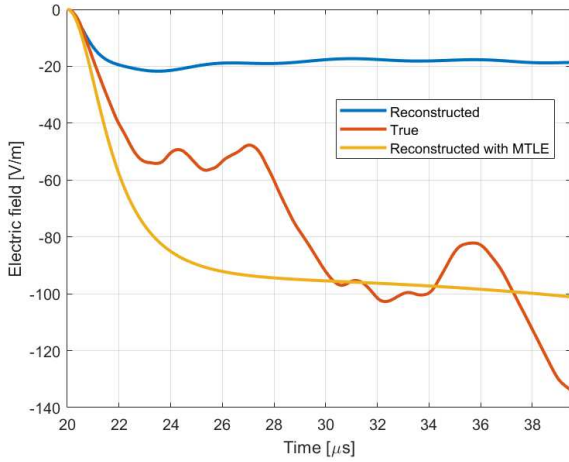


Figure 5. Comparison among the vertical electric fields obtained by considering either the tortuous channel ('True'), or the vertical channel with the equivalent attenuation function $T(z')$ ('Reconstructed'), or the vertical channel with a typical exponential decay ('Reconstructed with MTLE'). The validation point is located at $x=0, y=6$ km, $z=0$, whereas the reconstruction has been obtained from an observation point located at $x=0, y=1$ km, $z=0$.

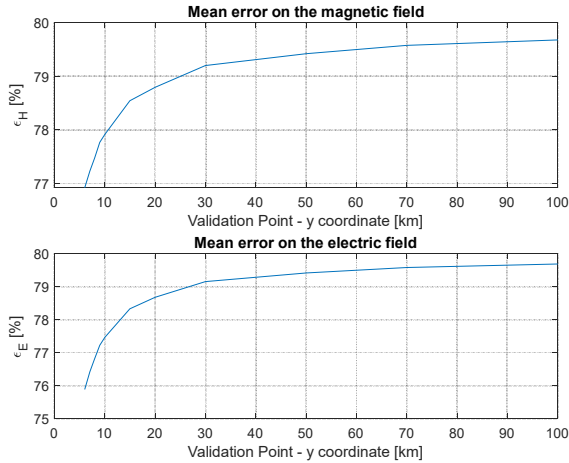


Figure 6. Relative mean error on the magnetic and electric fields, by considering a reconstruction obtained from a sensor at $x=0, y=1$ km, $z=0$ and $N=95$ validation points located at equispaced y -distances from 6 to 100 km.

Then, by applying the iterative procedure of step 5 in Section II, a solution below the threshold error can be found if the observation point P is moved to $x=0, y=5$ km, $z=0$. The new equivalent attenuation function is shown in Figure 7, whereas the comparison among the magnetic and the electric fields (computed for validation points located at $y=6$ km and $y=100$ km) are proposed in Figure 8-Figure 9 and Figure 10-Figure 11, respectively. As can be observed, the reconstructed field is closer to the true one; in particular, the reconstructed field represents a sort of average of the true one, or in other words, it behaves as a low-pass filter, excluding high-frequency oscillations, which is somehow reasonable since the reconstruction procedure has been implemented by limiting the maximum frequency to 25 kHz. For the sake of completeness, it should be noted that, if a classical model

with a typical attenuation decay (MTLE) is used, the results are drastically different in terms of EM field waveforms. Finally, Figure 12 shows the new relative mean error ϵ for the same validation points as in Figure 6: such error is now much lower than before and, in particular, lower than the threshold value of 20%. However, we also note that, in both cases, the relative mean error turns out to be an increasing function of the distance of the validation points from the channel base: indeed, in spite of the fact that at larger distances some asymmetries of the tortuous channel tend to fade (as well as some of its discrepancies with respect to a vertical channel), the fixed position of the reconstruction point also involves an increasing distance between the latter and the validation points, thus impairing the validity of the equivalent attenuation function.

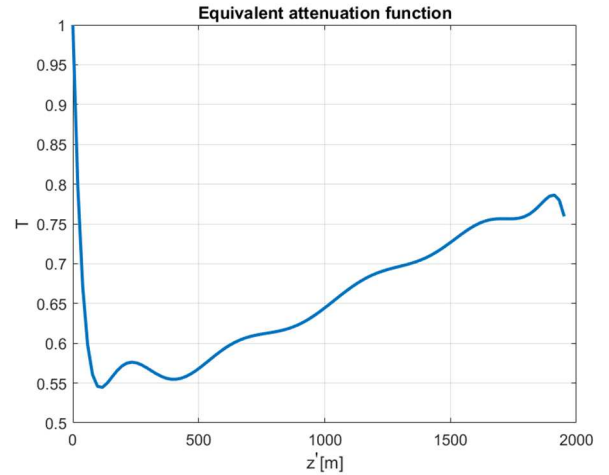


Figure 7. Equivalent attenuation function computed by considering the observation point $P(0, 5$ km, $0)$.

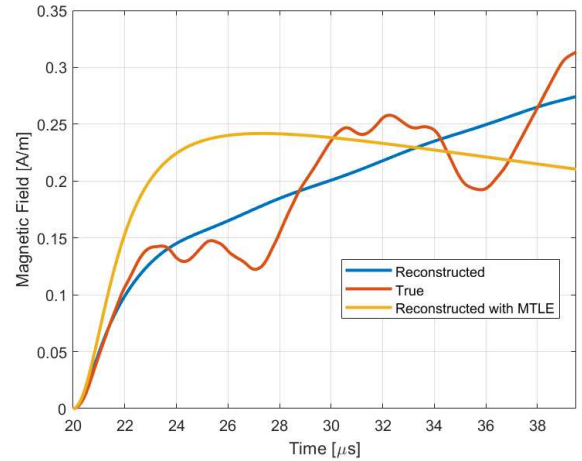


Figure 8. Comparison among the azimuthal magnetic fields obtained by considering either the tortuous channel ('True'), or the vertical channel with the equivalent attenuation function $T(z')$ ('Reconstructed'), or the vertical channel with a typical exponential decay ('Reconstructed with MTLE'). The validation point is located at $x=0, y=6$ km, $z=0$, whereas the reconstruction has been obtained from an observation point located at $x=0, y=5$ km, $z=0$.

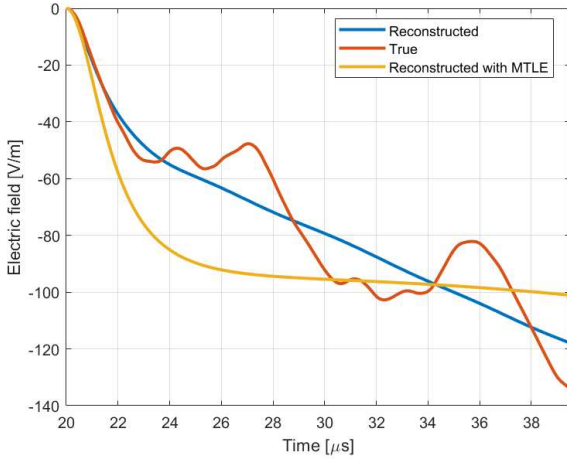


Figure 9. Comparison among the vertical electric fields obtained by considering either the tortuous channel ('True'), or the vertical channel with the equivalent attenuation function $T(z)$ ('Reconstructed'), or the vertical channel with a typical exponential decay ('Reconstructed with MTLE'). The validation point is located at $x=0, y=6$ km, $z=0$, whereas the reconstruction has been obtained from an observation point located at $x=0, y=5$ km, $z=0$.

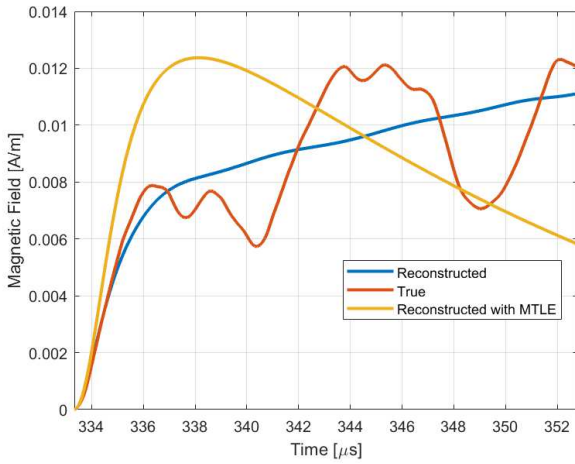


Figure 10. Comparison among the azimuthal magnetic fields obtained by considering either the tortuous channel ('True'), or the vertical channel with the equivalent attenuation function $T(z)$ ('Reconstructed'), or the vertical channel with a typical exponential decay ('Reconstructed with MTLE'). The validation point is located at $x=0, y=100$ km, $z=0$, whereas the reconstruction has been obtained from an observation point located at $x=0, y=5$ km, $z=0$.

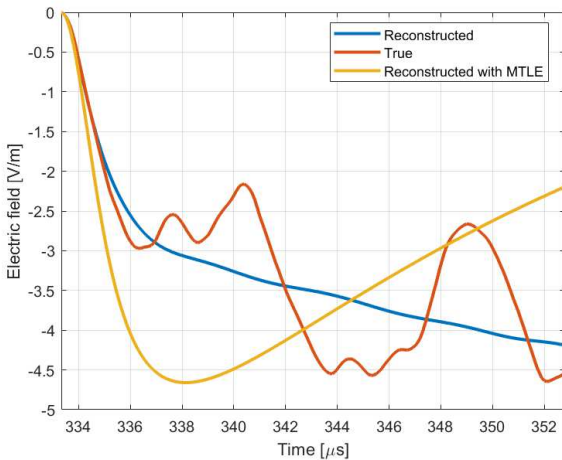


Figure 11. Comparison among the vertical electric fields obtained by considering either the tortuous channel ('True'), or the vertical channel with the equivalent attenuation function $T(z)$ ('Reconstructed'), or the vertical

channel with a typical exponential decay ('Reconstructed with MTLE'). The validation point is located at $x=0, y=100$ km, $z=0$, whereas the reconstruction has been obtained from an observation point located at $x=0, y=5$ km, $z=0$.

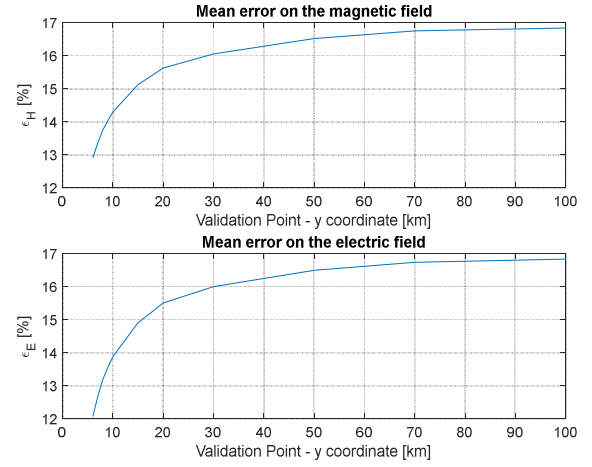


Figure 12. Relative mean error on the magnetic and electric fields, by considering a reconstruction obtained from a sensor at $x=0, y=5$ km, $z=0$ and $N=95$ validation points located at equispaced y -distances from 6 to 100 km.

IV. CONCLUSIONS

Dealing with tortuous lightning channels usually requires a huge computational effort to calculate the EM field radiated by them, since the whole channel should be first discretized into a sum of linear segments and then the total field has to be evaluated by means of the superposition principle. This work presents a methodology for computing an equivalent attenuation function that can include the effect of channel tortuosity in a vertical lightning channel. The proposed function is evaluated by means of an inverse procedure based on the Tikhonov regularization technique. The preliminary results presented here show a decent agreement between the EM fields generated by the tortuous lightning channel and the ones generated by the vertical channel with the equivalent attenuation function, provided that the latter is reconstructed from measurements taken by a sensor located at least 5 km far from the channel base.

Further developments might concern the possibility of increasing the versatility and effectiveness of the equivalent attenuation function, either by taking into account a larger frequency range (in order to reduce the low-pass filter effect highlighted in Section III) or by considering multiple reconstruction points distributed all around the channel (in order to obtain an attenuation function that is equally valid in all directions). Moreover, since the tortuous channel can be considered as a sequence of tilted segments that make the current arrive at different heights with varying delays, a similar approach might be conceived for finding an equivalent speed of the return stroke as a function of the curvilinear abscissa of the channel.

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