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Evaluation of the Return-Stroke Current Attenuation Function from the Vertical Electric Field by Means of the Regularization of an Inverse Problem

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***Abstract*—The behavior of the return-stroke current along the lightning channel is extremely important for the evaluation of the electromagnetic fields radiated at various distances, since such fields may cause disturbances or faults on electric and electronic equipment. It is commonly known that lightning current attenuates while propagating along the channel and several models have been adopted in an attempt to reproduce the characteristics of the electromagnetic fields at different distances. This work proposes a method that can reconstruct the attenuation of the current along the channel, starting from the knowledge of the channel-base current and of the vertical electric field measured by a sensor located five km away from the discharge location. The method relies upon the Tikhonov regularization technique applied to an ill-posed problem in the frequency domain and is mainly based on the hypothesis of long-time measurement windows.**

Keywords*—Lightning, reconstruction, inverse problems, electromagnetic fields, attenuation function.

I. Introduction

Lightning activity is a crucial issue in terms of people safety and infrastructures security [1]. When dealing with the protection of transmission and distribution systems [2], [3] or tall structures (such as communication towers and wind turbines), a proper knowledge of the lightning current, both at the channel base and of its behavior along the lightning channel, is extremely important due to the radiated electromagnetic fields that may be coupled with the aforementioned structures, thus possibly leading to critical damages [3].

The channel-base current has been widely measured on different instrumented towers [4]–[6], which have provided an extensive database in terms of statistics of the main parameters (such as the peak current, the front duration, the maximum derivative and the specific energy) and have consequently led to numerous channel-base current expressions [7]–[10]. On the other hand, the evaluation of the lightning current behavior at different heights is much more critical and cannot be obtained by means of direct measurements. For this reason, the most widely adopted ways for describing the evolution of the current along the channel are the so called Transmission Line Engineering Models (TLEM) [11], which basically express the current at different heights (z') as the measured channel-base current, delayed in time according to the return stroke velocity (v), and attenuated by means of an attenuation function (P), as formalized by the following equation:

$$I\left(z',t\right)=P\left(z'\right)I\left(0,t-\frac{|z'|}{v}\right) , \qquad (1)$$

where the return-stroke speed has been assumed equal to the wave propagation speed of the current.

Several researchers have proposed different models for *P*, based on either empirical assumptions or the attempt of reproducing the most significant features of the measured electromagnetic fields. Among them, the most famous functions are the Transmission Line model (TL) [12], the Modified Transmission Line Linear model (MTLL) [13] and the Modified Transmission Line Exponential model (MTLE) [14]. While the TL model assumes no attenuation along the channel, which is far from the reality, the MTLL and MTLE models propose a linear and exponential decay respectively, which can be good approximations able to reproduce the electromagnetic fields. Recently, other approaches such as the MTLTCOS, MTLTSIN, MTLT and MTLT2 models have been proposed with satisfactory results [15].

All these models have been postulated in advance and then validated by fitting their parameters to the experimental measurements of the electromagnetic fields. However, in order to evaluate the attenuation function of the lightning current, a second (and non-parametric) approach can be conceived. Following the work presented in [16], this article introduces a method for reconstructing the attenuation function from measurements of the radiated vertical electric field and of the channel-base current, as well as from the knowledge of some parameters, such as the speed *v* of the return stroke and the channel height *H*. In this framework, the attenuation function *P* is the unknown of an inverse ill-posed problem obtained from the discretization of the integral equations relating the lightning current to the radiated fields.

The inverse problem is formulated in the frequency-domain [17]–[19], in order to filter out (by considering low frequencies) the measurement noise affecting both the channel-base current and the radiated electric field; the presence of such noise is included in our study by means of an appropriate numerical simulation. The solution of the inverse problem is computed via the regularization technique adopted in [16] and here applied to a case where the channel-base current is represented by the expression proposed in [20]. As will be detailed later, the method is applied to cases where the attenuation function is known a priori, in order to prove its effectiveness in reconstructing such function under the only assumption of a wide window of current and field recording (i.e., 1500 μs).

In the following, Section II presents the theoretical formulation of the problem, Section III outlines its numerical treatment and the regularization procedure, Section IV and V shows the data and the reconstructions obtained for the attenuation function, and finally some conclusions are drawn in Section VI.

II. Theoretical formulation of the problem

Let us consider a lightning stroke characterized by a vertical and rectilinear channel whose height is *H* and let us define Q_1 as a generic point on the channel and Q_2 the measurement point of the electric field, as shown in Figure 1.

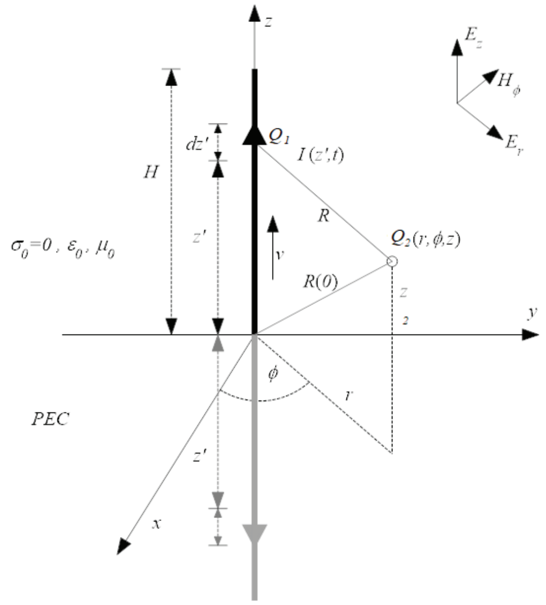

Figure 1. Geometry of the problem.

Under the assumption of a Perfect Electric Conducting ground (PEC), the vertical component of the electric field can be expressed as follows [21]:

$$E_z(r,z,t) = \frac{1}{4\pi\varepsilon_0}\int_{-H}^{H}\left[\frac{2(z-z')^2-r^2}{cR^4} I\left(z',t-\frac{R}{c}\right) - \frac{r^2}{c^2R^3}\frac{\partial I\left(z',t-\frac{R}{c}\right)}{\partial t} + \frac{2(z-z')^2-r^2}{R^5}\int_0^t I\left(z',\tau-\frac{R}{c}\right)d\tau\right]dz'. \quad (2)$$

where *r* and *z* are the coordinates of the measurement point Q_2, *c* is the speed of light in vacuum and $R=\sqrt{r^2+(z-z')^2}$. Note that, for the evaluation of the vertical electric field, the hypothesis of a PEC ground is commonly accepted [22]. According to [21], the Fourier Transform (FT) of (2) (for $\omega \neq 0$) is

$$\hat{E}_z(r,z,\omega) = \frac{1}{4\pi\varepsilon_0}\int_{-H}^{H}\left[\frac{\left(2(z-z')^2-r^2\right)\left(c^2+j\omega cR\right)}{j\omega c^2R^5} + \frac{\omega^2r^2R^2}{j\omega c^2R^5}\right]e^{-\frac{j\omega R}{c}}\hat{I}(z',\omega)\,dz'. \quad (3)$$

where $\hat{I}(z',\omega)$ is the FT of (1).

However, according to [23], a FT can be properly computed if an infinite time interval is considered, which is obviously impossible from a practical point of view. For this reason, and according to the expressions proposed in [16] and [23], (3) should be replaced by the following equation:

$$\hat{E}_z^T(r,z,\omega)=\frac{1}{4\pi\varepsilon_0}\int_{-H}^{H}\frac{\left(2(z-z')^2-r^2\right)c-j\omega r^2 R}{c^2R^4}e^{\frac{-j\omega R}{c}}\hat{I}(z',\omega)dz'$$
$$+\int_{-H}^{H}\frac{\left(2(z-z')^2-r^2\right)}{j\omega R^5}\left[e^{\frac{-j\omega R}{c}}\hat{I}(z',\omega)-e^{-j\omega T}\hat{I}(z',0)\right]dz'. \quad (4)$$

where T is the measurement time of the vertical electric field. In turn, T should satisfy the following condition [23]:

$$T\geq T_0+\frac{\sqrt{r^2+(H-z)^2}}{c}, \qquad T_0=\frac{H}{v}+T_c, \quad (5)$$

with T_c being the measurement time of the channel-base current $I(0,t)$. Note that the validity of (4) is guaranteed if the current $I(0,t)$ has a compact support in the interval [0, T_c], i.e., in practice, if it can be considered zero with a good approximation when $t\geq T_c$. For this reason, in the following, it will be mandatory to choose a time window large enough, i.e., such that the current will vanish at the end of it.

Finally, the FT of (1) is

$$\hat{I}(z',\omega)=P(z')\hat{I}(0,\omega)e^{-j\omega\frac{|z'|}{v}} \quad (6)$$

which, if substituted into (4), leads to

$$\hat{E}_z^T(r,z,\omega)=\frac{\hat{I}(0,\omega)}{4\pi\varepsilon_0}\int_{-H}^{H}\left\{\frac{\left(2(z-z')^2-r^2\right)c-j\omega r^2R}{c^2R^4}e^{-j\omega\left(\frac{R}{c}+\frac{|z'|}{v}\right)}\right.$$
$$\left.\frac{\left(2(z-z')^2-r^2\right)}{j\omega R^5}\left[e^{-j\omega\left(\frac{R}{c}+\frac{|z'|}{v}\right)}-e^{-j\omega T}\frac{\hat{I}(0,0)}{\hat{I}(0,\omega)}\right]\right\}P(z')dz' \quad (7)$$

The aim of the proposed method is to find $P(z')$ by solving a discretized version of (7) and assuming the knowledge of $\hat{I}(0,\omega)$, $\hat{E}_z^T(r,z,\omega)$, the location of Q_2 (i.e., r and z), the height H, and the return stroke speed v.

III. Numerical Treatment, Inversion and Regularization

A proper numerical treatment requires the transformation of (7) into a linear algebraic system. In the following we will only recall the main steps, and the reader is referred to [16] for a complete analysis.

First of all, it is necessary to decompose P into a linear combination of N basis functions as follows:

$$P(z')\approx\sum_{n=1}^{N}p_n\varphi_n(z'), \quad (8)$$

with p_n being the coefficients and φ_n the basis functions, which will be chosen as the Gegenbauer polynomials of order 1/2 [19], [24], [25], while, according to [16], $N=12$. Due to the lack of attenuation when $z'=0$, the coefficients have to satisfy the following condition:

$$\sum_{n=1}^{N}p_n\varphi_n(0)=1. \quad (9)$$

Secondly, the right-hand-side of (7) can be regarded as a linear integral operator, denoted by A_z and acting on the attenuation function P, so that (7) can be rewritten in the more compact form

$$\sum_{n=1}^{N}p_n\left[A_z(\varphi_n)\right](r,z,\omega)=\frac{\hat{E}_z^T(r,z,\omega)}{\hat{I}(0,\omega)}, \quad (10)$$

which, to be solved for the N unknown coefficients p_n, basically requires at least N equations corresponding to N different frequencies [16].

The solution of the algebraic system coming from (10) requires proper regularization procedures, since the problem is highly ill-posed. Due to the noise affecting A_z and the right-hand side (RHS) of (10), the Tikhonov regularization technique, coupled with the L-curve method, has been adopted [26], since it does not require any previous knowledge on the noise; the whole inversion procedure has been implemented in MATLAB by means of the routines described in [27].

IV. Choice of the Data

In order to test our approach, it is necessary to choose the parameters appearing in A_z (and summarized in Table 1), as well as to compute the data related to the RHS of (10), i.e., the FT of the channel-base current and of the vertical electric field.

Table 1. Choice of the parameters related to the computation of A_z.

Parameter	Value
H	4.0 km
v	150 m/µs
r	5.0 km
z	0 m
T_c	1500 µs

As proposed above, the measurement point Q_2 is located at ground level and 5.0 km away from the channel. Moreover, according to (5), we have $T=1548$ µs.

The channel-base current is chosen so as to be representative of a typical measurement in Morro do Cachimbo (Brazil), whose time evolution can be well described by the sum of seven Heidler's functions [20]. Since any real measurement is affected by noise, we added a 10% uniform noise to the signal $I(0,t)$ and then obtained a discretized version of $\hat{I}(0,\omega)$ by means of a Fast Fourier Transform (FFT), where the discretized step in the time-domain is 10 ns, corresponding to the data of [28]. The time-domain and the frequency-domain spectra of the channel-base current with and without the noise are shown in panels (a) and (c) of Figure 2.

The vertical electric field is computed in the time-domain according to (2) and for two different cases, corresponding to two attenuation functions (i.e., the MTLE model, with parameter λ=2000 m, and the MTLL model). Next, a 5% uniform noise is added to $E_z(r,z,t)$ and then the FFT is applied with a 100 ns time step (according to the measurements of [28]) in order to obtain $\hat{E}_z^T(r,z,\omega)$. The time-domain and the frequency-domain spectra of the vertical electric field with and without the noise, for the MTLE model are shown in panels (b) and (d) of Figure 2.

For the reconstruction procedure, the first 80 frequencies obtained from the FFT will be considered.

V. Reconstruction of the Attenuation Function

The reconstructions of the MTLE and MTLL models, obtained by implementing the procedure outlined in Section III, are displayed in Figure 3 and 4: these results can be considered satisfactory, considering a mean standard error below 5% in both cases.

The reconstructions are obtained by using a long-time window, i.e., 1500 µs. As a term of comparison, analogous reconstructions are proposed by also considering a reduced time window, i.e., 100 µs. The results, displayed in Figure 5 and 6, show a significant deterioration of the reconstruction quality: this effect is caused by the failure of the main assumption of the approach, i.e., the need of considering the current as vanishing at the end of its time interval, which is a consistent hypothesis if $T_c = 1500$ µs, whereas it is not true if $T_c = 100$ µs, as shown in Figure 2(a).

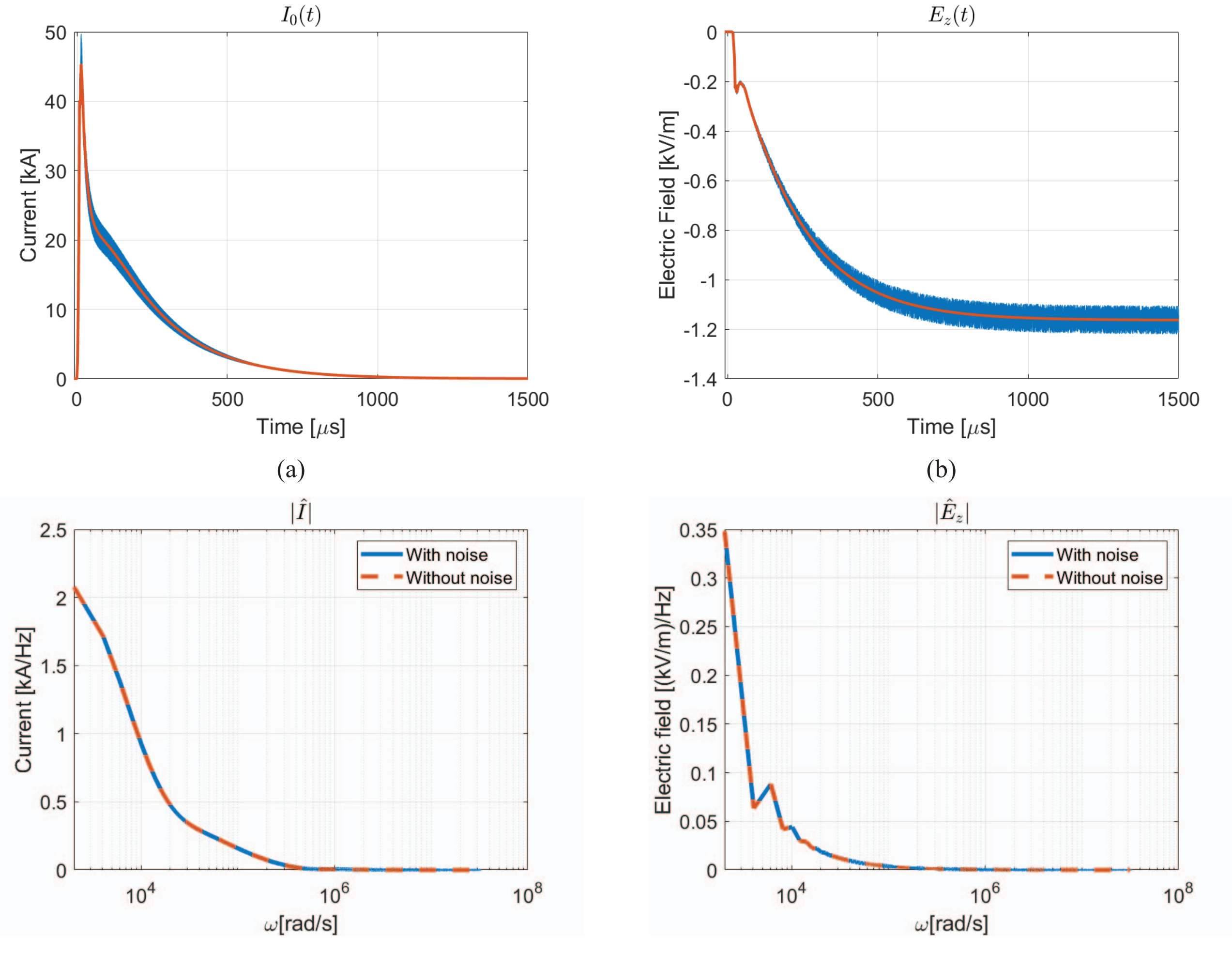

Figure 2. (a) Channel-base current in the time domain, (b) vertical electric field in the time domain, (c) channel-base current in the frequency domain, (d) vertical electric field in the frequency domain, considering the MTLE model and the presence of uniform noise.

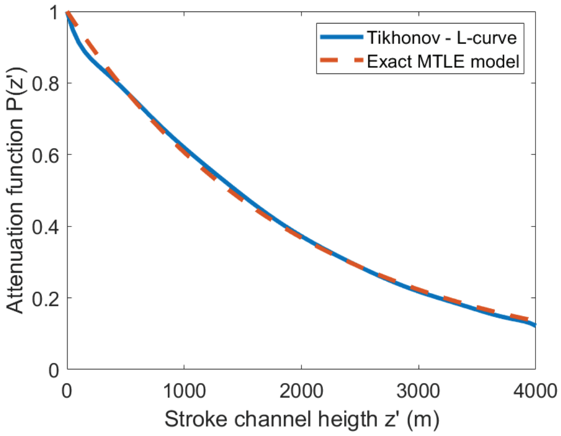

Figure 3. Reconstruction of the MTLE model for $T_c = 1500$ μs.

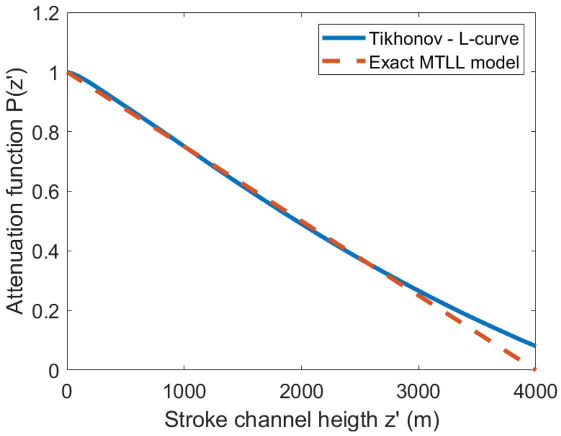

Figure 4. Reconstruction of the MTLL model for $T_c = 1500$ μs.

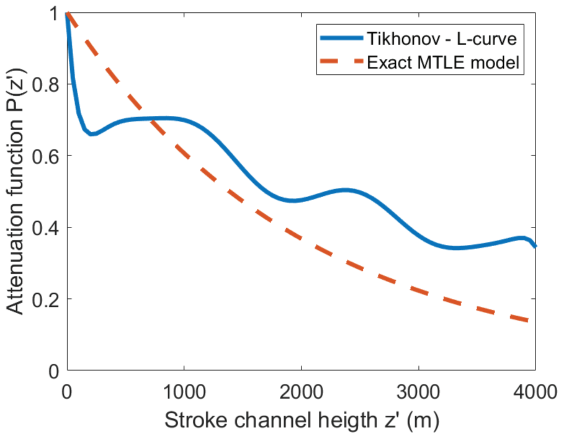

Figure 5. Reconstruction of the MTLE model for $T_c = 100$ μs.

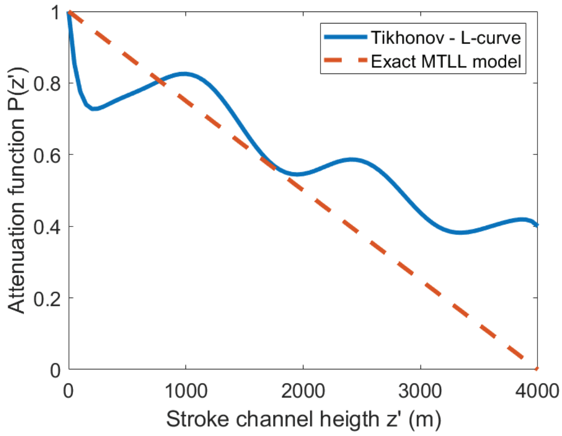

Figure 6. Reconstruction of the MTLL model for $T_c = 100$ μs.

VI. Conclusions

This work proposed a method for the reconstruction of the return-stroke attenuation function from the measurement of the vertical electric field, the channel-base current and the knowledge of some important parameters, such as the return-stroke speed and the channel height. The approach is based on the resolution of an ill-posed problem in the frequency domain by means of the Tikhonov regularization technique: the latter also allows coping with the measurement noise blurring the vertical electric field and the channel-base current data. The main assumptions of this work consist in the need of a long-time measurement window and in a correct estimation of the channel parameters, such as the return-stroke speed and the channel height.

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