Published in the *Proceedings of the 50th Seminar of the European Group of Risk and Insurance Economists*, Malaga, Spain, 17-20th September 2023

Long-term care insurance: The importance of distinguishing between severe and mild dependency

Christophe Courbage*
Haute école de gestion Genève - HES-SO (christophe.courbage@hesge.ch)

Cornel Oros
Université de Poitiers (cornel.oros@univ-poitiers.fr)

Peter Zweifel
University of Zurich (peter.zweifel@uzh.ch)

Abstract: Extant work on long-term care (LTC) and its insurance has neglected an important fact: Benefits of LTC insurance as well as the amount of public subsidization of LTC differ between severe and mild dependency. The objective of this paper is to revisit earlier results regarding the link between LTC insurance and informal care considering different levels of dependency together. It first models the optimal levels of insurance and of informal care for mild and severe dependency. It shows that the effect of potential intergenerational moral hazard and the crowding out of LTC insurance by public subsidization depend on the severity of dependency. The effects of a change in the child opportunity cost and inheritance rate are also considered.

Keywords: long-term care; informal care; long-term care insurance; crowding out

1. Introduction

For decades, observers have been puzzled by the small market for private long-term care (LTC) insurance. At first sight, LTC insurance is not very different from health insurance, whose private component is substantial in many countries. There have been two main explanations of this puzzle.

The first dates back to Pauly (1990), who pointed out intra-family moral hazard as a cause for the sluggish development of private LTC insurance. Intra-family moral hazard refers to the disincentives for children as potential informal caregivers to provide care when the parent has LTC insurance. It occurs as LTC insurance has the effect of protecting the bequest from the cost of LTC. This moral hazard effect works both ways in that the person at risk for LTC (typically a parent), may go without LTC insurance, anticipating informal care provided by the child which will obviate an expensive stay in the nursing home (Zweifel and Strüwe, 1996, 1997). While the interaction between the two decision-makers was originally couched in a principal-agent framework with the parent as the principal, Courbage and Zweifel (2011) argued that the child usually is aged 50 and beyond so should be modeled as an independent agent. This approach calls for the derivation of a non-cooperative Nash equilibrium where the parent controls the amount of LTC insurance and the child, the amount of caring effort. It

* Christophe Courbage acknowledges the financial support of RCSO E&M (HES-SO).

predicts a two-sided intergenerational moral hazard effect. Empirical evidence concerning the relationship between insurance and informal care is mixed being either positive or negative (see e.g. Mellor (2001), Costa-Font (2010), Xu and Zweifel (2014), Costa-Font and Courbage (2015)).

The second explanation for the small market for private LTC insurance emphasizes the crowding-out effect of public subsidization. Indeed, the OECD (n.d.) documents that in most industrial countries, support provided by the government or social insurance covers the major part of LTC both in the case of mild (care at home sufficient) and severe dependency (stay in a nursing home required). For instance, in the United States, Medicaid covers LTC expenses of low-income individuals. Created in 1965, it was expanded in 2014, triggering research on a potential crowding out of private LTC insurance. Brown and Finkelstein (2007), while noting the high loadings contained in premiums, argue that the spending-down requirement of Medicaid discourages the purchase of private LTC insurance by implicitly taxing individuals seeking to protect their wealth by buying coverage. Other empirical works from different countries come with mixed results (see e.g. Costa-Font and Courbage (2015)). Also, this strand of literature does not look into the interactions between individuals at risk of LTC and those potentially providing care in any detail.

The objective of this contribution is to bring potential intragenerational moral hazard and crowding out together, as in Courbage and Zweifel (2011), Zweifel and Courbage (2015) and Bascans et al. (2017), to investigate the link between LTC insurance and informal care but distinguishing between the state of severe dependency and its insurance from that of mild dependency and its insurance, thus adding realism to the analysis. Both public subsidization and private LTC insurance make this distinction, but is not considered in the previous literature.

Section 2 is devoted to a description of the model, followed by the most simple scenario in Section 3, where the person in potential need of LTC (the parent henceforth) purchases amounts of LTC coverage without taking the future behavior of the potential caregiver (the child henceforth) into account. The finding is that the difference between the optimal insurance coverage of nursing home and of care at home is driven by both the difference in the severity of dependency cost and in the marginal utilities of wealth in states of dependency.

In Section 4, the setting becomes that of a non-cooperative game, where the parent is assumed to be able to predict the child's future reaction to changes in LTC insurance coverage. The reaction functions indicate that the parent increases LTC insurance coverage of nursing home care for severe dependency in reaction to more effort from the child. In case of mild dependency, the parent increases or decreases insurance coverage for home care in response to more informal care depending on the amount of difference between the levels of two types of insurance. The reaction functions of the child indicate that he/she decreases or increases the amount of informal care when the parent opts for more coverage in the case of severe dependency depending on the child degree of absolute risk aversion. In the case of mild dependency, the child increases informal care in response to an increase in parent's coverage. These results tend to contradict the notion of two-sided intergenerational moral hazard in both the case of mild dependency and of severe dependency.

Section 5 contains an investigation of whether an increase in public subsidization of nursing home care or of care at home indeed shifts the Nash equilibrium and in which direction. Results shows that crowding-out and crowding-in effects of LTC insurance and informal care depend on the type of dependency considered. This is confirmed also in the case of a change in inheritance rate and child's opportunity cost, pointing to complementarity between insurance and informal care for severe dependency. The final Section 6 contains a review of the findings and of the limitations of the analysis.

2. The model

The model setup mainly follows Courbage and Zweifel (2011), Bascans et al. (2017) and Cremer and Roeder (2017). It considers a parent and a child characterized by state-dependent VNM utility functions defined over wealth. Their utility functions are increasing and concave reflecting risk-averse behavior.

The parent is retired so does not earn a labor income and has accumulated an amount of wealth w_0. They face the risk of becoming dependent to a severe or a mild degree. In the case of severe dependency, the parent moves to a nursing home, which entails a cost *N*. In the case of mild dependency, the parent can stay at home but incurs home care expenses amounting to L. The duration of all states is normalized to one. The government subsidizes the cost of a stay in the nursing home at a rate α_N and the cost of home care, at a rate α_H . Therefore, $1-\alpha_N$ and $1-\alpha_H$ respectively denote the share of LTC cost paid by the parent. Next, p is the probability of the parent being dependent. Given dependency, the probability of a severe level is q while the probability of a mild level is $1-q$. The patent´s state-dependent utility functions are $u(w)$ in the absence of dependency, $v_H(w)$ when in need of home care, and $v_N(w)$ when in the nursing home, respectively, with $u(w) > v_H(w) > v_N(w)$. This ranking can be justified by the argument that they reflect a decreasing degree of freedom of how to spend wealth on consumption.

The parent can also receive informal care *e* from the child, which has two effects. First, it lowers the probability of a nursing home admission such that $q'(e) < 0$ and $q''(e) > 0$. Second, it reduces the cost of care at home in the event of mild dependency such that $L'(e) < 0$ and $L''(e) > 0$.

The parent may decide to buy LTC insurance to protect against the cost of LTC, which pays the indemnity I_H in the case of mild dependency requiring home care and I_N in the case of severe dependency requiring admission to a nursing home. The actuarially fair premium equals $p\{q(e)I_N + (1-q(e))I_H\}$ and is waived when dependency sets in, as occurs in most LTC insurance contract [1]. We therefore assume that the LTC insurer can observe the amount of informal care e provided[2].

The child´s VNM utility functions acting as a caregiver are $\bar{v}_H(z)$ if the parent is mildly dependent and $\bar{v}_N(z)$ if he or she is severely dependent, with $\bar{v}_N(z) < \bar{v}_H(z)$ and *z* denoting child wealth. This ranking may reflect altruism on the part of the child or the mere fact that spending on consumption with a parent who is confined to a nursing home yields less utility than in the less constraining environment of one´s home. The child may provide informal care e at a wage or opportunity cost of θ per unit which has two effects as stressed earlier. It reduces the probability of a nursing home admission and reduces the cost of care at home in the event

[1] In reality, LTC insurance premiums contain a substantial loading. However, neglecting the loading simplifies calculations but does not substantially affect the results.

[2] We could also assume that the insurer is not able to observe the effort of the child and considers in the premium an average probability of severe dependency and nursing home use equals to $\bar{q}$ and such that $\bar{q} > q(e)$. Results are rather similar but more complex to interpret.

of mild dependency. Child wealth consists of an exogenous initial component z_0, from which θe is deducted and the share b of the parent's final wealth as bequest (inversely proportional to the inheritance tax) is added[3].

The timing of the model is as follows.

- At $t = 0$, the government announces its policy, i.e. the levels of α_N and α_H.
- At $t = 1$, before knowing whether being dependent or not, the parent chooses the optimal amounts of LTC insurance for each level of dependency, with an estimate of the probabilities p and q only. With regard to the predicted behavior of the child in response to the parent´s choice of LTC coverage, several scenarios are possible. The most simple is to neglect future child behavior altogether. This is not unrealistic because an LTC insurance decision needs to be set early (i.e. at an age when the child is still young) to avoid a hefty increase in premium, as advised by Brodsky (2020). The traditional variant is to assume that the parent can predict the future amount of caring e provided by the child.
- At $t = 2$, the state of nature is revealed and the child decides on the optimal quantity of informal care to provide in the event of dependency. This effort reduces both the amount of formal home care and the probability of entering into a nursing home.

The scenario where the parent is not able to anticipate the child's behavior to a change in insurance is considered first.

3. No anticipation of the child's behavior

The objective of this section is to determine the optimal amounts of LTC insurance purchased by a parent who neglects the child´s reaction to it. Expected utility is given by

$$P = (1-p)u\Big(w_0 - p\big(q(e)I_N + \big(1-q(e)\big)I_H\big)\Big) + pq(e)v_N(w_0 - N + I_N + \alpha_N N) + p(1-q(e))v_H(w_0 - L(e) + I_H + \alpha_H L(e)) \quad (1)$$

Indemnities I_N and I_H are chosen according to the first-order conditions for an interior optimum:

$$P_{I_N} := \frac{\partial P}{\partial I_N} = pq(e)\{v_N'(C) - (1-p)u'(A)\} = 0 \quad (2)$$

$$P_{I_H} := \frac{\partial P}{\partial I_H} = p(1-q(e))\{v_H'(B) - (1-p)u'(A)\} = 0 \quad (3)$$

with $A := w_0 - p\big(q(e)I_N + \big(1-q(e)\big)I_H\big)$, $B := w_0 - (1-\alpha_H)L(e) + I_H$, and $C := w_0 - (1-\alpha_N)N + I_N$.

In each of the two states of dependency, the optimal level of insurance, I_N^* or I_H^*, is such that its marginal expected benefit due to higher wealth equals its expected marginal cost, i.e. the

[3] Labor income is neglected because most informal LTC is provided by women, who are often in retirement age themselves (Administration for Community Living, 2020). Also, in most countries inheritances are taxed at a rate that varies with the closeness of family ties, a complication that is neglected as well for simplicity.

reduction in wealth and hence utility due to the extra premium paid when dependency does not materialize.

Division of eq. (2) by $pq(e)$ yields $v'_N(C) = (1-p)u'(A)$ and of eq. (3) by $p(1-q(e)$, $v'_H(B) - (1-p)u'(A)$. This implies:

$$v'_N(C) = v'_H(B) \tag{4}$$

The equality of marginal utilities permits to infer the optimal levels of wealth and amounts of coverage in the two states of dependency. Three cases need to be distinguished.

(1) $v'_N(w) = v'_H(w)$, i.e. the marginal utility of risky wealth is the same across the two states of dependency. Then, $v'_N(C) = v'_H(B)$ is equivalent to $C = B$, i.e. the optimal amount of insurance makes wealth equal across the two states of dependency. This in turn implies

$$I_N^* - (1-\alpha_N)N = I_H^* - (1-\alpha_H)L(e) \Leftrightarrow I_N^* - I_H^* = (1-\alpha_N)N - (1-\alpha_H)L(e)$$

It is reasonable to assume that the out-of-pocket cost of LTC is often higher for nursing home than for home care[4], i.e. $(1-\alpha_N)N - (1-\alpha_H)L(e) > 0$, one has $I_N^* > I_H^*$. This is intuitive: The more costly of the two states calls for extra insurance coverage, which precludes an equality of the two indemnities.

(2) $v'_N(w) > v'_H(w)$, i.e. the marginal utility of risky wealth is higher given severe dependency than given mild dependency. Since $v''_N(w) < 0$ and $v''_H(w) < 0$, the condition, $v'_N(C) = v'_H(B)$ is satisfied only if $C > B$, i.e.

$$I_N^* - (1-\alpha_N)N > I_H^* - (1-\alpha_H)L(e) \Leftrightarrow I_N^* - I_H^* > (1-\alpha_N)N - (1-\alpha_H)L(e)$$

If one continues to assume $(1-\alpha_N)N - (1-\alpha_H)L(e) > 0$, insurance coverage in the case of severe dependency optimally exceeds coverage in the case of mild dependency, again precluding an equality of indemnities. Since $C > B$ rather than $C = B$ as before, the difference between I_N^* and I_H^* is greater than in case (1). The two channels of influence, viz. the higher cost of nursing home and the higher marginal utility of wealth reinforce each other. However, this does not necessarily result in over-insurance since the level of wealth in case of dependency does not exceed the one in absence of dependency.

(3) $v'_N(w) < v'_H(w)$, i.e. the marginal utility of wealth is lower when dependent in a nursing home than at home. Since $v''_N(w)$ and $v''_H(w)$, then $v'_N(C) = v'_H(B)$ is satisfied only if $B > C$, i.e.

$$I_N^* - (1-\alpha_N)N < I_H^* - (1-\alpha_H)L(e) \Leftrightarrow I_N^* - I_H^* < (1-\alpha_N)N - (1-\alpha_H)L(e)$$

[4] According to AdvancedCare (2021), a nursing home stay cost between US$ 6,844 and 7,598 per month in 2021, whereas care at home cost between US$ 81 and 4,920 per month, depending on the amount of professional help. Thus, one can safely assume $N > L(e)$, which translates into $(1-\alpha_N)N > (1-\alpha_H)L(e)$, unless public subsidization of nursing home care exceeds that of care at home by far. U.S. Medicaid e.g. fully covers nursing home care (American Council on Aging, 2022a), thus $(1-\alpha_N) = 0$ while care at home is covered only in part (American Council on Aging, 2022b), thus reversing the above inequality, but this applies only to about 10% of the population (Medicaid.gov, 2021).

In that case, the joint optimal choices of insurance are such that insurance makes wealth in the state of dependency in nursing home lower than in the state of dependency at home care. As we assume $(1-\alpha_N)N-(1-\alpha_H)L(e)>0$, $I_N^*-I_H^*$ could be of any sign but possibly such as $I_N^*<I_H^*$ even if the cost of nursing home is higher than of homecare. In this case, the two decision channels, i.e. severity of dependency cost and marginal utilities in states of dependency, are divergent and whether one channel dominates the other explains individual preference between nursing home and home care insurance.

If the cost severity channel is cancelled, i.e. $(1-\alpha_N)N=(1-\alpha_H)L(e)$, results are only driven by differences in marginal utilities in the two states of dependency. In that case, $v_N'(w)>v_H'(w)\Leftrightarrow I_N^*>I_H^*$.

There is still some debate about the comparison between marginal utilities of wealth when dependent in a nursing home or at home. Finkelstein et al. (2013) find marginal utility in poor health (severe dependency in the present context) to be lower than in good health (mild dependency, no dependency). This is supported by recent empirical work showing that marginal utility is higher when receiving care at home versus in a nursing home (see e.g. Achou et al., 2023). Yet, according to Fischer et al. (2018), this does not apply to a risky wealth which occur as soon as insurance imposes a co-payment as in our model. In that case, the concept of 'pain of risk bearing' pioneered by Eeckhoudt and Schlesinger (2006) is relevant. Risk-averse people can be safely assumed to minimize this pain by avoiding the accumulation of risks. Conversely, the pain of risk bearing should be particularly marked if both of the two assets considered, 'health' and 'wealth', take on unexpectedly low values. This leads to the opposite conclusion, i.e. the marginal utility of (risky) wealth is higher in the sick than in the healthy state (see Zweifel et al. (2021), ch. 3.2.2 for further details).

Conclusion 1. The difference between the optimal insurance coverage of nursing home and of care at home is driven by both the difference in the severity of dependency cost and in the marginal utilities in states of dependency.

Hence, already in the simplest scenario, in which the parent decides about LTC insurance coverage without taking the child´s response into account, the distinction between severe dependency requiring admission to a nursing home and mild dependency requiring care at home proves relevant because the coverage of nursing home care is optimally different of the coverage of care at home.

4. Parent-child interaction

This section analyses the interaction between the two decision-makers in the guise of a set of Nash equilibria. The parent and the child interact in the guise of a non-cooperative game.

4.1. The reaction functions of the parent

Here, a change $de>0$ in the child´s caring effort constitutes an exogenous shock impinging on the parent. In the Appendix, the two first-order conditions derived in eqs. (2) and (3) are totally differentiated w.r.t. *e*. Applying the implicit function rule and Cramer's rule, and

assuming a negative definite Hessian matrix, one has $sgn\left(\frac{dI_N^*}{de}\right) = sgn(-P_{I_N e}P_{I_H I_H} + P_{I_H e}P_{I_N I_H})$ and $sgn\left(\frac{dI_H^*}{de}\right) = sgn(-P_{I_N I_N}P_{I_H e} + P_{I_H I_N}P_{I_N e})$.
In the case of severe dependency, the parent's reaction function is given by:

$$\frac{dI_N^*}{de} = p\big(1 - q(e)\big)v_H''(B)\big\{q'(e)(I_N - I_H) + \big(1 - q(e)\big)(1 - \alpha_H)L'(e)\big\} > 0 \text{ if } I_N^* \geq I_H^* \qquad (5)$$

When coverage of a stay in the nursing home optimally exceeds coverage of care at home, the parent is predicted to increase it further in response to an increase in the child´s caring effort. In the case of $v_N'(w) \geq v_H'(w)$, we previously found that $I_N^* \geq I_H^*$. In the case where $v_N'(w) < v_H'(w)$, it could happen that $I_N^* < I_H^*$. Yet, assuming $(1 - \alpha_N)N - (1 - \alpha_H)L(e) > I_N^* - I_H^* > -\big(1 - q(e)\big)(1 - \alpha_H)L'(e)/q'(e)$ still ensures that $\frac{dI_N^*}{de} > 0$.

This prediction of a complementary relationship between insurance and informal care is contrary to earlier work (e.g. Courbage and Zweifel (2011)), which however does not distinguish between the two components of LTC insurance, focusing exclusively on $q'(e) < 0$, i.e. the reduced probability of admission to a nursing home thanks to the child´s effort.
The curvature of the parent's reaction function can be derived considering that

$$\frac{dI_N^*}{de} \to 0 \text{ since } q'(e) \to 0 \text{ and } L'(e) \to 0 \text{ when } e \to \infty$$

Graphically, the parent´s reaction function in the case of severe dependency is represented in Figure 1 below.
Concerning the reaction function of the parent in the case of mild dependency, one obtains:

$$sgn\left(\frac{dI_H^*}{de}\right) = p(1 - q(e))\{p\, q(e)(1 - \alpha_H)L'(e)v_N''(C)v_H''(B) + p^2 q(e)^2(1 - p)(1 - \alpha_H)L'(e)v_H''(B)u''(A) - p^2(1 - p)q(e)q'(e)(I_N - I_H)u''(A)v_N''(C)\} \qquad (6)$$

After some rearrangement, one has:

$$\frac{dI_H^*}{de} < 0 \text{ if and only if } I_N^* - I_H^* < T, \text{ with } T = \frac{(1-\alpha_H)L'(e)v_H''(B)\left[v_N''(C) + p(1-p)q(e)u''(A)\right]}{p(1-p)q'(e)v_N''(C)u''(A))} \qquad (6bis)$$

The sign of the reaction function is thus conditioned by the amount of difference between the two types of insurance. For a relatively small difference between I_N^* and I_H^*, i.e. $I_N^* - I_H^* < T$, the optimal level of insurance decreases in response to an increase in informal effort. Considering that the out-of-pocket cost of LTC in nursing home is higher than in home care, this situation can be related to case (1) in the previous section, i.e. $v_N'(w) = v_H'(w)$. Conversely, when $I_N^* - I_H^*$ becomes important, exceeding the threshold T (case (2) in the previous section), the agents' decisions become complementary. The parent increases his or her optimal level of insurance in response to an increase of informal effort provided by the child.
Following the same logic as above, the curvature of the parent's reaction function in the case of mild dependency is given by:

$$\frac{dI_H^*}{de} \to 0 \text{ since } q'(e) \to 0 \text{ and } L'(e) \to 0 \text{ when } e \to \infty$$

Graphically, the parent´s reaction functions in the case of mild dependency are represented in Figure 2 below.

Conclusion 2. The parent´s reaction function in terms of LTC insurance coverage of nursing home care (severe dependency) has positive slope w.r.t. child effort *e*. It has positive or negative slope in terms of coverage of care at home (mild dependency) according to the amount of difference between the two types of insurance.

The difference between the levels of insurance coverage in the two states of dependency drives the reaction of the parent´s response to a change in the amount of informal care provided. Clearly, this finding provides important additional insights that are only available when LTC insurance coverage is differentiated between severe and mild dependency.

4.2. The reaction functions of the child

The expected utility of the child is given by[5]

$$\begin{aligned} C &= q(e)\bar{v}_N\big(z_0 - \theta e + b(w_0 - (1-\alpha_N)N + I_N)\big) \\ &\quad + (1-q(e))\bar{v}_H(z_0 - \theta e + b(w_0 - (1-\alpha_H)L(e) + I_H)) \end{aligned} \tag{7}$$

The first-order condition is expressed as follows:

$$\begin{aligned} C_e = \frac{\partial C}{\partial e} &= q'(e)(\bar{v}_N(D) - \bar{v}_H(E)) - (1-q(e))b(1-\alpha_H)L'(e)\bar{v}_H'(E) - \theta\big(q(e)\bar{v}_N'(D) + \\ &\quad \big(1-q(e)\big)\bar{v}_H'(E)\big) = 0 \end{aligned} \tag{8}$$

with $D = z_0 - \theta e + b(w_0 - (1-\alpha_N)N + I_N)$ and $E = z_0 - \theta e + b(w_0 - (1-\alpha_H)L(e) + I_H)$.

The two first terms represent the marginal benefit of informal care. Note that the first term indicates a gain in utility only if $\bar{v}_N(D) < \bar{v}_H(E)$ so that reducing the probability of the parent entering a nursing home entails such a gain in utility. The second term denotes the extra benefit from a higher inheritance due to reduced spending on formal care in the case of mild dependency. The last term represents the marginal cost of informal care in terms of a reduction in expected utility.

By totally differentiating eq. (8) with respect to I_N and applying the implicit function theorem, one obtains:

$$\frac{de^*}{dI_N} = b[\bar{v}_N'(D)q'(e) - \theta q(e)\bar{v}_N''(D)] = b\bar{v}_N'(D)\left[q'(e) - \theta q(e)\frac{\bar{v}_N''(D)}{\bar{v}_N'(D)}\right] \tag{9}$$

[5] Note that *C* in this section differs from the *C* denoting the parent´s final wealth in the case of severe dependency in Section 3.

$\frac{de^*}{dI_N} < 0$ iif $R_a^{\bar{v}_N} < -\frac{q'(e)}{\theta q(e)} = \underline{R}$, (9bis)

where $R_a^{\bar{v}_N} := -\frac{\bar{v}_N''(D)}{\bar{v}_N'(D)}$ denotes the coefficient of absolute risk aversion pertaining to the child when the parent has severe dependency. The behavior of the child is thus driven by the degree of his/her absolute risk aversion. For a relatively small value of absolute risk aversion, children reduces their informal efforts in response to an increase in insurance coverage against the cost of a stay in the nursing home, thus suggesting the existence of an intergenerational moral hazard effect. Conversely, a high degree of absolute risk aversion is associated with a complementary relationship between informal care and insurance as the child increases his/her provision of informal care in a response to a higher level of LTC insurance coverage in the case of nursing home care.

When e increases, the first term of the bracket of eq. (9) goes to zero whereas the second one becomes excessively small. One thus has

$$\frac{de^*}{dI_N} \to 0 \text{ since } q'(e) \to 0 \text{ when } e \to \infty$$

The child´s reaction functions in the case of severe dependency are represented in Figure 1 below. The two cases are indicated as (a) and (b), respectively.

Next, total differentiation of eq. (8) w.r.t. I_H yields:

$$\frac{de^*}{dI_H} = -b\bar{v}_H'(E)\left[q'(e) + \big(1-q(e)\big)\big(\theta + b(1-\alpha_H)L'(e)\big)\frac{\bar{v}_H''(E)}{\bar{v}_H'(E)}\right] \quad (10)$$

$$\frac{de^*}{dI_H} > 0 \text{ iif } R_a^{\bar{v}_H} > \frac{q'(e^*)}{\big(1-q(e)\big)\big(\theta+b(1-\alpha_H)L'(e^*)\big)}$$

It can be shown that $\theta + b(1-\alpha_H)L'(e^*) > 0$. Indeed, in accordance with Bascans et al. (2017) in the case of only mild dependency with care at home, the optimal effort is shown to be such that the marginal cost of effort (θ) equals the marginal benefit represented by a gain in inheritance due to the parent spending less on formal care in case of mild dependency ($b(1-\alpha_H)L'(e)$). When, severe dependency is considered, the marginal cost of effort is still the same (θ) but the marginal benefit is now higher as it also includes the gain of utility from reducing the probability of nursing home entry. Therefore, $\theta + b(1-\alpha_H)L'(e^*) > 0$, and hence, $\frac{de^*}{dI_H} > 0$ for any risk averse individuals.

This result suggests a complementary relationship between informal care and insurance as the child increases the amount of informal care in response to a higher level of home care insurance. It contradicts the notion of intrafamily hazard in the case of mild dependency.

As before, the curvature of the child's reaction function is derived from:

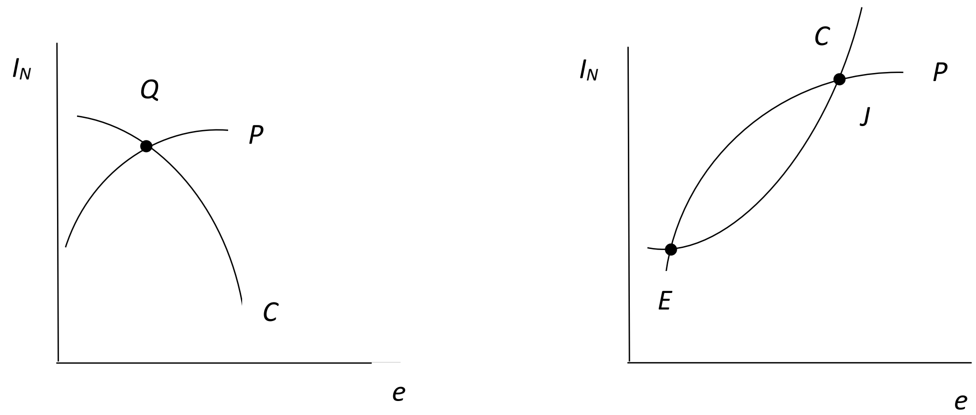
$$\frac{de^*}{dI_H} \to 0 \text{ since } q'(e) \to 0 \text{ when } e \to \infty$$

The child´s reaction functions in the case of mild dependency are represented in Figure 2 below.

Conclusion 3. The child´s reaction functions in terms of informal care provided have positive slope w.r.t. an increase in the parent´s LTC coverage in the event of mild dependency. It has positive or negative slope w.r.t. an increase in the parent´s LTC coverage in the event of severe dependency according to the value of the coefficient of absolute risk aversion.

By distinguish between severe and mild dependency, the agents 'reactions functions and the Nash equilibria are illustrated in Figures 1 and 2 respectively.
Figure 1. Reaction functions of parent and child and Nash equilibria in the case of severe dependency

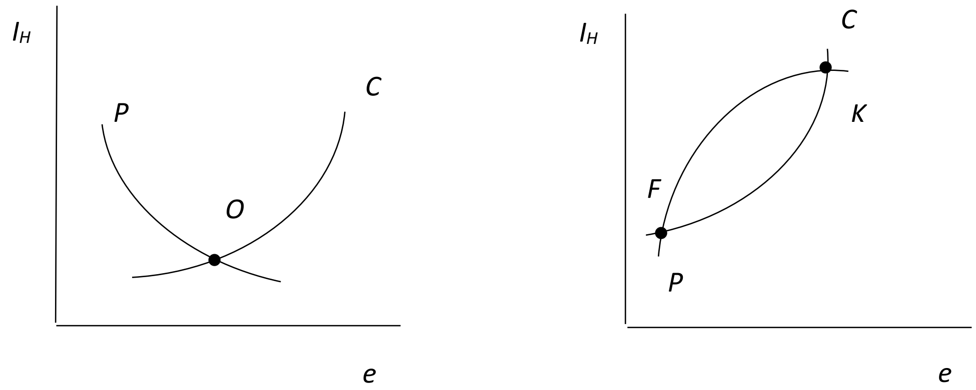
(a) $R_a^{\bar{v}_N} < \underline{R}$ (b) $R_a^{\bar{v}_N} > \underline{R}$



The figure suggests that a single Nash equilibrium exists in case (a), whereas two Nash equilibria are usually predicted in case (b). However, equilibrium *J* proves to be unstable, because when the parent (presumably the first mover) selects an off-equilibrium, the adjustment process ends at *E* not *J*. Thus small values of both *I* and e at point E are predicted.

Figure 2. Reaction functions of parent and child and Nash equilibria in the case of mild dependency

(a) $I_N - I_H < T$ (b) $I_N - I_H > T$



In Figure 2, two Nash equilibria need again be distinguished for the case of mild dependency. Similar to the previous case, a single Nash equilibrium can be identified in case (a) and two

Nash equilibria in case (b), of which the one at point K is unstable. The stable one at F again corresponds to small values of both I and e.

5. Displacements of Nash equilibria

In this section, exogenous shocks associated with public policy are considered, with the exception of an increase in the child´s opportunity cost θ in view of its importance. Changes in public policy analyzed are the subsidization of nursing home care (affecting α_N), the subsidization of care at home (affecting α_H), and the after-tax share of the bequest b.

The graphical analysis of these exogeneous shocks as well as calculations are provided in the Appendix. Several effects can be signed by comparing $R_a^{\bar{v}_N}$ with the absolute index of prudence defined by Kimball (1990) as $P_a^{\bar{v}_N} := -\frac{\bar{v}_N'''(D)}{\bar{v}_N''(D)}$. Under classical assumptions with respect to individual behavior, i.e. decreasing absolute risk aversion, $P_a^{\bar{v}_N} > R_a^{\bar{v}_N}$ (see Kimball, 1990).

Public subsidization of nursing home care (α_N)

The displacements of Nash equilibria caused by a subsidization of nursing home care (α_N) are indicated in Figures 3 and 4 of the Appendix.

In the case of severe dependency, the child´s reaction function shifts from C to C'. In panel (a) of Figure 3, the child´s degree of absolute risk aversion is below the threshold $\underline{R}$ defined in (9bis) and above the degree of absolute prudence. The equilibrium moves from Q to Q', implying a decrease in both I and e. In panel (b), the child is characterized by a degree of absolute risk aversion above the threshold $\underline{R}$ defined in (9bis) and below the degree of absolute prudence. Here, increases in both I and e are predicted. Thus, public subsidization may create a crowding-in or a crowding-out effect on both insurance and informal care depending on the child´s degree of risk aversion, a parameter about which little is known in general.

In the case of mild dependency, only the parent´s reaction function moves, from P to P' in Figure 4. This time, the difference in coverage appears to be crucial. If nursing home coverage exceeds coverage for care at home by less than a threshold value T defined in (6bis), the equilibrium changes from O to O' in panel (a) and from F to F' in panel (b). However, both changes are associated with decreases in both I and e. Given mild dependency, public subsidization of nursing home care is therefore predicted to crowd out both LTC insurance as well as informal care.

Conclusion 4. Given severe dependency, an increase in the public subsidization of nursing home care increases (decreases) both insurance coverage of nursing home care and informal care if the child´s degree of risk aversion is low (high) and above (below) the degree of absolute prudence. Given mild dependency, it decreases both insurance coverage against nursing home care and informal care.

Public subsidization of care at home (α_H)

The displacements of Nash equilibria are shown in Figures 5 and 6 of the Appendix.

In the case of severe dependency, the parent´s reaction function shifts from P to P'. When the intensity of the child's absolute risk aversion is small (as in panel (a) of Figure 5), an increase in public subsidization causes an increase in I but a reduction of e. When the child is characterized by a high degree of absolute risk aversion (as in panel (b)), public subsidization is predicted to crowd out both I and e.

In the case of mild dependency (Figure 6), the parent´s reaction function moves from P to P', while the child´s reaction functions shifts from C to C'. In this case, public subsidization of care at home is found to crowd out LTC insurance but to crowd in informal care.

Conclusion 5. Given severe dependency, an increase in public subsidization of care at home is predicted to increase (decrease) insurance coverage against the cost of nursing home care if the child´s degree of absolute risk aversion is low (high.). As to amount of informal care, a decrease is predicted. Given mild dependency, the prediction is a decrease in insurance for care at home but an increase in informal care.

Increase in the opportunity cost of the child (θ)

The displacements of Nash equilibria in this case are displayed in Figures 7 and 8 of the Appendix.

Given severe dependency, the child´s reaction function shifts from C to C'. When the degree of absolute risk aversion on the part of the child is small and above the degree of absolute prudence (panel (a) of Figure 7), an increase in the child's opportunity cost generates a double crowding-out effect on both LTC insurance and informal care. Conversely, when the degree of the child's absolute risk aversion is high and below the degree of absolute prudence, an increase in both LTC insurance and the amount of informal care is predicted.

Given mild dependency, only the child´s reaction function shifts, from C to C. When the difference between I_N and I_N is relatively small, an increase of child's opportunity crowds in (crowds out) LTC insurance (informal care). When the difference between I_N and I_N is large, crowding-in effects are predicted for both LTC insurance and informal care.

Conclusion 6. Given severe dependency, an increase in the opportunity cost of the child decreases (increases) both insurance for nursing home and informal care if the child´s degree of risk aversion is low (high) and above (below) the degree of absolute prudence. Given mild dependency, it increases insurance for home care and decreases (increases) informal care when the difference in insurance levels is rather low (high).

Change in the inheritance rate (b)

The displacements of Nash equilibria in this case are indicated in Figures 9 and 10 in Appendix.

In the case of severe dependency, only the child´s reaction function shifts from C to C'. When the absolute risk aversion of the child is small and above the degree of absolute prudence, an increase in inheritance rate generates an increase in both LTC insurance and informal care. Conversely, when the child's absolute risk aversion becomes large and below the degree of absolute prudence, an increase in inheritance rate leads to a reduction in both LTC insurance and informal care.

In the case of mild dependency, the displacement of the child´s reaction function from C to C' triggers to a rise (reduction) in informal care (LTC insurance) consecutively to an increase in inheritance rate if the intensity of the difference between I_N and I_H is small. When this intensity becomes important, an increase in inheritance rate causes a reduction in both LTC insurance and informal care.

Conclusion 7. Given severe dependency, an increase in the inheritance rate increases (decreases) both insurance for nursing home and informal care if the child´s degree of risk aversion is low (high) and above (below) the degree of absolute prudence. Given mild dependency, it decreases insurance for home care and increases (decreases) informal care when the difference in insurance levels is rather low (high).

Results of the comparative statics are summarised below.

Table 1. Effect of exogeneous shocks on LTC insurance and informal care in case of two types of dependency

	Severe dependency		Mild dependency	
	I_N^*	e^*	I_H^*	e^*
α_N	- iff $\underline{R} < R_a^{\bar{v}_N} < P_a^{\bar{v}_N}$	- iff $\underline{R} < R_a^{\bar{v}_N} < P_a^{\bar{v}_N}$	-	-
α_H	- iif $\underline{R} < R_a^{\bar{v}_N}$	-	-	+
θ	+ iff $\underline{R} < R_a^{\bar{v}_N} < P_a^{\bar{v}_N}$	+ iff $\underline{R} < R_a^{\bar{v}_N} < P_a^{\bar{v}_N}$	+	- iff $I_N - I_H < T$
b	- iff $\underline{R} < R_a^{\bar{v}_N} < P_a^{\bar{v}_N}$	- iif $\underline{R} < R_a^{\bar{v}_N} < P_a^{\bar{v}_N}$	-	+ iff $I_N - I_H < T$

One can now compare the results of the case with two dependency levels together to the one with only one dependency level, being either severe dependency or mild dependency as addressed in Courbage and Zweifel (2011) or Bascans et al. (2017), respectably. The outcome of this comparison is summarised below.

Table 2. Effect of exogeneous shocks on LTC insurance and informal care in case of one type of dependency

	Only severe dependency (Courbage and Zweifel, 2011)		Only mild dependency (Bascans et al., 2017)	
	I_N^*	e^*	I_H^*	e^*
α_N	-	+		
α_H			-	0 (+)
θ	0	-	+	-
b	0	?	-	+

Interestingly, when only one type of dependency is considered, an increase in public subsidisation of either nursing home care or home care leads to a decrease in insurance coverage and of an increase in informal care, making insurance and informal care substitutes (see Table 2).

Yet, when two types of dependency are considered together, predictions differ especially when it comes to severe dependency. Indeed, on the one hand, an increase in public subsidisation of nursing home influences in the same direction both insurance for nursing home and informal care. It could even happen that for a low level of risk aversion of the child, increasing public subsidies of nursing home increases both insurance for severe dependency and informal care, but decreases insurance for mild dependency and informal care. Insurance and informal care would become complements in that case.

On the other hand, increasing public subsidisation of home care is predicted to have opposite effects on insurance and informal care. In the case of severe dependency and low risk aversion of the child, it leads to more insurance and less informal care. While in the case of mild dependency, it decreases insurance and increases informal care. In that case, insurance for home care and informal care would become substitutes.

Hence, as a way to influence LTC insurance purchase, it might be more appropriate for the government to modify the subsidisation rate of nursing home rather than of home care if the intent is to increase both insurance and informal care. Indeed, a change in the subsidisation of nursing home impacts insurance and informal in the same direction, either positively for severe dependency and low risk aversion of the child, or negatively for mild dependency. While a change in the rate of subsidisation of home care leads to a crowding out of insurance and crowding in of informal care.

Finally, turning to a change in the inheritance rate or the child's opportunity cost, their impacts are similar but opposite. In the case of severe dependency, an increase in the inheritance tax or an increase in the opportunity cost, max increase or decrease both LTC insurance and informal care. The predicted outcome depends on whether the child's degree of risk aversion is low or high, but it suggests a complementary relationship between insurance and informal care. This result strongly contrasts with the situation when only one type of dependency is considered. In the case of mild dependency, both a substitutability or complementary relationship between insurance and informal care may occur. This time, the prediction hinges on the size of the difference in the levels of insurance in the two dependency states.

6. Conclusion

The objective of this short paper is to revisit earlier theoretical results regarding the link between LTC insurance and informal care when two levels of dependency are distinguished. In particular, it aims to find out whether public subsidization of LTC crowds out both private LTC insurance and informal care in this case. LTC coverage differs between coverage in the event of severe dependency calling for of admission to a nursing home and of mild dependency, where care at home is sufficient. It is governed by the count of ADL (Activities of Daily Living) limitations in most policies. In the first scenario, the person in potential need of LTC (the parent) neglects the behavior of the potential caregiver (the child) when deciding about the extent of insurance coverage.

It turns out that the degree of coverage for severe dependency is different from the one for mild dependency depending on LTC cost and marginal utility of wealth in the two states of dependency, which justifies the distinction made (Conclusion 1). Next, the parent anticipates the child's reaction to changes in LTC coverage, calling for the determination of Nash equilibria. The parent´s reaction function has positive slope w.r.t. child effort in the case of severe dependency and positive or negative slope in the case of mild dependency according to the difference between the two types of insurance. The child's reaction functions have positive or negative slope w.r.t. an increase in the parent´s LTC coverage in the event of severe dependency according to the value of the coefficient of absolute risk aversion on the part of the child. These reaction functions give rise to one or two equilibria of which only one is stable (Conclusions 2 and 3). The shapes of the reaction functions tend to contradict the notion of two-sided intergenerational moral hazard in both the case of mild dependency and of severe dependency.

Next, these equilibria are displaced by exogenous shocks in particular increases in the public subsidization of a stay in the nursing home and of care at home, and also changes in the after-tax share of the bequest the child can count on, and his or her opportunity cost of time when providing informal care.

Increases in the public subsidization of nursing home care is predicted to lead to a change in the same direction of LTC insurance and insurance in the case of severe dependency, the direction depending on the child's level of risk aversion. In the case of mild dependency, it is predicted to lead to a decrease in both insurance and informal care. Therefore, there is a crowding in or crowding out of both LTC insurance and informal in the case of severe dependency, and a crowding out of both insurance and informal care in case of mild dependency (Conclusion 4). Hence, insurance and informal care are complements.

Increases in the public subsidization of home care are predicted to lead to opposite effects on insurance and informal care, highlighting in this case the substitutability between the two. In case of severe dependency and low risk aversion of the child, it leads to more insurance and less informal care. But in the case of mild dependency, it decreases insurance while increasing informal care (Conclusion 5).

Hence, crowding in or crowding out effects of subsidies depends on the degree of dependency. In the aim of modifying insurance and informal care, changing the rate of subsidisation of nursing home might be more appropriate than modifying the rate of subsidisation of home care or vice versa depending on the government objectives.

Finally, change in the inheritance rate and the child's opportunity cost have similar but opposite effects. In case of severe dependency, their impacts are driven by the level of child's risk aversion. They indicate a complementary relationship between insurance coverage and informal care. In the case of mild dependency, both a substitutability or complementary relationship between insurance and informal care may occur. This time, the prediction hinges on the size of the difference in the levels of insurance coverage in the two dependency states (Conclusions 6 and 7). Once again, it shows that exogenous changes may well have impacts that differ between LTC insurance covering nursing home and covering home care.

There are a number of limitations to this analysis that need to be pointed out. The first is that the parent-child interaction is one-to-one rather than between two parents on the one side and several potential caregivers on the other. Also, the effect of altruism is limited in that the parent is willing to bequeath his or her wealth to the child in its entirety and that the child has a higher utility of wealth when the parent is at home rather than in the nursing home. Next, the share of the bequest the potential caregiver can expect is fixed rather than a function of the closeness of the relationship with the parent. Finally, the degree of public subsidization of LTC is considered exogenous whereas it may well depend on the public expense engendered and ultimately on the relative political influence of the aged and the young population.

This said, the present work provides new insights into the interaction between parents in potential need of LTC and their children as potential caregivers, which is usually characterized by intergenerational moral hazard and crowding out effects. The finding here is that this need not to be the case once the distinction between severe and mild dependency and their associated levels of LTC coverage is made. In particular, the hypothesis of two-sided intergenerational moral hazard needs to be questioned. Also, the effects of public subsidisation have been found to differ on whether this is targeted towards nursing home or home care, highlighting the importance of distinguishing between the two levels of dependency when studying LTC insurance and implementing LTC financing strategies.

References

Achou, B., De Donder, P., Glenzer, F., Lee, M., Leroux, M-L. (2023). At Home Versus in a Nursing Home: Long-Term Care Settings and Marginal Utility, CESifo Working Paper No. 10482.

Administration for Community Living (2020). Who will provide your care? https://acl.gov/ltc/basic-needs/who-will-provide-your-care.

AdvancedCare (2021). Home care vs. Nursing home : Making the correct decision. https://advancedcare.health/home-care-vs-nursing-home/.

American Council on Aging (2022a). Medicaid coverage of nursing home care. When, where and how much they pay. https://www.medicaidplanningassistance.org/medicaid-and-nursing-homes/.

American Council on Aging (2022b). Medicaid and home health care & non-medical, in-home care. https://www.medicaidplanningassistance.org/in-home-care/.

Bascans, J.M., Courbage, C., Oros, C. (2017). Public support and the interaction between long-term care insurance and informal care, International Journal of Health Economics and Management, 17(2), 113-133.

Brodsky M. (2020). Compare top long-term care insurance companies. https://www.consumeraffairs. com/insurance/ltc.html.

Brown H, Finkelstein A (2007). Insuring long-term care in the United States. Journal of Economic Perspectives 23(4), 119-142.

Brown H, Finkelstein A (2011). Why is the market for long-term care insurance so small? Journal of Public Economics 91(10), 1967-1991.

Costa-Font J, Courbage C (2015). Crowding out of long-term care insurance: Evidence from European expectations data. Health Economics 24(51), 74-88.

Courbage C, Eeckhoudt L. (2012). On insuring and caring for parents´ long-term care needs. Journal of Health Economics 31(6), 842-850.

Courbage C, Zweifel P (2011). Two-sided intergenerational moral hazard, long-term care insurance, and nursing home use. Journal of Risk and Uncertainty 43(1), 65- 80.

Cremer H, Roeder K (2017). Long-term care and lazy rotten kids. Journal of Public Economics Theory 19(3), 583-602.

Eeckhoudt L, Schlesinger H (2006). Putting risk in its proper place. American Economic Review 96(1), 280-289.

Finkelstein A, Luttmer EFP, Notowidigdo M (2013). What good is wealth without health? The effect of health on the marginal utility of consumption. Journal of the European Economic Association 11(s1), 221-258.

Indeed (2021). What is the average hourly wage in the US? https://www.indeed.com/career-advice/pay-salary/average-hourly-wage-in-us.

Kimball, M.S. (1990). Precautionary Saving in the Small and in the Large. Econometrica, 61, 53-73.

Medicaid.gov (2921). Share of Medicaid & CHIP population by age. https://www.medicaid.gov/state-overviews/scorecard/share-of-medicaid-chip-population-age/index.html.

Mellor, J.M. (2001). Long-Term Care and Nursing Home Coverage: Are Adult Children Substitutes for Insurance Policies?. Journal of Health Economics, 20(4), 527–47.

OECD (n.d.). Social protection for older people with long-term care needs. https://www.oecd.org/health/health-systems/social-protection-for-older-people-with-ltc-needs.htm.

Orem T, Davis C (2022). Inheritance tax: What it is and how to avoid it. https://www.nerdwallet.com/article/taxes/inheritance-tax.

Pauly M (1990). The rational nonpurchase of long-term care insurance. Journal of Political Economy 98(1), 153-168.

Rand Corporation (2017). Average American´s risk of needing nursing home care is higher than previously estimated. https://www.rand.org/news/ press /2017/08/28/index1.html.

Xu X, Zweifel P (2014). Bilateral intergenerational moral hazard: Empirical evidence from China. Geneva Papers on Risk and Insurance: Issues and Practice 39(4), 651-667.

Zweifel P (2020). Innovation in long-term care insurance: Joint contracts for mitigating relational moral hazard. Insurance: Mathematics and Economics 93, 116-123.

Zweifel P, Eisen R, Eckles DL (2021). Insurance Economics. Boston: Springer.

Zweifel P, Courbage C (2015). Double crowding-out effects of means-tested public provision for long-term care. Risks 3(1), 61-76.

Zweifel P, Strüwe W (1996). Long-term care insurance and bequests as instruments for shaping intergenerational relationships. Journal of Risk and Uncertainty 12(1), 1265-1276.

Zweifel P, Strüwe W (2017). Long-term care insurance in a two-generation Model. Journal of Risk and Insurance 65(1), 13-32.

Appendix

$sgn\left(\frac{dI_N^*}{d\alpha_N}\right) = -P_{I_N\alpha_N}P_{I_HI_H} < 0$

$sgn\left(\frac{dI_H^*}{d\alpha_N}\right) = P_{I_N\alpha_N}P_{I_HI_N} > 0$

$sgn\left(\frac{dI_N^*}{d\alpha_H}\right) = P_{I_H\alpha_H}P_{I_HI_N} > 0$

$sgn\left(\frac{dI_H^*}{d\alpha_H}\right) = -P_{I_NI_N}P_{I_H\alpha_H} < 0$

$P_{I_NI_N} = p\, q(e)[v_N'' + (1-p)p\, q(e)u''] < 0$

$P_{I_NI_H} = p^2(1-p)q(e)\big(1-q(e)\big)u'' < 0$

$P_{I_HI_H} = p\,(1-q(e))[v_H'' + (1-p)p(1-\ q(e))u''] < 0$

$P_{I_N\alpha_N} = p\, q(e)Nv_N'' < 0$

$P_{I_H\alpha_N} = 0$

$P_{I_N\alpha_H} = 0$

$P_{I_H\alpha_H} = p\,\big(1-q(e)\big)L(e)v_H'' < 0$

$P_{I_Ne} = p^2(1-p)q(e)q'(e)(I_N - I_H)u''(A) \qquad sgn(P_{I_Ne}) = sgn\,(I_N - I_H)$

$P_{I_He} = p\big(1-q(e)\big)[-(1-\alpha_H)L'(e)v_H''(B) + p(1-p)q'(e)(I_N - I_H)u''(A)]$

Shock on the subsidization of nursing home care $(\boldsymbol{\alpha_N})$

Concerning the parent, total differentiation of eqs. (5) and (6) yields:

$\frac{d}{d\alpha_N}\left(\frac{dI_N^*}{de}\right) = 0$

$\frac{d}{d\alpha_N}\left(\frac{dI_H^*}{de}\right) = p^2q(e)\big(1-q(e)\big)Nv_N'''(C)\{(1-\alpha_H)L'(e)v_H''(B) - p(1-p)q'(e)(I_N - I_H)u''(A)\}$

Considering a prudent behaviour of the parent ($v_N''' > 0$), we have:

$\frac{d}{d\alpha_N}\left(\frac{dI_H^*}{de}\right) > 0$ iff $I_N - I_H < \frac{|(1-\alpha_H)L'(e)v_H''(B)|}{|p(1-p)q'(e)u''(A))|} = T'$

Given that : $T = \frac{|(1-\alpha_H)L'(e)v_H''(B)[v_N''(C)+p(1-p)q(e)u''(A)]|}{|p(1-p)q'(e)v_N''(C)u''(A))|}$, $T > T'$ but the difference is very small. Consequently, the expressions become:

$\frac{dI_H^*}{de} < 0$ and $\frac{d}{d\alpha_N}\left(\frac{dI_H^*}{de}\right) > 0$ if $I_N - I_H < T$

$\frac{dI_H^*}{de} > 0 \quad \frac{d}{d\alpha_N}\left(\frac{dI_H^*}{de}\right) < 0$ if $I_N - I_H > T$

On the part of the child, total differentiation of eqs. (9) and (10) yields:

$\frac{d}{d\alpha_N}\left(\frac{de^*}{dI_N}\right) = Nb^2[\bar{v}_N''q'(e) - \theta q(e)\bar{v}_N''']$

$\frac{d}{d\alpha_N}\left(\frac{de^*}{dI_N}\right) < 0$ if $P_a^{\bar{v}_N} > -\frac{q'(e)}{\theta q(e)} = \underline{R}$ where $P_a^{\bar{v}_N} = -\frac{\bar{v}_N'''(D)}{\bar{v}_N''(D)}$ represents the index of absolute prudence defined by Kimball.

According to the nature of the relationship between $P_a^{\bar{v}_N}$ and $R_a^{\bar{v}_N}$, we can distinguish between two cases:

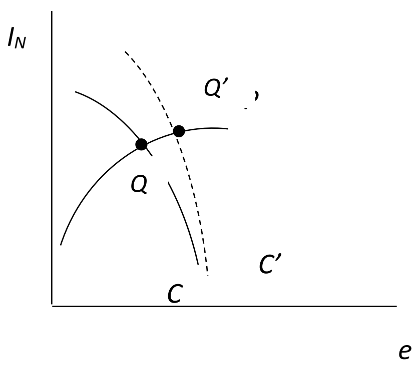
- $R_a^{\bar{v}_N} > P_a^{\bar{v}_N}$. In this case, we have: $\frac{de^*}{dI_N} < 0$ and $\frac{d}{d\alpha_N}\left(\frac{de^*}{dI_N}\right) > 0$ iif $R_a^{\bar{v}_N} < \underline{R}$

- $R_a^{\bar{v}_N} < P_a^{\bar{v}_N}$ In this case, our relations write: $\frac{de^*}{dI_N} > 0$ and $\frac{d}{d\alpha_N}\left(\frac{de^*}{dI_N}\right) < 0$ iif $R_a^{\bar{v}_N} > \underline{R}$,

$\frac{d}{d\alpha_N}\left(\frac{de^*}{dI_H}\right) = 0$

Figure 3. Displacements of Nash equilibria caused by a shock in nursing home public subsidization (α_N) in the case of severe dependency

(a) $R_a^{\bar{v}_N} < \underline{R}$ and $R_a^{\bar{v}_N} > P_a^{\bar{v}_N}$



(b) $R_a^{\bar{v}_N} > \underline{R}$ and $R_a^{\bar{v}_N} < P_a^{\bar{v}_N}$

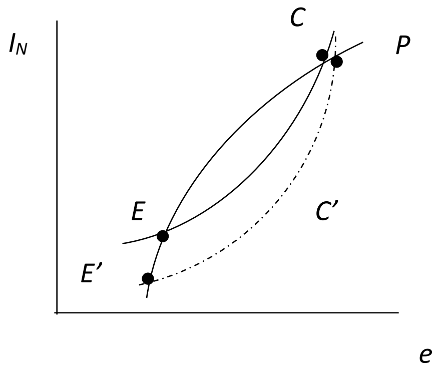
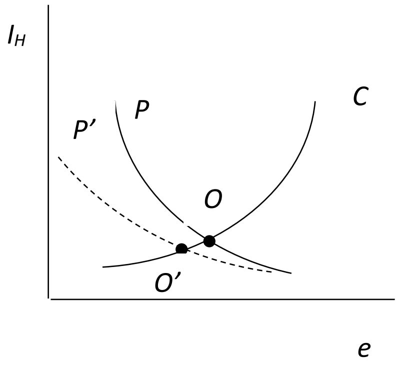
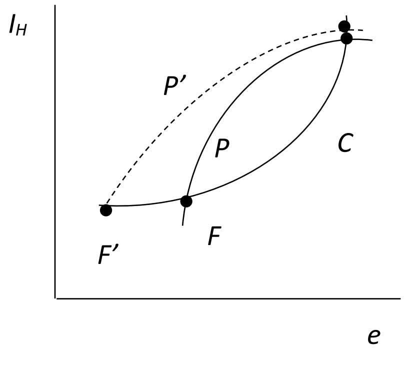


Figure 4. Displacements of Nash equilibria caused by a shock in nursing home public subsidization (α_N) in the case of mild dependency

(a) $I_N - I_H < T$



(b) $I_N - I_H > T$



Shock in the home care public subsidization (α_H)

Using the same approach as indicated above, the effects of an increase of α_H on the agents' reactions function are:

$$\frac{d}{d\alpha_H}\left(\frac{dI_N^*}{de}\right) = p\big(1-q(e)\big)\big\{v_H'''(B)L(e)\big[q'(e)(I_N - I_H) + \big(1-q(e)\big)(1-\alpha_H)L'(e)\big] - \big(1-q(e)\big)L'(e)v_H''(B)\big\} < 0$$

$$\frac{d}{d\alpha_H}\left(\frac{dI_H^*}{de}\right) = p^2 q(e)\big(1-q(e)\big)L'(e)[(1-\alpha_H)v_H'''(B)L(e) - v_H''(B)][v_N''(C) + p(1-p)q(e)u''(A)] > 0$$

Concerning the child, we obtain:

$$\frac{d}{d\alpha_H}\left(\frac{de^*}{dI_N}\right) = 0$$

$$\frac{d}{d\alpha_H}\left(\frac{de^*}{dI_H}\right) = -b^2 q'(e)L(e)\bar{v}''_H(E) < 0$$

Figure 5. Displacements of Nash equilibria caused by a shock in home care public subsidization (α_H) in the case of severe dependency

(a) $R_a^{\bar{v}_N} < \underline{R}$ (b) $R_a^{\bar{v}_N} > \underline{R}$

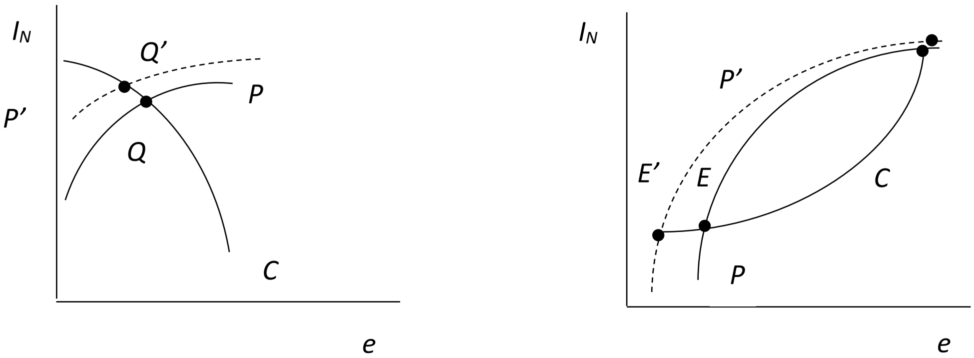
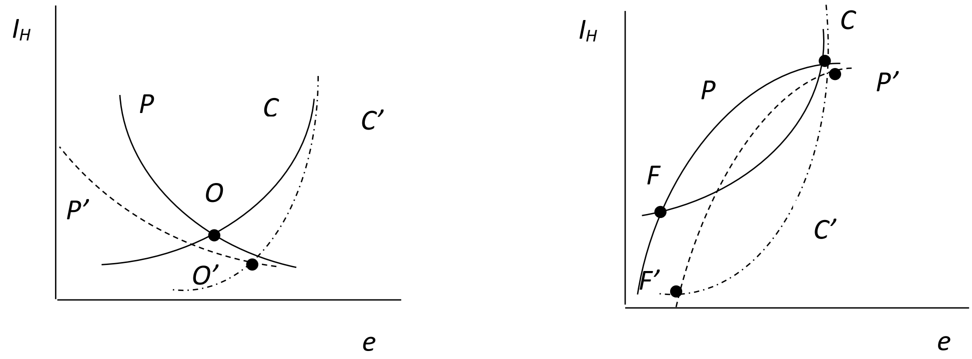


Figure 6. Displacements of Nash equilibria caused by a shock in home care public subsidization (α_H) in the case of mild dependency

(a) $I_N - I_H < T$ (b) $I_N - I_H > T$



Shock on the opportunity cost of the child (θ)

The reactions functions become:

$$\frac{d}{d\theta}\left(\frac{dI_N^*}{de}\right) = 0$$

$$\frac{d}{d\theta}\left(\frac{dI_H^*}{de}\right) = 0$$

$$\frac{d}{d\theta}\left(\frac{de^*}{dI_N}\right) = -be[\bar{v}_N'' q'(e) - \theta q(e)\bar{v}_N'''] - bq(e)\bar{v}_N''$$

$\frac{d}{d\theta}\left(\frac{de^*}{dI_N}\right) > 0$ iff $\bar{v}_N'' q'(e) - \theta q(e)\bar{v}_N''' < 0$ which means iff $P_a^{\bar{v}_N} > -\frac{q'(e)}{\theta q(e)} = \underline{R}$

According to the nature of the relationship between $P_a^{\bar{v}_N}$ and $R_a^{\bar{v}_N}$, we can distinguish between two cases:

- $R_a^{\bar{v}_N} > P_a^{\bar{v}_N}$. In this case, we have: $\frac{de^*}{dI_N} < 0$ and $\frac{d}{d\theta}\left(\frac{de^*}{dI_N}\right) < 0$ iif $R_a^{\bar{v}_N} < \underline{R}$
- $R_a^{\bar{v}_N} < P_a^{\bar{v}_N}$ In this case, our relations write: $\frac{de^*}{dI_N} > 0$ and $\frac{d}{d\theta}\left(\frac{de^*}{dI_N}\right) > 0$ iif $R_a^{\bar{v}_N} > \underline{R}$,

$$\frac{d}{d\theta}\left(\frac{de^*}{dI_H}\right) = bq'(e)e\bar{v}''_H(E) > 0$$

Figure 7. Displacements of Nash equilibria caused by a shock in opportunity cost (θ) in the case of severe dependency

(a) $R_a^{\bar{v}_N} < \underline{R}$ and $R_a^{\bar{v}_N} > P_a^{\bar{v}_N}$ (b) $R_a^{\bar{v}_N} > \underline{R}$ and $R_a^{\bar{v}_N} < P_a^{\bar{v}_N}$

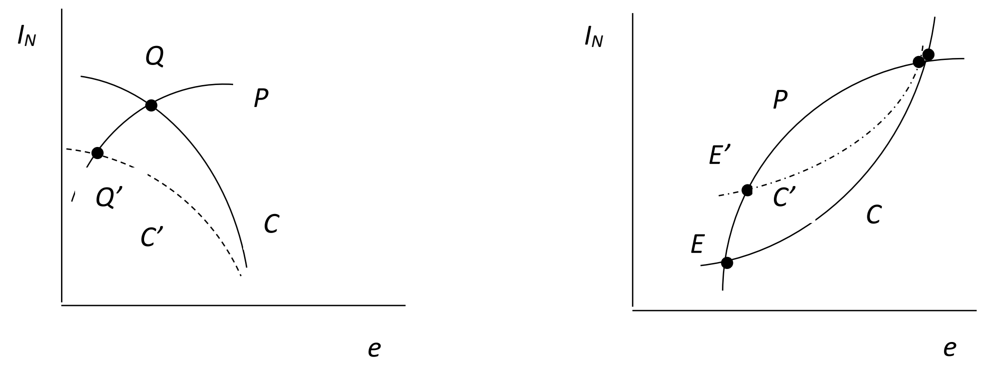
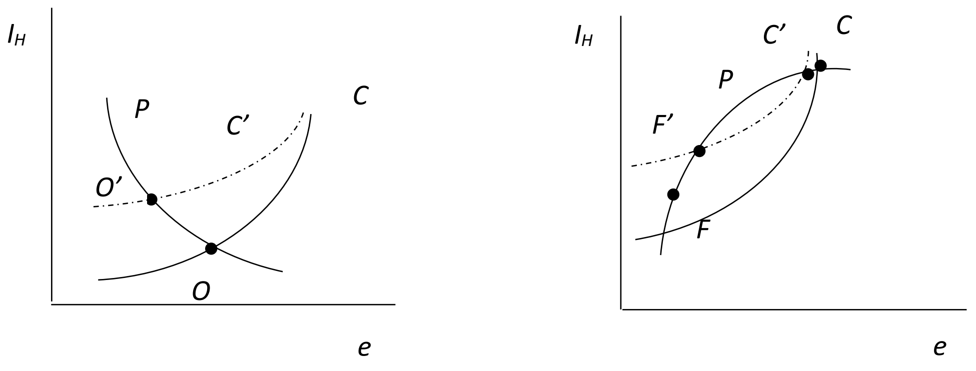


Figure 8. Displacements of Nash equilibria caused by a shock in opportunity cost (θ) in the case of mild dependency

(a) $I_N - I_H < T$ (b) $I_N - I_H > T$



Shock on the inheritance rate (b)

$$\frac{d}{db}\left(\frac{dI_N^*}{de}\right) = 0$$

$$\frac{d}{db}\left(\frac{dI_H^*}{de}\right) = 0$$

$$\frac{d}{db}\left(\frac{de^*}{dI_N}\right) = bC[\bar{v}_N'' q'(e) - \theta q(e) \bar{v}_N''']$$

$$\frac{d}{db}\left(\frac{de^*}{dI_N}\right) < 0 \text{ if } P_a^{\bar{v}_N} > -\frac{q'(e)}{\theta q(e)} = \underline{R}$$

$$\frac{d}{d\alpha_N}\left(\frac{de^*}{dI_N}\right) < 0 \text{ if } P_a^{\bar{v}_N} > -\frac{q'(e)}{\theta q(e)} = \underline{R}$$

According to the nature of the relationship between $P_a^{\bar{v}_N}$ and $R_a^{\bar{v}_N}$, we can distinguish between two cases:

- $R_a^{\bar{v}_N} > P_a^{\bar{v}_N}$. In this case, we have: $\frac{de^*}{dI_N} < 0$ and $\frac{d}{db}\left(\frac{de^*}{dI_N}\right) > 0$ iif $R_a^{\bar{v}_N} < \underline{R}$
- $R_a^{\bar{v}_N} < P_a^{\bar{v}_N}$ In this case, our relations write: $\frac{de^*}{dI_N} > 0$ and $\frac{d}{db}\left(\frac{de^*}{dI_N}\right) < 0$ iif $R_a^{\bar{v}_N} > \underline{R}$,

$$\frac{d}{db}\left(\frac{de^*}{dI_H}\right) = -bBq'(e)\bar{v}''_H(E) < 0$$

Figure 9. Displacements of Nash equilibria caused by a shock in the inheritance rate (b) in the case of severe dependency

(a) $R_a^{\bar{v}_N} < \underline{R}$ and $R_a^{\bar{v}_N} > P_a^{\bar{v}_N}$

(b) $R_a^{\bar{v}_N} > \underline{R}$ and $R_a^{\bar{v}_N} < P_a^{\bar{v}_N}$

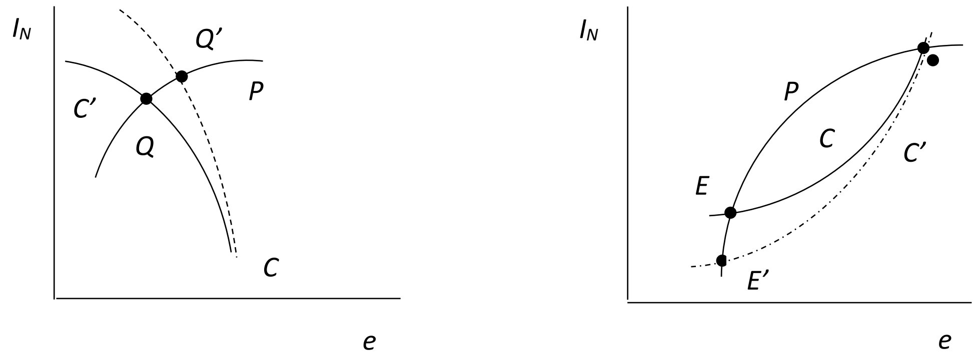
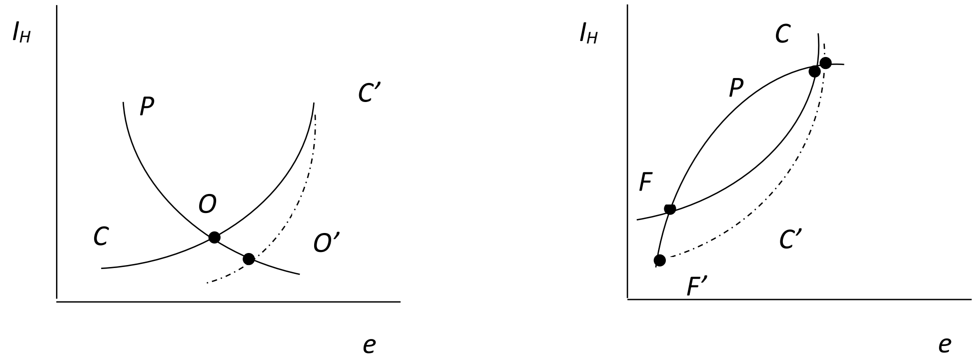


Figure 10. Displacements of Nash equilibria caused by a shock in the inheritance rate (b) in the case of mild dependency

(a) $I_N - I_H < T$

(b) $I_N - I_H > T$



	$\frac{dI_N^*}{de}$	$\frac{dI_H^*}{de}$	$\frac{de^*}{dI_N}$	$\frac{de^*}{dI_H}$
α_N	0	+ if $I_N - I_H < T$ - if $I_N - I_H > T$	+ iif $R_a^{\bar{v}_N} > P_a^{\bar{v}_N}$ and $R_a^{\bar{v}_N} < \underline{R}$ - iif $R_a^{\bar{v}_N} < P_a^{\bar{v}_N}$ and $R_a^{\bar{v}_N} > \underline{R}$	0
α_H	-	+	0	-
θ	0	0	- iif $R_a^{\bar{v}_N} > P_a^{\bar{v}_N}$ and $R_a^{\bar{v}_N} < \underline{R}$ + iif $R_a^{\bar{v}_N} < P_a^{\bar{v}_N}$ and $R_a^{\bar{v}_N} > \underline{R}$	+
b	0	0	+ iif $R_a^{\bar{v}_N} > P_a^{\bar{v}_N}$ and $R_a^{\bar{v}_N} < \underline{R}$ - iif $R_a^{\bar{v}_N} < P_a^{\bar{v}_N}$ and $R_a^{\bar{v}_N} > \underline{R}$	-