Sensitivity and Reliability Analyses Applied to day-to-day Geotechnical Engineering Using Meta-Models Coupled with 3D Finite Elements

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ABSTRACT: On one hand, finite element analyses have nowadays become a standard in order to design complex soil-structure interaction works, especially when dealing with sensitive structures in the vicinity of the project. On the other hand, reliability design and full probabilistic analysis are more and more used in practice. In this contribution we aim to show that day-to-day 3D finite element probabilistic analyses can now be performed in a reasonable amount of time, using adequate meta-models like Polynomial Chaos Expansions (PCE) or Polynomial Chaos Kriging (PCK). We illustrate this on a study for a tunnel which will be constructed in Lausanne, Switzerland, for a new underground line, still under development. A 3D finite element mesh is constructed, including real stratigraphy, excavation steps, support and bolting. Probability density functions are defined for soil parameters, loads... A sensitivity analysis is conducted first in order to select the relevant input parameters, and then a reliability analysis is performed, the quantity of interest being the settlement of a sensitive building due to the tunnel excavation. Special attention is put in this paper on the selection of the most adequate meta-model, on the error measurement (generalization of the leave-one out error) and on the size of the experimental design needed to construct a reliable surrogate model.

1. INTRODUCTION

In mechanics, the displacement of a structure depends, among other variables, on the material parameters. The finite element method is quite accurate, but in practice, especially in geotechnics, the material properties are not known exactly. Indeed, even with a perfectly accurate model, the observed displacement in a project will hardly match the predicted one, because of the uncertainty on soil properties and on other factors. A probabilistic analysis allows to obtain a probabilistic distribution for the prediction of the model, in order to take this uncertainty into account.

With complex 3D finite element models, each calculation is time expensive, but gives accurate results. Thus, intuitive methods for uncertainty quantification like crude Monte Carlo cannot be used directly, because of the computation time of the finite element model.

State-of-the-art algorithm can be used to approximate the relationship between the input variables and the quantity of interest, trained on a reasonable number of finite element simulations. With this surrogate model, with which new

realizations can be computed instantly, it is possible to do accurate reliability and sensitivity analyses, if the approximation method is accurate.

2. UNCERTAINTY QUANTIFICATION COUPLED WITH A FINITE ELEMENT SOFTWARE

2.1. Uncertainty in geotechnics

Uncertainty has always been a part of geotechnical calculations. Since information comes from boreholes, in-situ and laboratory testing and experience, some level of uncertainty will always remain when defining soil properties. That explains the semi probabilistic methods used nowadays. If the uncertainty of the soil properties can be quantified, with experience or statistical methods, it is interesting to propagate this uncertainty on the results (displacement, internal forces, safety factors…). Most of the time it shows that some variables contribute more to the uncertainty on the results than others, and that even with uncertainty in the properties of the soils, e.g., displacements can be estimated. It means that one can gain real understanding in the modelling.

The range of variation depends on the parameters, see ISSMGE-TC304 (2021) and it is often characterized by the mean parameter μ, the standard deviation σ, or the mean parameter μ and coefficients of variation $CoV = \mu / \sigma$, and finally the type of distribution. Some typical values for the coefficient of variation are given in Table 1, according to Phoon and Kulhawy (2008).

Table 1: Typical range of CoV of soil parameters

	Variability	*CoV (%)*
Undrained shear strength c_u	*Low*	*10-30*
	Medium	*30-50*
	High	*50-70*
Friction angle φ	*Low*	*5-10*
	Medium	*10-15*
	High	*15-20*

Without on-site information, the distributions are often supposed to be Gaussians (for small CoV) or Lognormal.

Once a probabilistic distribution is assigned to input parameters, the output value of a model is also probabilistic. It means the results of the finite element model will be random variables.

2.2. Metamodeling

2.2.1. Description

Let us consider a geotechnical finite element model, with m random input parameters (as the friction angle, the cohesion etc…). The FE model can be seen as a function M which associates a quantity Y (the displacement of a node, the moment in a beam element etc… : a key quantity for the engineers) to the inputs parameters $X = \{x_1, \dots, x_m\}$, as described in Eq. (1) (see Marelli et al. (2022))

$$X \in D_X \subset \mathbb{R}^m \rightarrow Y = M(X) \in \mathbb{R} \qquad (1)$$

This function is only known pointwise: one can get samples from *X*, run the FE model with the corresponding inputs, and get the quantity *Y*. As each run of the model is time expensive, one has to gain the maximum of information on the random variable *Y* with the minimum of samples, in order to later perform a sensitivity and reliability analysis.

The metamodels aim to approximate the equation Eq. (1), based on a small number of samples that have already been computed. Say that one has got N samples of X, $\{X_1, \dots X_N\}$, and computed $\{Y_1, \dots, Y_N\} = \{M(X_1), \dots, M(X_N)\}$. Then the metamodel is a function which surrogates M based on this experimental design, and which can be evaluated in no time.

2.2.2. Polynomial Chaos Expansion (PCE)

The polynomial chaos expansion of the random variable Y, assuming that it has a finite variance, is defined as Eq. (2) (Ghanem and Spanos (2003), Blatman (2009))

$$Y = M(X) = \sum_{a \in \mathbb{N}^m} y_\alpha \psi_\alpha(X) \qquad (2)$$

where the ψ_α (X) are multivariate polynomials orthonormal, $a \in \mathbb{N}^m$ is a multi-index that identifies the components of the multivariate

polynomials ψ_α and the $y_\alpha \in \mathbb{R}$ are the coordinates of the polynomials.

This expansion is then truncated to $\mathcal{A} \subset \mathbb{N}^m$ a set of selected multi-indices and the approximation is given in Eq. (3)

$$M(X) \approx \sum_{a \in \mathcal{A}} y_\alpha \psi_\alpha(X) \tag{3}$$

The complete mathematical description is available in Marelli and Lüthen (2022).

2.2.3. Polynomial Chaos Kriging

Polynomial-Chaos-Kriging (PC-Kriging) is a state-of-the-art metamodeling algorithm, which combines PCE and Kriging process (Schöbi et al (2022)). More precisely, the PCK is a Kriging process (an efficient tool to surrogate local variation), and the trend is represented with the orthonormal polynomials (PCE, to catch the global behavior). The expansion is given in Eq. (4)

$$M^{PCK}(x) = \sum_{a \in \mathcal{A}} y_\alpha \psi_\alpha(X) + \sigma^2 Z(x, \omega) \tag{4}$$

where $\sum_{a \in \mathbb{N}^m} y_\alpha \psi_\alpha(X)$ is a weighted sum of orthonormal polynomials, σ^2 and $Z(x, \omega)$ are the variance and the zero mean, unit variance, stationary Gaussian process, respectively.

2.2.4. Accuracy of a metamodel

A common tool to assess the accuracy of the surrogate model is the leave-one-out cross-validation error ε_{LOO}, see Marelli and Lüthen (2022). It is computed by creating several metamodels on reduced sets of the experimental design, as in Eq. (5)

$$\varepsilon_{LOO} = \frac{\sum_{i=1}^{N} \left(M(x^{(i)}) - M^{PC\backslash i}(x^{(i)}) \right)^2}{\sum_{i=1}^{N} (M(x^{(i)}) - \mu_Y)^2} \tag{5}$$

where $M^{PC\backslash i}(x^{(i)})$ is the metamodel created on the experimental design, without the ith sample, and $M(x^{(i)})$ the FE model result.

This cross-validation error can be extended to the leave-k-out cross-validation error, which consists in creating metamodels based on several subset of the experimental design, of size k, in Allen (1974). A graphical comparison can then be made on the rest of the samples in the experimental design.

2.3. Probabilistic analysis based on metamodeling

2.3.1. Uncertainty propagation

Once an accurate metamodel is created, it is possible to draw many samples for the inputs, and to compute an approximation of the results that the FE model would give. This allows to draw histograms of the quantity of interest, and to compute relevant information about the output distribution, like moments and percentiles.

2.3.2. Reliability analysis

The reliability analysis aims to calculate the probability of failure, i.e., the probability that the output gets higher or lower than a fixed value, the threshold. As described in Marelli and Schöbi (2022), one can define two domains in the input space: the safe region D_s, and the failure one D_f. The limit state function g is then a function which has positive values in the safe domain and negative values in the failure domain (Eq. (6))

$$x \in D_s \Leftrightarrow g(x) > 0 \tag{6}$$

$$x \in D_f \Leftrightarrow g(x) \leq 0$$

The probability of failure P_f quantifies then the probability that the random variable X belongs to D_f, i.e., Eq. (7)

$$P_f = P\big(g(X \leq 0)\big) = \int_{D_f} f_x(x) \, dx \tag{7}$$

The integral in Eq. (7) can then be computed with the Monte Carlo method, by drawing samples and computing the ratio between the number of samples which belong to the failure domain, over the total number of samples (Eq. (8))

$$P_{f,MC} = \frac{\#Nfail}{\#Nsample} \tag{8}$$

The probability of failure calculated with Monte Carlo converges to the exact probability of failure, independently from the system's dimension: it is the main advantage of this algorithm. The drawback is the convergence rate, in $1/\sqrt{N}$, where N is the sample size. Moreover,

the accuracy depends on the probability of failure: a larger sample is necessary to obtain convergence when the probability of failure is smaller.

2.3.3. Sensitivity analysis

The sensitivity analysis aims to estimate the influence of each input variable, on the variability of the quantity of interest. The variance of Y can be decomposed in terms attributable to each of the *m* inputs and interactions between inputs (Marelli et al., 2022). The partial variances $D_{i_1}, \ldots, D_{is}$ are computed with the following formula (Eq. (9))

$$D_{i_1,\ldots,i_s} = \int_{D_{i_1}} \cdots \int_{D_{i_s}} f^2_{i_1,\ldots,i_s}(x_{i_1}, \ldots, x_{i_s}) dx_{i_1} \ldots dx_{i_s} \quad (9)$$

where $1 < i_1 < \ldots < i_s \leq m, s = 1, \ldots, m$ and the total variance is defined as (Eq. (10))

$$D = \int_D f^2(x)\, dx - f_0^2 \quad (10)$$

Finally, the Sobol Total indices can then be defined (Eq. (11))

$$S_i^T = \sum_{i \subset \{i_1,\ldots,i_s\}} \frac{D_{i_1,\ldots,i_s}}{D} \quad (11)$$

3. REAL CASE APPLICATION

This paragraph presents an application of this method on a 3D complex finite element model.

3.1. Context

A new underground metro line is to be constructed in Lausanne in Switzerland. The project has to be carefully designed, because of the dense urban area, and it is currently under development. The official description of the project is available on the canton of Vaud website (www.vd.ch/métros). Around one of the future stations, pipe umbrellas are used (Figure 1), particularly because sensitive buildings are close to the top of the tunnel.

Figure 1 : The metro line (red) with pipe umbrella, under a building (black)

A 3D finite element model was created (Figure 2 and Figure 3) with the finite element software ZSOIL to assess the effect of the excavation process. This software is dedicated to static and dynamic analyses of soil, roc, structures and soil-structure interaction (ZSOIL.PC (2023)). The model contains 400 000 8-nodes Brick elements.
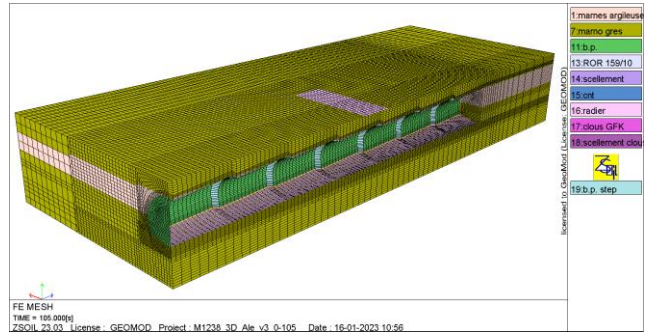

*Figure 2 : Complete 3D FE model, with 400 000 B8 elements, of size 135m * 50m *20m, and 105 time steps*

The elements taken into account in the 3D model are:

- 7 pipe umbrellas, each 14m long, with ROR 159/10 in order to control the displacements
- A supporting system (steel profiles and shotcrete) modelled with shells, interface elements between soil and support are also taken into account.
- Excavation process: the supporting system appears when 1m is excavated in front
- Stratigraphy, composed of 2 layers of sandstone marl, and one layer of clayey

marl. They are modelized with elastic perfectly plastic materials. The parameters of each layer are given in Table 2.
- The building's foundation is modelled with a 2D shell element, loaded by 170kPa.
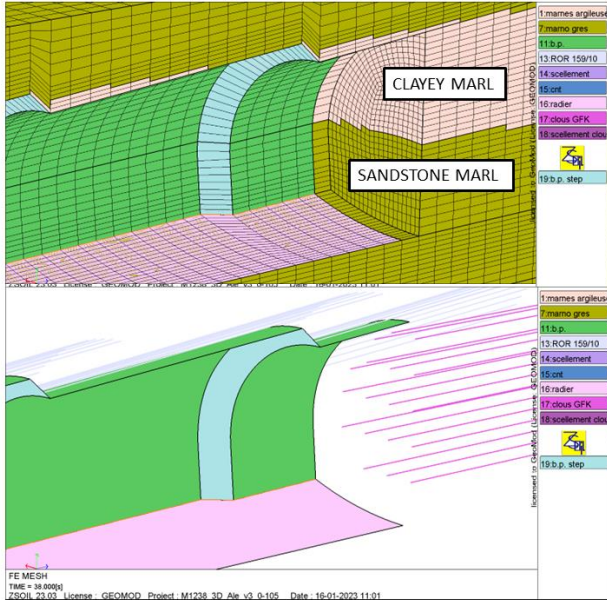

Figure 3 : The complex 3D model takes into account the excavation process and the pipe umbrella (view of the structural elements)

The FE model runs in 4 hours, and the settlement of the building (Figure 4) is 1.15cm with the mean parameters.

3.2. Uncertainty in the geotechnical parameters
Once the model is created and optimized, it is possible to define the uncertainty of the parameters. The whole probabilistic implementation and algorithm is done with UQLab, a Matlab based probabilistic toolbox (Marelli and Sudret (2014)).

The elastic modulus of the two soils, and the cohesion and friction angles are chosen to be probabilistic, in discussion with the geotechnical engineers. The distributions are Gaussians, except for the cohesion (Lognormal), and every variable is considered independent.

Concerning the structure, the load of the building is also set to be probabilistic. The aim is

to assess its influence over the settlement, the chosen coefficient of variation is 10%:

It means 7 input variables are considered for the probabilistic analysis (Table 2). In this method every uncertain parameter of the model can be set to probabilistic, with a realistic distribution.

Table 2 : Description of the inputs considered in the probabilistic analysis. The type "N" means a Gaussian distribution, and "Ln" is Lognormal

Parameter		Type	Mean	CoV %
Clayey marl	E_1	N	50 MPa	30
	c_1	Ln	150 kPa	20
	φ_1	N	22°	10
Sandstone marl	E_2	N	900 MPa	30
	c_2	Ln	1300 kPa	20
	φ_2	N	35°	10
load (building)		N	170 kPa	10

Once the inputs are carefully chosen, the experimental design can be created. 100 samples of the 7 inputs are drawn, with the Latin Hypercube Sampling (LHS) method, named X_{ed}. For each sample, the finite element model is run, with parallel computations, to speed up the calculation time. It takes between 4 and 5 days of computation for the 100 samples. For practical cases, it is recommended to get at least 50 samples, and then to get new samples until the convergence in the ε_{LOO} is reached.

Then, the quantity of interest, i.e., the settlement of the building Y_{ed} is extracted from the results files (see Figure 4).
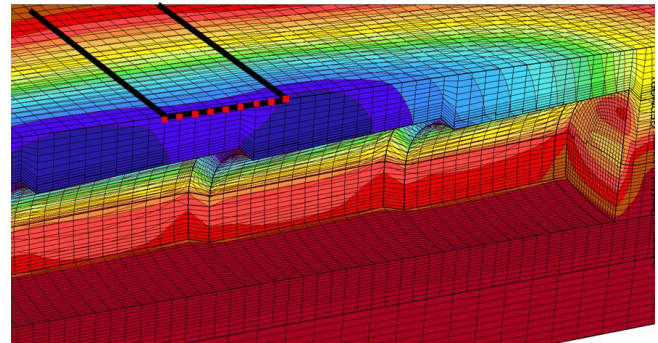

Figure 4 : Vertical displacement computed with ZSOIL. The nodes of interest are in red. A blue color corresponds to a 1.15cm settlement, and the red to 0cm

3.3. Metamodeling with PCE and PCK

When the experimental design $(X_{ed}, Y_{ed} = M(X_{ed}))$ is computed, the metamodels are created to surrogate the FE model.

With the whole experimental design, i.e., 100 samples, the leave-one-out error is 0.65% for the PCE, and 0.39% for the PCK.

A key subject is to assess how many calculations of the FE model are needed to get an accurate metamodel. The Figure 5 shows that a convergence in the leave-one-out error is reached when the metamodel is computed on at least 50 (PCK) or 60 (PCE) samples. From this first result, a first guess would be that 50 samples can be enough with a PCK, and 60 with a PCE.
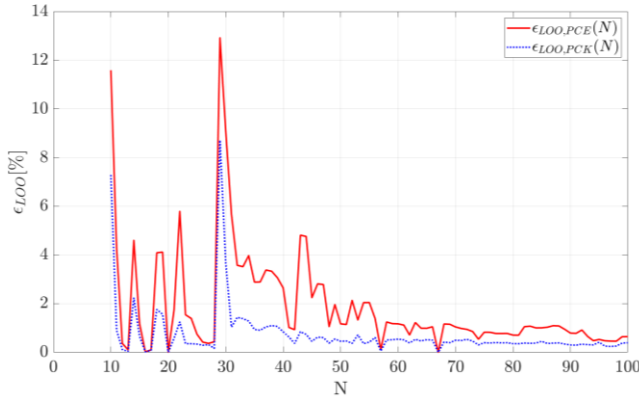

Figure 5 : Visualization of the ϵ_{LOO} for PCE and PCK in function of the size N of the experimental design

To ensure the accuracy of the metamodels, it is possible to compare the predictions with the true response of ZSOIL, on a validation set. The experimental design is then split into two set: the first 60 samples are used to create the metamodels, and the last 40 samples plays the role of validation. The results are shown in Figure 6: the mean difference is 0.2 mm (PCE) and 0.14 mm (PCK) on those 40 settlements.
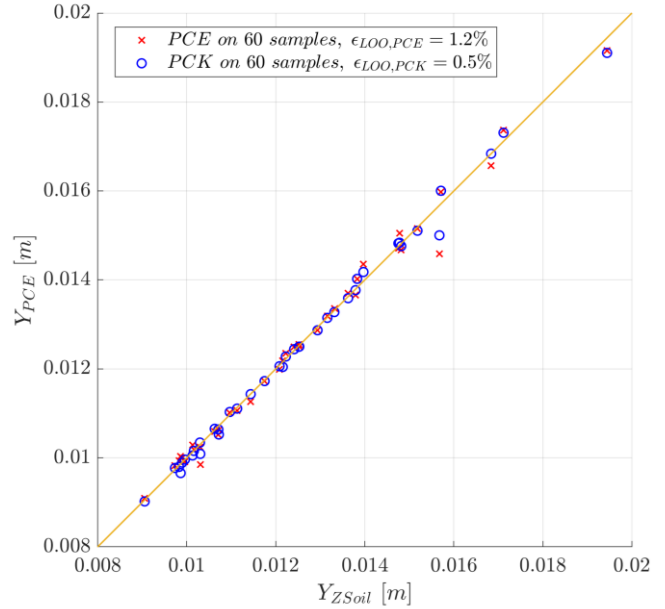

Figure 6 : Comparison of the settlement predicted by metamodels, and the FE settlement

Finally, the leave-k-out graphical validation is done, with 30, 60, and 80 samples (Figure 7)
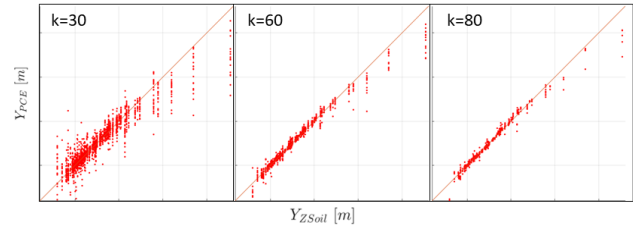

Figure 7 : leave-k-out validation for the PCE is a way to increase the validation set

Those three visualizations confirm that the experimental design is large enough to create an accurate PCE or PCK. Thanks to those metamodels, the uncertainty propagation can be done with many samples, because it does not require a lot of computational resources.

3.4. Reliability and sensitivity analyses

The steps of the probabilistic analysis are the following:

- Draw a large sample of the inputs
- Compute the metamodel response
- Visualize the histogram of the simulated settlement

- Do the sensitivity analysis
- Compute the probability of failure

For the Monte Carlo analysis, 10^6 samples of the inputs are generated with the LHS method. To compute the response of the metamodels to this set of input, it takes a few seconds.

Then it is possible to visualize the histogram of the predicted settlement (Figure 8). The results are given in Table 3.
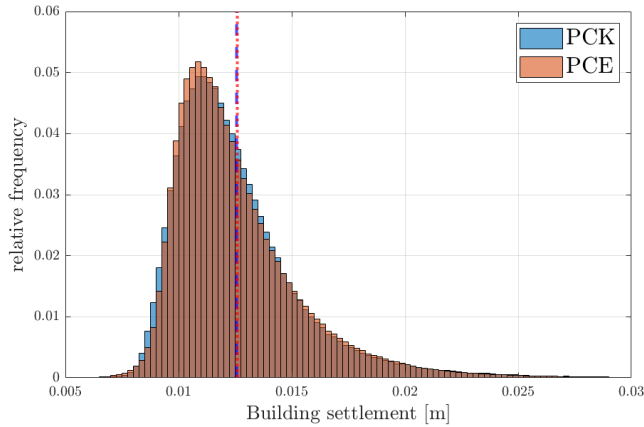

Figure 8 : PDF of the settlement, generated with Monte Carlo. The PCE and PCK give nearly the same smooth histogram. The means of the histograms are the vertical lines

The failure criterion is given by the geotechnical engineer: the vertical displacement of the building should not be bigger than 2 cm. It is computed with the Monte Carlo simulation, on 10^6 samples with the metamodels (Table 3).

Table 3 : Summary of the Monte Carlo simulation, and statistical quantities of the random variable Y (settlement)

Model	N sample	Mean [m]	Std [m]	$P_{f,MCS}$
PCE	10^6	0.0126	$2.9e^{-3}$	2.53 %
PCK	10^6	0.0126	$2.9e^{-3}$	2.67 %

The Sobol indices given in Figure 9 show that the modulus of the clayey marl is the most influent input by far. This layer in ZSOIL is thus a key parameter in the model to predict accurately the settlement.

Also, the load of the building plays nearly no role in the variation of the settlement. This means that it is not necessary to spend too much time modelling the exact load of the building.
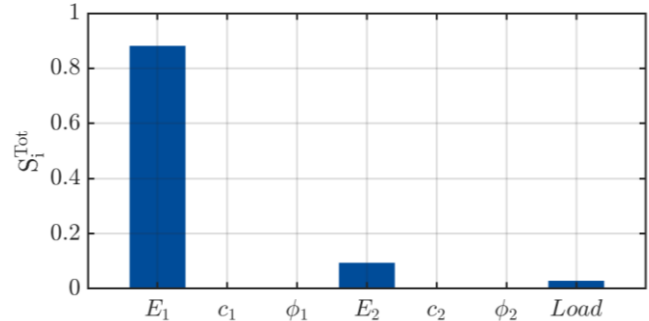

Figure 9 : Sobol total indices, which quantifies the influence of each input over the settlement

4. CONCLUSION
The method and the tools described in this article show that with efficient metamodels like Polynomial Chaos Expansion, it is possible to do a probabilistic analysis, including uncertainty propagation, reliability analysis, and sensitivity analysis, on a complex 3D finite element model. Probabilistic analyses needed around 1 week to be set up and computed, it can therefore be used in day-to-day practice (in the whole project, the main part remains the construction of the FE model). It helps the geotechnical engineers to understand the situation, by completing the deterministic calculations.

5. ACKNOWLEDGEMENTS

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