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A Dissimilarity Measure for On-Line Signature Verification Based on the Sigma-Lognormal Model

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***Abstract*—The Sigma-Lognormal model of the Kinematic Theory of rapid human movements allows us to represent on-line signatures with an analytical neuromuscular model. It has been successfully used in the past to generate synthetic signatures in order to improve the performance of an automatic verification system. In this paper, we attempt for the first time to build a verification system based on the model parameters themselves. For describing individual lognormal strokes, we propose eighteen features which capture cognitive psychomotor characteristics of the signer. They are matched by means of dynamic time warping to derive a dissimilarity measure for signature verification. Promising initial results are reported for an experimental evaluation on the SUSIG visual sub-corpus, which contains some of the most skilled forgeries currently available for research.**

***Keywords*—on-line signature verification; Kinematic Theory of rapid human movements; Sigma-Lognormal model**

I. Introduction

Signatures are widely used biometrics for personal authentication. In contrast to *off-line* images of signatures, modern digitizers such as tablet computers and smartphones capture *on-line* signatures, that is the trajectory of the pen tip over time possibly enriched with additional information such as the pressure of the pen [1]. The time dimension allows an analysis of movement patterns in addition to static images, which usually leads to a much higher performance for automatic signature verification [2].

Many models have been proposed to analyze human movement patterns in general and handwriting in particular, including coupled oscillator models [3], minimum jerk models [4], and models relying on neural networks [5] to name just a few.

Among them, the Kinematic Theory of rapid human movements is a unique framework based on the lognormal law [6], [7]. It includes a family of analytical models for representing movements based on neuromuscular strokes with lognormal velocity [8]. The Delta-Lognormal model represents single rapid movements by means of two strokes in opposite direction. Similarly, the Omega-Lognormal model represents oscillatory movements with an alternating sequence of opposed strokes. Finally, the Sigma-Lognormal model has been proposed to represent complex movements like signatures using a vectorial sum of lognormal strokes [9].

Robust algorithms have been developed for estimating the lognormal parameters from observed trajectories [10], [11]. They achieve an excellent reconstruction quality of the observed movement provided that the movement is skilled and unimpaired. On the other hand, it has been shown recently that aging, for example, leads to a deviation from lognormality in handwriting movements when the control of the fine motricity begins to decline and on the other hand, as children improve in learning handwriting, their movements tend toward lognormality [12].

Apart from its powerful potential in biomedical and neuroscience applications, one of the most successful applications of the Kinematic Theory has been the synthetic generation of handwriting based on the analytical model, for example gestures [13], signatures [14], [15], and also unconstrained handwriting [16]. The synthetic specimens could be used as learning samples to improve an automatic recognition system. This is particularly interesting for signature verification, where only few reference signatures are available per user.

In this paper, we go a step further and aim to build a signature verification system based on cognitive psychomotor characteristics captured by the model itself. Such characteristics have been linked recently with brain stroke risk factors [17], which highlights the promising potential of the model in the context of biometric verification. We propose a new dissimilarity measure between two signatures based on their Sigma-Lognormal representation. Eighteen features are suggested for describing an individual stroke and the stroke sequences are matched by means of dynamic time warping. Initial results are reported for the highly skilled forgeries of the SUSIG visual sub-corpus [18].

The remainder of this paper is organized as follows. The data set and the model parameter extraction are discussed in Section II. Afterwards in Section III, the proposed dissimilarity measure for signature verification is introduced. Finally, experimental results are reported in Section IV and conclusions are drawn in Section V.

II. Model Extraction

A. Data Set

On-line signatures from the SUSIG visual sub-corpus [18] are considered in this paper. It includes signatures from 94 users captured with Interlink Electronics's ePad-ink tablet. This tablet has a pressure-sensitive LCD screen which shows the signer what he or she is writing.

For every user, highly skilled forgeries were created based on animations of the signature to imitate. The animations were shown on the LCD screen so that the forger could trace over the genuine signature in several attempts. This acquisition protocol has allowed to generate some of the most skilled forgeries currently available for research.

B. Sigma-Lognormal Model

The Sigma-Lognormal model ($\Sigma\Lambda$) [9] represents on-line signatures $s = (s_1, \ldots, s_N)$ as a sequence of strokes. Each stroke s_i has lognormal speed

$$|\vec{v}_i(t)| = \frac{D_i}{\sqrt{2\pi}\sigma_i(t - t_{0_i})} \exp\left(-\frac{(\ln(t - t_{0_i}) - \mu_i)^2}{2\sigma_i^2}\right) \quad (1)$$

with respect to the initialization time t_{0_i}, the input command D_i which corresponds with the covered distance when executed in isolation, and the two parameters μ_i and σ_i related to the logtime delay and the logresponse time of the neuromuscular system responding to the command.

The angular position of the movement along a pivot direction is expressed with respect to the start angle θ_{s_i} and the end angle θ_{e_i}. In total, each stroke is represented by six parameters

$$s_i = (D_i, t_{0_i}, \mu_i, \sigma_i, \theta_{s_i}, \theta_{e_i}) \quad (2)$$

which allow a reconstruction of the observed velocity by means of vectorial summation:

$$\vec{v}_r(t) = \sum_{i=1}^{n} \vec{v}_i(t) \quad (3)$$

The quality of the reconstruction is measured as a signal-to noise ratio taking into account the observed velocity $\vec{v}_o(t)$ and the reconstructed velocity $\vec{v}_r(t)$

$$SNR = 10 \log \left(\frac{\int_{t_s}^{t_e} |\vec{v}_o(\tau)|^2 d\tau}{\int_{t_s}^{t_e} |\vec{v}_o(\tau) - \vec{v}_r(\tau)|^2 d\tau} \right) \quad (4)$$

where t_s is the start time and t_e is the end time of the pen tip trajectory.

C. Parameter Extraction

Recently, a robust algorithm for the extraction of the Sigma-Lognormal model from the observed pen tip trajectory has been introduced in [11]. It iteratively adds lognormal strokes to the model in order to maximize the SNR.

Each pen-down component is analyzed separately as suggested in [16]. The pen tip is stopped artificially at the beginning and at the end of each component to ensure zero velocity for an improved extraction of the first and the last stroke. Furthermore, signal preprocessing includes an interpolation with cubic splines, resampling at 200Hz, and low pass filtering with a Chebyshev filter to remove high-frequency components introduced by the digitizer.

Afterwards, one stroke after the other is extracted from the preprocessed observed velocity $\vec{v}_o(t)$ in three steps. First, s_i is localized in the speed profile $|\vec{v}_o(t)|$ based on local minima and maxima. Secondly, the stroke parameters $s_i = (D_i, t_{0_i}, \mu_i, \sigma_i, \theta_{s_i}, \theta_{e_i})$ are estimated based on the analytical Robust XZERO solution [11] as well as non-linear least squares curve fitting. Thirdly, s_i is added to the result and $\vec{v}_i(t)$ is subtracted from $\vec{v}_o(t)$. The three steps are repeated until the SNR cannot be further improved.

A reconstruction example is illustrated in Figure 1. Individual strokes are shown in the trace as well as in the velocity profile. Virtual target points are marked with a circle. They would have been reached if the strokes were executed in isolation rather than computing the vectorial sum in Equation 3. The reconstructed velocity profile is very accurate with an average SNR of 18.5dB for the three pen-down components.

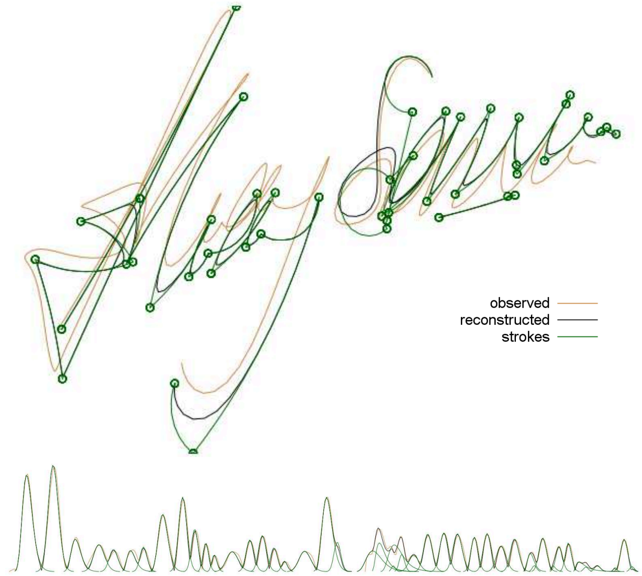

Fig. 1. Reconstructed trace and velocity profile of an on-line signature.

III. Signature Verification

For automatic signature verification, we represent the questioned signature $q = (q_1, \ldots, q_N)$ and the reference signatures $r = (r_1, \ldots, r_M) \in R$ with a sequence of strokes based on the Sigma-Lognormal model. Then, we compute a dissimilarity $\hat{d}_R(q)$ between the questioned signature q and the set of reference signatures R, which is compared with a threshold in order to accept or reject the questioned signature.

In the following, features for describing an individual stroke are presented in Section III-A and the dissimilarity measure $\hat{d}_R(q)$ is derived in Section III-B based on dynamic time warping.

A. Stroke Features

Eighteen features are proposed to characterize a stroke $s_i = (D_i, t_{0_i}, \mu_i, \sigma_i, \theta_{s_i}, \theta_{e_i})$. The first seven features correspond directly with model parameters

- $f_1 = D_i$
- $f_2 = \mu_i$
- $f_3 = \sigma_i$
- $f_4 = \sin(\theta_{s_i})$
- $f_5 = \cos(\theta_{s_i})$
- $f_6 = \sin(\theta_{e_i})$
- $f_7 = \cos(\theta_{e_i})$

considering Cartesian coordinates $(\sin(\alpha), \cos(\alpha))$ for angular parameters. For the initialization time t_{0_i}, we compute a feature in comparison with the preceding stroke s_{i-1}

- $f_8 = \Delta t_0 = t_{0_i} - t_{0_{i-1}}$

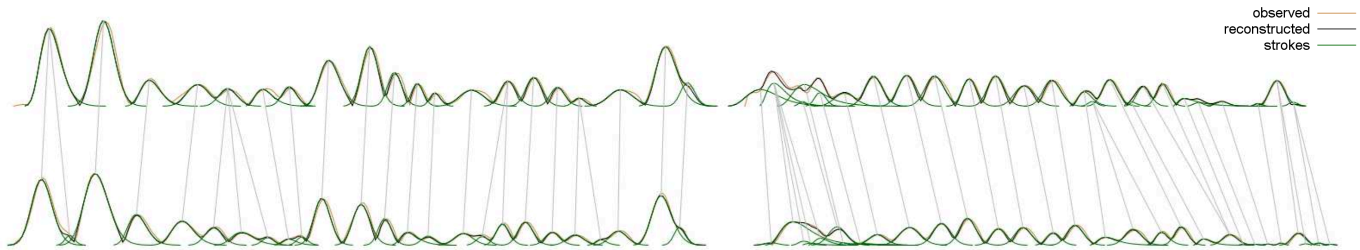

Fig. 3. Stroke alignment using dynamic time warping.

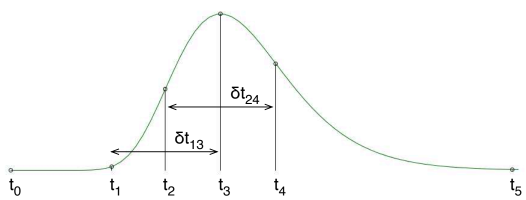

Fig. 2. Characteristic times of a lognormal stroke.

The remaining features are calculated with respect to five characteristic times $t_{1_i}, \ldots, t_{5_i}$ of a lognormal stroke [11]. They are illustrated in the velocity profile in Figure 2. The times t_{2_i}, t_{3_i}, and t_{4_i} are the zeroes of the first and second derivative of the lognormal Equation 1 and correspond respectively to the mode $t_{3_i} = t_{0_i} + \exp(\mu_i - \sigma_i^2)$ and the inflection points of the lognormal stroke. The other times $t_{1_i} = t_{0_i} + \exp(\mu_i - 3\sigma_i)$ and $t_{5_i} = t_{0_i} + \exp(\mu_i + 3\sigma_i)$ are chosen such that the interval $[t_{1_i}, t_{5_i}]$ contains 99.97% of the area under the lognormal curve. Based on these characteristic times, the remaining ten features are defined as

- $f_9 = v_2 = |\vec{v}_i(t_{2_i})|$
- $f_{10} = v_3 = |\vec{v}_i(t_{3_i})|$
- $f_{11} = v_4 = |\vec{v}_i(t_{4_i})|$
- $f_{12} = \delta t_{05} = t_{5_i} - t_{0_i}$
- $f_{13} = \delta t_{15} = t_{5_i} - t_{1_i}$
- $f_{14} = \delta t_{13} = t_{3_i} - t_{1_i}$
- $f_{15} = \delta t_{35} = t_{5_i} - t_{3_i}$
- $f_{16} = \delta t_{24} = t_{4_i} - t_{2_i}$
- $f_{17} = \Delta t_1 = t_{1_i} - t_{1_{i-1}}$
- $f_{18} = \Delta t_3 = t_{3_i} - t_{3_{i-1}}$

They capture detailed timing characteristics of the neuromuscular Sigma-Lognormal model.

B. Dissimilarity Measure

In order to compute a distance $d(q, r)$ between the questioned signature $q = (q_1, \ldots, q_N)$ and a reference signature $r = (r_1, \ldots, r_M) \in R$ with a different number of strokes, we consider the dynamic time warping distance (DTW) [19]

$$d(q, r) = \min_p \sum_{i=1}^{|p|} |f_k(q_{p_{i,1}}) - f_k(r_{p_{i,2}})| \tag{5}$$

with respect to one of the features f_k, $k \in 1, \ldots, 18$, and the time warping path p, which is illustrated in Figure 3.

Based on the DTW distance $d(q, r)$, the minimum distance

$$d_R(q) = \min_{r \in R} d(q, r) \tag{6}$$

to the set of reference signatures R is computed. Finally, this value is normalized

$$\hat{d}_R(q) = \frac{d_R(q)}{\mu_d} \tag{7}$$

with the mean score $\mu_d = \frac{1}{|R|} \sum_{i=1}^{|R|} d_{R \setminus r_i}(r_i)$ computed over all reference signatures to make it comparable across different users in the database.

IV. Experimental Evaluation

In this section, we present the results of a preliminary evaluation of the proposed method for skilled forgery detection on the SUSIG visual sub-corpus (see Section II-A).

A. Setup

All available genuine signatures and skilled forgeries are used in the trial. For each of the 94 users, the first 5 signatures are used as references and the remaining 15 for evaluation. In total, we consider $94 \cdot 5 = 470$ reference signatures, $94 \cdot 15 = 1,410$ genuine signatures, and $94 \cdot 10 = 940$ skilled forgeries.

The performance is evaluated in terms of equal error rate (EER), that is the point in the receiver operating characteristic (ROC) where the false acceptance rate equals the false rejection rate.

B. Model Quality

The extraction algorithm (see Section II-C) for the Sigma-Lognormal model achieves an SNR of 19.87 ± 2.40dB for the SUSIG visual sub-corpus. This is a good reconstruction quality when compared with 15dB which is generally considered as sufficient for human movement analysis [10]. 96.77% of all signatures were reconstructed with an SNR above this threshold.

C. Verification Results

Table I lists the EER results for the best seven out of eighteen investigated features. The main observation is that the best performing features on this data set are those related to timing differences, both within the same stroke and between two consecutive strokes. The overall best performance is achieved with the feature Δt_3, that is the difference between the mode of two consecutive strokes.

Rank	Feature	EER
1.	Δt_3	5.11%
2.	Δt_1	5.43%
3.	δt_{24}	8.94%
4.	δt_{13}	8.94%
5.	δt_{05}	13.83%
6.	δt_{15}	14.47%
7.	Δt_0	15.11%

TABLE I. EER RESULTS FOR THE SUSIG VISUAL SUB-CORPUS.

Furthermore, it is interesting to notice that features with respect to the four characteristic times t_1, t_2, t_3, and t_4 lead to a significantly better performance than features related to the initialization time t_0 and the end time t_5. These two parameters are particularly difficult to estimate with our current model extraction algorithm when several strokes overlap in time [20].

When compared with the state of the art, the EER of 5.11% obtained with the proposed method is in the ballpark of the best results reported for this difficult verification task. In [21], Sae-Bae and Memon report an EER of 6.08% with a recent histogram-based system. By fine-tuning the system to the data set, an EER of 4.37% is achieved. In [22], Yanikoglu and Kholmatov propose a verification based on Fourier descriptors and report an EER of 6.20%. When combined with a second DTW-based verification system, an EER of 3.03% is obtained.

V. CONCLUSIONS

In this paper, we have introduced one of the first pattern recognition systems which is directly based on the Sigma-Lognormal model. Instead of using the model to generate synthetic movements, cognitive psychomotor characteristics of the signer are derived from the model itself and are integrated into a signature verification system.

A preliminary evaluation on the SUSIG visual sub-corpus has demonstrated that the proposed method is able to achieve state-of-the-art results for skilled forgery detection. Difficult forgeries are taken into account that were created by tracing animated genuine signatures on an LCD screen.

In order to build a complete system that includes the proposed Sigma-Lognormal verifier, future work includes a more comprehensive experimental evaluation, the combination of complementary features, and also the combination of complementary verification systems. The Sigma-Lognormal verifier is expected to have a particular advantage for detecting highly skilled forgeries when compared with other approaches. Even if the trace signals and the velocity signals are very similar, the model might be able to distinguish nuanced differences in the fine motor control.

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