Published in *Energy Economics*, vol. 109, 25 p., which should be cited to refer to this work.
DOI: 10.1016/j.eneco.2022.105929

Electricity Market Design and Implementation in the Presence of Asymmetrically Informed Strategic Producers and Consumers: A Surrogate Optimization-based Mechanism*

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Abstract

We consider electricity networks where the agents (producers and consumers) are strategic and possess asymmetric information about the networks' status. The network model accounts for power losses, line congestion, and financial transmission rights (FTRs). We propose a mechanism (a set of rules for energy production and consumption) that takes into account the network model, the agents' strategic behavior and their informational asymmetries, and has the following properties at all Nash equilibria of the game induced by it: (i) it is budget balanced (the sum of taxes received by the agents and the sum of subsidies paid to the agents is equal to zero); (ii) it implements the optimal power flow (OPF) dispatch (equivalently it implements the social welfare maximizing dispatch); (iii) it is price efficient (the price received per unit of energy production is equal to the price paid per unit of energy consumption, and they are both equal to the price corresponding to the OPF dispatch); (iv) it is individually rational (the strategic agents voluntarily participate in the mechanism). We also propose a tâtonnement-process (an algorithm), based on a best-estimate method, and prove that it converges to a Nash equilibrium of the game induced by the proposed mechanism. The allocations (energy production and consumption) resulting at each step of the tâtonnement-process are a feasible solution of the OPF problem.

Keywords— Electricity market, financial transmission right (FTR), optimal power flow (OPF), mechanism design, local public goods, tâtonnement-process.

*This work was supported in part by Army Research Office (ARO) grant W911NF-17-1-0232. The authors would like to thank Farzaneh Farhadi for useful discussions and her comments on an earlier version of this paper.

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1 Introduction

1.1 Background and Motivation

In the past decades, the electricity industry around the world has been moving from a traditional structure of an electricity network owned by one public utility towards a competitive market structure including a number of independent strategic agents, i.e., producers and consumers. By this restructuring, the functional operations of generators, flexible demands and transmission lines have been separated, and multiple self-interested strategic agents have been created [1]. Even though it appears that free trading is the objective of restructuring, the presence of an independent system operator (ISO) is necessary, as she provides certain critical coordinating services for the reliability and the security of the electricity network [2]. As Hogan pointed out in [3], *"the special nature of electricity systems leads to the need for a seeming contradiction in terms: coordination for competition"*.

The efficient design of electricity markets has been discussed in many debates, studies, and experiments. For example, the price spikes of California ISO's energy market in June 2000 have been analyzed in [4, 5, 6, 7]. There is a general agreement that the design of the California market had major flaws, and the lesson learned from the California crisis is that *"electricity has unusual physical attributes that make the design of well-functioning competitive wholesale power markets a significant challenge"*, and *"market power problems must be addressed both initially and as evidence about actual market performance and supplier behavior emerges as the market operates"* [7]. However, it is widely debatable whether the reason behind the California market crisis is the exercised of market power. Nevertheless, numerous models[1] have been introduced to study the impact of market power on the social welfare outcome and on the electricity network's performance.

The discussions on efficient market design appearing in the literature have not been limited to the California crisis and the exercise of market power; they have identified and discussed the key issues/difficulties associated with the design of efficient electricity markets. These issues/difficulties are: (i) the information asymmetries among the market participants and the ISO; (ii) the presence of strategic producers/consumers that possess market power; and (iii) the presence of complex externalities associated with the energy flow across a constrained transmission network. We briefly discuss each one of these difficulties.

(i) One of the key feature of electricity markets are the information asymmetries that exist among the participating agents and the ISO. The agents, including producers and consumers, have private information on their production costs, their utilities, their technologies, and their budgets. These asymmetries results in several challenges for the ISO. One such challenge is the missing money problem, which discourages investors from investing in the market as they cannot collect back the costs of investment.

(ii) Strategic agents (producers and consumers) who possess market power can manipulate the market so as to increase their own payoffs at the expense of the social welfare. Market power (gaming) can be

[1] The most famous model is the supply function equilibrium (SFE) model, based on the work of Klemperer and Meyer [8]. For information about the SFE and other models see [9]-[10], and references therein.

even exercised by producers who own small capacities in an electricity market. Consider for example, a part of a network that is isolated due to congestion. Then, a small producer becomes the monopolist in this part of the network and can exercise market power. Other reasons resulting market power for small producers are discussed in [11].

(iii) An important and unique feature of electricity networks is the fact that the flows must obey Kirchhoff's voltage and current laws. Furthermore, the flows are limited by thermal and stability constraints on the transmission lines. These constraints along with Kirchhoff's voltage and current laws result in externalities in free trading among producers and consumers. Even when the network's participants are non-strategic as in [12], when the congestion and transmission line limitations are neglected, bilateral agreements among agents do not result in an optimal dispatch; furthermore, the merchandising surplus is positive and budget balance is not satisfied.

There has been considerable debate on whether the competitive electricity markets that address the above difficulties should be organized around *"bid-based pools with financial transmission rights (FTRs)"* or *"bilateral agreements with tradable physical transmission rights (PTRs)"* [13]. Experience indicates that bilateral agreements among participants in electricity networks are not likely to be successful. In [3], it has been suggested that the best approach is a spot-pricing bid-based market with voluntary participation of producers and consumers; moreover, FTRs should be preferred to PTRs. In [13], the social welfare superiority of FTRs as compared with PTRs has been shown. [2]

Understanding and overcoming the above difficulties ((i)-(iii)) provide the motivation for the research reported in this paper. Our approach to efficient electricity market design is organized around *"bid-based pools with FTRs"*, and it is based on mechanism design for local public goods problems (as explained in Section 2.5).

1.2 Literature Survey

Our approach consists of two components; modeling and market design. Our model takes into account the lines' thermal and stability constraints, Kirchhoff's voltage and current laws, FTRs, and a convex direct-current (DC) approximation to losses. Thus, it is similar to the models of Miehling et al. and Wu et al. (see [16, 17] and references therein). There are two key differences between our model and that of [16, 17]: (i) our model incorporates FTRs (a feature missing from that of [16, 17]); (ii) in our model the networks' agents behave strategically, in [16, 17] the agents are non-strategic. Our market design is based on mechanism design for local public goods. The design of mechanisms for public goods and local public goods problems has been investigated within the context of economic theory and engineering. In engineering, public goods and local public goods problems arise in many areas, including wired and wireless networks, cyber security, energy networks, etc.

[2]PTRs can be withheld from the market; thus, the effective transmission capacity is reduced and the production inefficiency is consequently induced. It is worth mentioning that the FTRs are basically issued so that the agents can hedge themselves against the fluctuations of the local market prices [14]. Moreover, the FTRs instrument is supposed to become the sole source of investors' revenues for transmission expansion [15]. However in [13], it has been proved that FTRs can be adopted by strategic agents for speculation purposes.

Within the context of economic theory, Groves and Ledyard [18], Hurwicz [19], Chen [20], Walker [21], and Kim [22] were among the first to propose mechanisms for public goods and local public goods problems. In [18], the agents participating in the mechanism are non-strategic, in [19, 20, 21, 22] they are strategic. The authors of [19, 20, 21, 22] present mechanisms that implement the Lindahl correspondence in Nash equilibria.

In wired networks, public goods problems arise in multi-rate multicast problems. Stoenescu et al. [23, 24] present a mechanism that achieves the social optimal allocation when the agents are non-strategic and have quasi-linear utilities. Kakhbod and Teneketzis [25] consider the multi-rate multicast problem with strategic agents having quasilinear utilities, and present a mechanism that implements in Nash equilibria the social welfare function. The mechanism of [25] is budget-balanced and individually rational.

In wireless networks, the interferences among the agents' signals and the channels' noise give rise to public goods or local public goods problems. Sharma and Teneketzis have formulated and solved power allocation problems in wireless networks both as public goods [26] and local public goods [27] problems. In [26, 27], they proposed mechanisms that implement the social welfare function in Nash equilibrium. Kakhbod and Teneketzis [28] investigated spectrum sharing problems in wireless networks. They proposed a mechanism that implements the social welfare function in Nash equilibria. The mechanisms proposed in [26, 27, 28] are also individually rational and budget-balanced.

Naghizadeh and Liu [29] formulated a cyber security problem as a public goods problem and proposed a mechanism that implements the social welfare function in Nash equilibria.

In the restructured electricity industry, researchers formulated the optimal power flow (OPF) problem as a public goods or local public goods problem with strategic/non-strategic agents. The existing literature can be divided into three categories. The first category consists of studies where the system's agents, producers and consumers, are assumed to be non-strategic [12, 16, 17, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40]. All these studies present centralized algorithms, decentralized markets, or public goods mechanisms/algorithms for the solution of the OPF problem. A detailed survey of these mechanisms appears in [16, 40]. The second category of problems consists of studies that take an implementation theory approach, i.e., mechanism design with strategic agents, to the OPF problem. Such studies appear in [41, 42, 43, 44, 45]. Our paper belongs to this category. The implementation theory approach to the OPF problem is distinctly different from the third category of the literature where the OPF problem is formulated as a non-cooperative game among strategic agents. In contrast to the mechanism design approach (where the game is designed), the non-cooperative games studied in the third category of literature are given, not designed. A non-cooperative game theoretic approach to the OPF appears in [8, 9, 10, 46, 47, 48, 49, 50, 51, 52]. The equilibria of these games are studied and analyzed; in general, the allocations corresponding to these equilibria are not social welfare maximizing.

Since our paper belongs to the second category of the existing literature, we provide a more detailed description of the results appearing in this category; such a description will allow us to compare the results of our paper with the existing results on the mechanism design (or implementation theory) approach to the OPF.

Silva et al. [41] propose a direct revelation mechanism to the OPF. Truthful strategy is a Nash equilibrium of the non-cooperative game induced by the proposed mechanism; furthermore, individual rationality is satisfied at that truthful Nash equilibrium. However, the budget balance criterion is not satisfied at that equilibrium.

Xu and Low [42] design a Vickrey–Clarke–Groves (VCG) mechanism for the wholesale electricity market problem. The mechanism achieves the social cost minimization at the truth-telling dominant strategy equilibrium; moreover, it guarantees dominant strategy incentive compatibility and ex-ante individual rationality. However, the budget balance is not satisfied. Moreover, the transmission network is approximated by the DC power flow equations.

Sessa et al. [43] also design a VCG mechanism for the electricity market problem considering the non-convexity of transmission networks' constraints. They find the conditions under which shill-bidding and collusion can occur. To resolve this issue, Karaca and Kamgarpour propose a coalition-proof mechanism in [44].

Rasouli and Teneketzis [45] propose a local public goods mechanism for the OPF that takes into account the lines' capacities, Kirchhoff's voltage and current laws, and a convex DC approximation to the losses. The mechanism implements the social welfare function in Nash equilibrium, it is individually rational, budget balanced, and price efficient. However, [45] does not provide any algorithm, a.k.a., tâtonnement-process, for computing a Nash equilibrium of the non-cooperative game induced by the proposed mechanism. Furthermore, in contrast to this paper, [45] adopts a simultaneous bilateral agreement approach to market design rather than a bid-based pool structure. Finally, FTRs are not taken into account.

In this paper, we propose a surrogate optimization-based mechanism that implements in Nash equilibria the solution of the OPF problem. The mechanism is organized around *"bid-based pools with FTRs"*. We note that in [53], Farhadi et al. have also used a surrogate optimization-based mechanism for resource allocation and routing in communication networks, thus it is different from the problem of this paper (since it does not address a local public goods problem).

1.3 Contribution

We propose a surrogate optimization-based mechanism for the OPF problem in electricity markets with strategic agents. Our model accounts for power losses (convex DC approximation), line congestion, and FTRs. The proposed mechanism possesses the following properties: (i) it is budget-balanced at the equilibrium; (ii) it implements in Nash equilibria the social welfare function (the sum of all the strategic agents' utilities); (iii) it is price-efficient; (iv) it is individually rational; (v) it has the off-equilibrium feasibility property; (vi) it has finite message space. Furthermore, we discover an algorithm/tâtonnement-process that satisfies the constraints of our model along with the constraints imposed by the decentralization of information and the agents' strategic behavior, and converges to a Nash equilibrium of the non-cooperative game induced by the proposed mechanism.

To the best of our knowledge, a mechanism with the above properties along with the associated algorithm that guarantees convergence to a socially optimal dispatch for the OPF problem with strategic agents does not currently exist in the literature.

1.4 Organization

The remainder of this paper is organized as follows. In Section 2, we introduce the model and objectives. In Section 3, we describe the structure of *"bid-based pools with FTRs"*. We present the proposed surrogate optimization-based mechanism in Section 4. We prove the properties of the mechanism in Section 5. We present the tâtonnement-process associated with the proposed mechanism in Section 6. In Section 7, we extend the result of Sections 4-5 to sparse networks. In Section 8, we evaluate the performance of the proposed mechanism and algorithm in a case study. We conclude in Section 9. We present the proofs of all technical results in Appendices A-J.

1.5 Notation

In this section, we present the key notation of the paper. Additional symbols are defined as needed throughout the text.

Indices

n,n',n''	Indices of nodes.
s,s'	Indices of strategic agents.

Sets

$\mathcal{N}$	Set of nodes; $\mathcal{N} := \{1, \ldots, \lvert\mathcal{N}\rvert\}$.
$\mathcal{S}$	Set of agents; $\mathcal{S} := \{1, \ldots, \lvert\mathcal{S}\rvert\}$.
$\mathcal{R}_n$	Set of nodes which are connected to node n with a transmission line; node n itself is also included; $\mathcal{R}_n := \{1, \ldots, \lvert\mathcal{R}_n\rvert\}$.
$\mathcal{N}_s^{(E)}$	Set of nodes of generators owned by agent s; $\mathcal{N}_s^{(E)} := \{1, \ldots, \lvert\mathcal{N}_s^{(E)}\rvert\}$.
$\mathcal{N}_s^{(D)}$	Set of nodes of flexible demands owned by agent s; $\mathcal{N}_s^{(D)} := \{1, \ldots, \lvert\mathcal{N}_s^{(D)}\rvert\}$.
$\Psi_n^{(E)}$	Set of agents that have generators on node n; $\Psi_n^{(E)} := \{1, \ldots, \lvert\Psi_n^{(E)}\rvert\}$.
$\Psi_n^{(D)}$	Set of agents that have flexible demands on node n; $\Psi_n^{(D)} := \{1, \ldots, \lvert\Psi_n^{(D)}\rvert\}$.
Δ_s	Set of nodes, in which generators and flexible demands of agent s are located or connected; $\mathcal{N}_s^{E} \cup \mathcal{N}_s^{(D)} \subseteq \Delta_s$; $\Delta_s := \{1, \ldots, \lvert\Delta_s\rvert\}$.
Φ_s	Set of transmission lines connecting to generators and flexible demands of agent s; $\Phi_s := \{1, \ldots, \lvert\Phi_s\rvert\}$.
Ω_n	Set of agents that have generators or flexible demands on node n or neighboring nodes; $\Psi_n^{(E)} \cup \Psi_n^{(D)} \subseteq \Omega_n$; $\Omega_n := \{1, \ldots, \lvert\Omega_n\rvert\}$.
$\Upsilon_{nn'}$	Set of agents that have generators or flexible demands connected to transmission line from node n to n'; $\Upsilon_{nn'} := \{1, \ldots, \lvert\Upsilon_{nn'}\rvert\}$.

$\mathcal{E}_n^{(s)}$	Production set of agent s at node n; $\mathcal{E}_n^{(s)} := [0, \overline{E}_n^{(s)}]$.
$\mathcal{D}_n^{(s)}$	Consumption set of agent s at node n; $\mathcal{D}_n^{(s)} := [0, \overline{D}_n^{(s)}]$.
$\mathcal{M}_s$	Message set of agent s.
$\mathbb{R}_{\geq 0}$	Set of non-negative numbers; $\mathbb{R}_{\geq 0} := [0, \infty)$.
$\mathbb{R}_{>0}$	Set of positive numbers; $\mathbb{R}_{>0} := (0, \infty)$.

Variables

θ_n	Voltage angle of node n in rad; $\theta_{nn'} := \theta_n - \theta_{n'}$.
V_n	Voltage magnitude of node n in per unit.
$e_n^{(s)}$	Production of agent s at node n in MW, $e_n^{(s)} \geq 0$.
$d_n^{(s)}$	Consumption of agent s at node n in MW, $d_n^{(s)} \geq 0$.
$u_n^{(s)}(d_n^{(s)})$	Utility of agent s from consuming $d_n^{(s)}$ at node n in \$.
$c_n^{(s)}(e_n^{(s)})$	Cost of agent s from producing $e_n^{(s)}$ at node n in \$.
t_s	Payment of agent s in \$; if net consumption is positive: $t_s > 0$, and if net production is positive: $t_s < 0$.
U_s	Total utility of agent s in \$.
W	The ISO's objective in \$.
$\alpha_{nn'}^{(s)}$	Share of agent s from issued financial transmission right of line from node n to n' in MW.
$F_{nn'}$	Real power flow of line from node n to node n' in MW.
$L_{nn'}$	Losses of line between nodes n and n' in MW.

Vectors

$\vec{m}_s$	Message of agent s in the mechanism.
$\vec{\mathbf{m}}$	Message profile of all agents; $\vec{\mathbf{m}} := (\vec{m}_s)_{s \in \mathcal{S}}$.

Parameters

O_n	Must-run demand at node n in MW.
$\overline{E}_n^{(s)}$	Maximum production of agent s at node n in MW.
$\overline{D}_n^{(s)}$	Maximum consumption of agent s at node n in MW.
$\overline{F}_{nn'}$	Maximum flow of line from node n to n' in MW.
$B_{nn'}$	Susceptance of line from node n to n' in MW/rad.
$G_{nn'}$	Conductance of line from node n to n' in MW/rad^2.

2 Model and Objective

2.1 Power Flow Model

We consider an electricity network with a number of nodes connected by transmission lines. Let $\mathcal{N} := \{1, \ldots, n, \ldots, |\mathcal{N}|\}$ denotes the set of nodes. The set of nodes connecting to node $n \in \mathcal{N}$ is denoted by $\mathcal{R}_n$. The transmission line between nodes n and n' has capacity $\overline{F}_{nn'}$. The capacity $\overline{F}_{nn'}$ captures both the thermal constraint and stability constraint requirements for the transmission line between nodes n and n'. It has an admittance $\sqrt{G_{nn'}^2 + B_{nn'}^2}$ consisting of conductance $G_{nn'}$ and susceptance $B_{nn'}$ components.

A node $n \in \mathcal{N}$ has an associated voltage angle denoted by θ_n. We assume that the network has no intermediary and isolated nodes, i.e., nodes without any production or consumption; we relax this assumption in Section 7. We do not assume the existence of a slack node (i.e., a reference node in the model) since the optimal dispatch of generators or flexible demands does not depend on the selection of a slack node, it only depends on the voltage angles differences $\theta_{nn'} := \theta_n - \theta_{n'}$.

We approximate the power flowing through a transmission line by a convex function of the voltage angles differences. We also use a convex DC approximation to model the losses in transmission lines (see [12, 16, 17]). The accuracy of this approximation is evaluated in Section 8. To derive the convex DC approximation, we consider the real power $F_{nn'}$ flowing from node n to node n', which is given by

$$F_{nn'}(\theta_{nn'}) := G_{nn'}.V_n^2 - G_{nn'}.V_n.V_{n'}.\cos\theta_{nn'} + B_{nn'}.V_n.V_{n'}.\sin\theta_{nn'}, \tag{1}$$

where V_n is the voltage magnitude of node n. We assume that V_n as equal to 1 per unit for all $n \in \mathcal{N}$, and the voltage angles differences $\theta_{nn'}$ are small enough. Based on these assumptions, $\sin\theta_{nn'}$ and $\cos\theta_{nn'}$ are equal to $\theta_{nn'}$ and $1 - \frac{1}{2}.\theta_{nn'}^2$, respectively. Then, the power $F_{nn'}$ flowing in the line connecting node n to node n' and the loss $L_{nn'}$ along that line are

$$F_{nn'}(\theta_{nn'}) \approx B_{nn'}.\theta_{nn'} + \frac{1}{2}.G_{nn'}.\theta_{nn'}^2, \tag{2}$$

$$L_{nn'}(\theta_{nn'}) \approx G_{nn'}.\theta_{nn'}^2, \tag{3}$$

respectively. [3]

2.2 Model's Information Structure and Key Assumptions

The energy market under consideration consists of independent agents or market participants and an ISO (she). Let $\mathcal{S}$ denote the set of independent agents or market participants. We assume that all agents are strategic. An agent $s \in \mathcal{S}$ (he) can have generators or flexible demands at different nodes. A node can be *"dense"*, that is, it can be shared by two or more agents. The sets of locations of generators and flexible demands owned by agent s are denoted by $\mathcal{N}_s^{(E)}$ and $\mathcal{N}_s^{(D)}$, respectively. In addition, the sets

[3] As a consequence of definition of $\overline{F}_{nn'}$ we must have $\max_{\theta_{nn'}} F_{nn'}(\theta_{nn'}) \geq \overline{F}_{nn'}$, because $\overline{F}_{nn'} := \min(\text{thermal capacity}, \max_{\theta_{nn'}} F_{nn'}(\theta_{nn'}))$.

of agents sharing node n, by having generators or flexible demands on that node, are denoted by $\Psi_n^{(E)}$ and $\Psi_n^{(D)}$. When agent s produces energy $e_n^{(s)}$ (respectively, consumes energy $d_n^{(s)}$) at node n, he incurs a cost $c_n^{(s)}(e_n^{(s)})$ (respectively, enjoys a utility $u_n^{(s)}(d_n^{(s)})$), where $e_n^{(s)} \in \mathcal{E}_n^{(s)}$ and $d_n^{(s)} \in \mathcal{D}_n^{(s)}$ ($\mathcal{E}_n^{(s)} := [0, \overline{E}_n^{(s)}]$ and $\mathcal{D}_n^{(s)} := [0, \overline{D}_n^{(s)}]$).

Let t_s denote the monetary payment made by agent s to the ISO; t_s is equal to his payment to the energy market minus his payoff in the FTR market. The payment of agent s to the energy market is due to his energy consumption and energy production. The total payment t_s can be either a positive or a negative number; if $t_s > 0$, then agent s pays money, whereas $t_s < 0$ implies that agent s receives money. The total utility of agent s is given by

$$\mathrm{U}_s\left((e_n^{(s)})_{n\in\mathcal{N}_s^{(E)}}, (d_n^{(s)})_{n\in\mathcal{N}_s^{(D)}}, t_s\right) := \sum_{n\in\mathcal{N}_s^{(D)}} u_n^{(s)}(d_n^{(s)}) - \sum_{n\in\mathcal{N}_s^{(E)}} c_n^{(s)}(e_n^{(s)}) - t_s. \tag{4}$$

Each agent s makes strategic decisions to maximize his own utility function U_s. The ISO's objective is to maximize the overall utility of consumption minus the total cost of production. Thus, the ISO's objective is defined by

$$\mathrm{W}\left((e_n^{(s)})_{n\in\mathcal{N}_s^{(E)}}, (d_n^{(s)})_{n\in\mathcal{N}_s^{(D)}}\right) := \sum_{s\in\mathcal{S}}\left(\sum_{n\in\mathcal{N}_s^{(D)}} u_n^{(s)}(d_n^{(s)}) - \sum_{n\in\mathcal{N}_s^{(E)}} c_n^{(s)}(e_n^{(s)})\right). \tag{5}$$

We make the following assumptions:

Assumption 1. *For every $s \in \mathcal{S}$, $n \in \mathcal{N}_s^{(E)}$, the cost function $c_n^{(s)}(e_n^{(s)})$ is continuously differentiable, strictly convex, and strictly increasing. Moreover, $c_n^{(s)}(0) = 0$.*

Assumption 2. *For every $s \in \mathcal{S}$, $n \in \mathcal{N}_s^{(D)}$, the utility function $u_n^{(s)}(d_n^{(s)})$ is continuously differentiable, strictly concave, and strictly increasing. Moreover, $u_n^{(s)}(0) = 0$.*

Assumption 3. *For the network under consideration, for any agent $s \in \mathcal{S}$, any feasible profile of production $(e_n^{(s)} \in \mathcal{E}_n^{(s)})_{n\in\mathcal{N}_s^{(E)}}$ and consumption $(d_n^{(s)} \in \mathcal{D}_n^{(s)})_{n\in\mathcal{N}_s^{(D)}}$ satisfying the network constraints (the network's topology and the lines' thermal capacity and stability constraints) and the physical laws (Kirchhoff's voltage and current laws) can be achieved.*

Assumption 4 (Agent s's information)**.** *Agent s knows:*

- *The functions $u_n^{(s)}(d_n^{(s)}), \forall n \in \mathcal{N}_s^{(D)}$ and $c_n^{(s)}(e_n^{(s)}), \forall n \in \mathcal{N}_s^{(E)}$. These functions are private information of agent s.*
- *The parameters $\overline{D}_n^{(s)}, \forall n \in \mathcal{N}_s^{(D)}$ and $\overline{E}_n^{(s)}, \forall n \in \mathcal{N}_s^{(E)}$.*
- *The sets Δ_s, and Φ_s along with the parameters $G_{nn'}$, $B_{nn'}$, and $\overline{F}_{nn'}$, $\forall n \in (\mathcal{N}_s^{(E)} \cup \mathcal{N}_s^{(D)})$, $\forall n' \in \mathcal{R}_n$.*

Assumption 5 (ISO's information)**.** *The ISO knows:*

- *The parameters* $G_{nn'}$, $B_{nn'}$, *and* $\overline{F}_{nn'}$, $\forall n \in \mathcal{N}, \forall n' \in \mathcal{R}_n$.

- *The parameter* O_n, *which denotes the must-run demand at node* n *in MW (see Section 1.5),* $\forall n \in \mathcal{N}$.

- *The parameters* $\overline{D}_n^{(s)}, \forall s \in \mathcal{S}, \forall n \in \mathcal{N}_s^{(D)}$ *and* $\overline{E}_n^{(s)}, \forall s \in \mathcal{S}, \forall n \in \mathcal{N}_s^{(E)}$.

- *The sets* $\mathcal{R}_n$, $\Psi_n^{(E)}$, $\Psi_n^{(D)}$, Ω_n, *and* $\Upsilon_{nn'}$.

Assumption 6. *There is at least one agent at each node* $n \in \mathcal{N}$. *Furthermore, an agent can own a number of generators or flexible demands at different nodes.* [4]

Assumption 7. *A line connecting two nodes is shared by more than one agent.*

2.3 Objective

Our objective is to design a mechanism (a set of rules) in equilibrium form that determines the interactions among the ISO and the strategic agents (consumers and producers) through the electricity network. Such a design must be organized according to *"bid-based pools with FTRs"*, must take into account the network constraints (the network's topology and the lines' thermal capacity and stability constraints), the physical laws (Kirchhoff's voltage and current laws), the model's information structure (described in Section 2.2), the agents' strategic behavior, and must possess the following additional properties:

(P1) Budget balance;

(P2) Implementation of the social welfare function (i.e., the sum of strategic agents' utilities) in Nash equilibria;

(P3) Price efficiency;

(P4) Individual rationality;

(P5) Off-equilibrium feasibility;

(P6) Finite message space.

In addition, we aim to determine a tâtonnement-process/algorithm that has the following properties: (i) it converges to a Nash equilibrium of the game induced by the mechanism; (ii) the allocations (productions and consumptions) resulting at each step of the process are feasible solutions of the OPF problem.

[4] We relax Assumption 6 in Section 7.

2.4 Explanation/Discussion of the Objective

We explain our above objective by first describing what we mean by a *"mechanism"* and then by describing the meaning of properties (P1)-(P6). Then, we discuss the suitability of Nash equilibrium as a solution/equilibrium concept.

A mechanism $(\mathcal{M}, h)$ consists of two components.

(i) A message space $\mathcal{M} := \prod_{s\in\mathcal{S}} \mathcal{M}_s$, that is, a communication alphabet through which the strategic agents send information to the ISO. $\mathcal{M}_s, s \in \mathcal{S}$, is the message space/communication alphabet of agent s.

(ii) An outcome function $h : \mathcal{M} \to \mathcal{E} \times \mathcal{D} \times \mathbb{R}^{|\mathcal{S}|}$, $\mathcal{E} := \prod_{s\in\mathcal{S}} \prod_{n\in\mathcal{N}_s^{(E)}} \mathcal{E}_n^{(s)}$, $\mathcal{D} := \prod_{s\in\mathcal{S}} \prod_{n\in\mathcal{N}_s^{(D)}} \mathcal{D}_n^{(s)}$, $h(\vec{\mathbf{m}}) := \left((e_n^{(s)})_{n\in\mathcal{N}_s^{(E)}}, (d_n^{(s)})_{n\in\mathcal{N}_s^{(D)}}, t_s\right)_{s\in\mathcal{S}}$, that determines, for each message $\vec{\mathbf{m}} := (\vec{m}_s)_{s\in\mathcal{S}}$, the power production $(e_n^{(s)})_{s\in\mathcal{S}, n\in\mathcal{N}_s^{(E)}}$, power consumption $(d_n^{(s)})_{s\in\mathcal{S}, n\in\mathcal{N}_s^{(D)}}$ along with monetary incentives $(t_s)_{s\in\mathcal{S}}$, that are taxes or subsidies provided to the agents after the message exchange process between the agents and the ISO terminates.

A mechanism induces a game among the agents. We consider Nash equilibrium as the solution concept of this game. At the end of this section, we discuss why Nash equilibrium is an appropriate solution concept.

We now explain the meaning of properties (P1)-(P6) stated above. Property (P1) implies that at equilibrium the mechanism is implementable without any monetary transfers to or from the ISO. That is, at equilibrium, the mechanism does not result in any money surplus or money deficit for the ISO. As a result of property (P1) at all Nash equilibria of the game induced by the mechanism the ISO's objective is equal to the social welfare function. Property (P2) means that the allocations (productions, consumptions) corresponding to all Nash equilibria of the game induced by the mechanism maximize the social welfare function, that is the sum of the consumers' utilities minus the sum of the producers' costs. Property (P3) implies that the price $P_n^{(s)}(\vec{\mathbf{m}}^*)$ agent s at node n is paid (or pays) per unit of power produced (respectively, consumed) at any Nash equilibrium $\vec{\mathbf{m}}^* \in \mathcal{M}$ of the game induced by the mechanism is equal to the sum of his marginal cost of production (respectively, his marginal utility of consumption), the saturation price for his limited production capacity (respectively, his limited consumption capacity), the congestion price of the lines to which he is connected, and the marginal loss price of the lines to which he is connected. Since the agents are strategic, they are interested only in maximizing their utilities (due to their own consumptions) or minimizing their costs (due to their own productions). Therefore, after examining the mechanism (announced by the ISO), they do not necessarily have to participate in the production/consumption process. Property (P4) implies that, after examining the mechanism, each strategic agent (producer or consumer) voluntarily participates in the energy production/consumption process. Property (P5) ensures that the allocations (productions and consumptions) corresponding to any message $\vec{\mathbf{m}} \in \mathcal{M}$ are a feasible solution of the OPF problem. Property (P6) implies that the complexity of the mechanism and the data communication requirements are not high.

A mechanism in equilibrium form does not specify how the Nash equilibria of the game induced by it are determined/computed. For this reason, we need a tâtonnement-process/algorithm that determines the above mentioned Nash equilibria. In practice, any tâtonnement-process stops after a finite number of iterations. For this reason, we require that the allocations (productions and consumptions) corresponding to each step of the process should be a feasible solution of the OPF problem (ISO's problem).

Nash equilibrium is the solution/equilibrium concept used in most studies of electricity networks with strategic agents (see [54, 55, 56, 45] and references therein). References [54, 55, 56] consider electricity markets with strategic agents and symmetric information and argue why symmetric information is a reasonable assumption. Reference [45] considers electricity markets with asymmetric information. In this work, we assume that the agents have asymmetric information; even if the agents monitor each others' technology and capacity they do not know each others' utilities/valuations, thus the game induced by the mechanism is one of asymmetric information where the environment is non-Bayesian. For both of the above information-structures, Nash equilibrium is an appropriate solution concept for the following reason. According to Nash, [57], Nash equilibrium can be interpreted in two ways: (i) as the solution concept/outcome of a game of complete/symmetric information where all agents are rational; (ii) as the result of a game of asymmetric information where strategic agents are involved in an unspecified message exchange process in which they grope their way to a stationary message and in which the Nash equilibrium is a necessary condition for stationarity. This is the so-called *"mass-action"* interpretation of Nash equilibrium (see also [58], page 644). In this paper, we adopt the second interpretation of Nash equilibrium.

2.5 Nature of Our Problem

In electricity networks, the action (energy production, energy consumption) an agent takes at a network node influences directly, due to the network interconnection among agents and the physical laws of power flow, the actions and utilities of agents at that node and the nodes in his immediate neighborhood. Therefore, the network interconnection among agents results in both negative (dispatch in the same direction) and positive (dispatch in the opposite direction) externalities. Furthermore, according to the network's information structure (Section 2.2), each agent knows the network topology and the presence of other agents only in his immediate neighborhood. Consequently, the design of electricity markets is a local public goods problem.

Before we present the mechanism that achieves our objective, we formulate the centralized OPF problem that determines the globally optimal dispatch for the model of Section 2, and define the market of FTRs and its interaction with the real-time energy market in a bid-based pool structure.

3 Structure of Bid-based Pools with FTRs

3.1 The Centralized OPF Problem

The centralized OPF problem the ISO would solve in a monopolistic setup if she knew the agents' utilities and costs functions (their private information) is given below,

$$\max_{\Xi} \mathrm{W} = \sum_{s\in\mathcal{S}} \left(\sum_{n\in\mathcal{N}_s^{(D)}} u_n^{(s)}(d_n^{(s)}) - \sum_{n\in\mathcal{N}_s^{(E)}} c_n^{(s)}(e_n^{(s)}) \right) \qquad (\mathbf{OPF})$$

subject to:

$$-O_n + \sum_{s\in\Psi_n^{(E)}} e_n^{(s)} - \sum_{s\in\Psi_n^{(D)}} d_n^{(s)} \geq \sum_{n'\in\mathcal{R}_n} \left(B_{nn'}.\theta_{nn'} + \frac{1}{2}.G_{nn'}.\theta_{nn'}^2 \right), \qquad \forall n \in \mathcal{N}, \qquad (6)$$

$$0 \leq e_n^{(s)} \leq \overline{E}_n^{(s)}, \qquad \forall s \in \mathcal{S}, \forall n \in \mathcal{N}_s^{(E)}, \qquad (7)$$

$$0 \leq d_n^{(s)} \leq \overline{D}_n^{(s)}, \qquad \forall s \in \mathcal{S}, \forall n \in \mathcal{N}_s^{(D)}, \qquad (8)$$

$$B_{nn'}.\theta_{nn'} + \frac{1}{2}.G_{nn'}.\theta_{nn'}^2 \leq \overline{F}_{nn'}, \qquad \forall n \in \mathcal{N}, \forall n' \in \mathcal{R}_n, \qquad (9)$$

$$\theta_{nn'} + \theta_{n'n} = 0, \qquad \forall n \in \mathcal{N}, \forall n' \in \mathcal{R}_n, \qquad (10)$$

where $\Xi := \left((e_n^{(s)})_{s\in\mathcal{S},n\in\mathcal{N}_s^{(E)}}, (d_n^{(s)})_{s\in\mathcal{S},n\in\mathcal{N}_s^{(D)}}, (\theta_{nn'})_{n\in\mathcal{N},n'\in\mathcal{R}_n} \right)$.

The objective of (**OPF**) represents the ISO's objective function, the sum of the consumers' utility functions minus the sum of the producers' cost functions. Constraint (6) describes the power flow at each network node. Constraints (7) and (8) reflect the fact that each generator or flexible demand has bounds on the amounts of power they are able to produce or consume. The power flow at each transmission line of the electricity network is limited by (9). Constraint (9) is also a stability constraint on the voltage angle differences $\theta_{nn'}$ between nodes n and n' ($n \in \mathcal{N}, n' \in \mathcal{R}_n$). Specifically, the power flow in the line connecting node $n \in \mathcal{N}$ to node $n' \in \mathcal{R}_n$ is a convex function of $\theta_{nn'}$; given the line's capacity $\overline{F}_{nn'}$, (9) determines the limits of $\theta_{nn'}$. Constraint (10) is a consistency condition for voltage angle differences.

Because of Assumptions 1 and 2 and constraints (6)-(10), the above problem (**OPF**) is a strictly concave optimization problem with a convex domain and has a unique solution.

3.2 The FTRs Market

The FTRs market operates separately from the energy market in the following way.

Shares of FTRs are first issued for every line/link in the network by the ISO. The total amount of FTRs shares for link nn' is equal to $\overline{F}_{nn'}$, the link's capacity. After the FTRs are issued by the ISO, they are allocated among the agents according to some auction mechanism that is independent of the mechanism that determines production and consumption in the electricity market. Furthermore, there is a secondary market where agents can bilaterally trade their FTRs. Examples of mechanisms determining allocations of FTRs can be found in [59, 14, 60]. An example of a secondary market for FTRs trading can be found in [61].

As a result of the FTRs auction mechanisms and secondary markets, a strategic agent s owns $\alpha_{nn'}^{(s)}$ MW shares of FTR in the transmission line from node n to node n', where $\alpha_{nn'}^{(s)} \in [0, \overline{F}_{nn'}]$ and $\sum_{s\in\mathcal{S}} \alpha_{nn'}^{(s)} = \overline{F}_{nn'}$ for all transmission lines of the electricity network.

The owner of FTRs can be either a strategic producer or a strategic consumer or someone who is neither a producer nor a consumer. In *"bid-based pools with FTRs"*, owners of FTRs use them for hedging and speculation purposes. Without any loss of generality, in this paper we will assume that owners of FTRs are either strategic producers or strategic consumers. This assumption simplifies the notation in our subsequent analysis.

The electricity market mechanism determines energy production, energy consumption along with prices and payments for energy production, energy consumption, and FTRs (such a mechanism appears in Section 4). The interaction of the market of FTRs with the real-time energy market is shown in Figure 1 below.

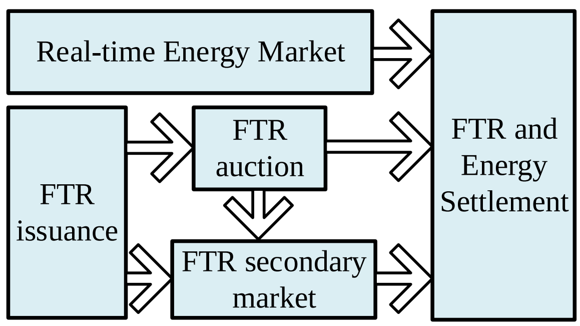

Figure 1: Interaction between FTRs and energy markets.

After energy production and consumption along with prices for production, consumption, and FTRs are determined, the owner s of FTRs at a certain line, say the line nn' connecting node n to node n', receives a payment that depends on the price per unit of FTRs in line nn', the amount $\alpha_{nn'}^{(s)}$ agent s owns in line nn', the losses in line nn', and the congestion in line nn' (see Sections 4 and 5).

4 Proposed Surrogate Optimization-based Mechanism

In this section, we propose a surrogate optimization-based mechanism that achieves the objective stated in Section 2.3. The mechanism is organized according to the structure of *"bid-based pools with FTRs"*.

In the proposed surrogate optimization-based mechanism, first of all, the ISO selects and announces two surrogate functions that replace the actual cost and utility functions. The surrogate functions are $f^{(E)}(y) : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ and $f^{(D)}(y) : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$, where $\mathbb{R}_{\geq 0} := [0, \infty)$.

Assumption 8. *The function $f^{(E)}(y)$ is strictly convex, strictly increasing, and continuously differentiable. Moreover, $f^{(E)}(0) = 0$.*

Assumption 9. *The function $f^{(D)}(y)$ is strictly concave, strictly increasing, and continuously differentiable. Moreover, $f^{(D)}(0) = 0$ and $\frac{\partial f^{(D)}}{\partial y}\Big|_{y=0} < \infty$.*

As stated in Section 2.4, a mechanism consists of a message space $\mathcal{M}$ and an outcome function h that are defined below.

4.1 Message Space

The message vector $\vec{m}_s$ of an agent s who owns a number of generators and flexible demands at different nodes is:

$$\vec{m}_s := (\vec{w}_s, \vec{v}_s, \vec{p}_s, \vec{q}_s)\,, \tag{11}$$

where

- $\vec{m}_s$ is the message vector of agent s in $\mathcal{M}_s \subseteq (\mathbb{R}_{\geq 0}^{3.|\Delta_s|-2.|\mathcal{N}_s^{(E)}|-2.|\mathcal{N}_s^{(D)}|}) \times (\mathbb{R}_{>0}^{|\mathcal{N}_s^{(E)}|+|\mathcal{N}_s^{(D)}|})$, where $\mathbb{R}_{>0} := (0, \infty)$,
- $\vec{w}_s := (w_n^{(s)})_{n \in \mathcal{N}_s^{(E)}}$, where $w_n^{(s)} \in \mathbb{R}_{>0}$ is the weight factor of a generator at node $n \in \mathcal{N}_s^{(E)}$. The surrogate function $w_n^{(s)} \times f^{(E)}(e_n^{(s)})$ is formed to use instead of $c_n^{(s)}(e_n^{(s)})$.
- $\vec{v}_s := (v_n^{(s)})_{n \in \mathcal{N}_s^{(D)}}$, where $v_n^{(s)} \in \mathbb{R}_{>0}$ is the weight factor of a flexible demand at node $n \in \mathcal{N}_s^{(D)}$. The surrogate function $v_n^{(s)} \times f^{(D)}(d_n^{(s)})$ is formed to use instead of $u_n^{(s)}(d_n^{(s)})$.
- $\vec{p}_s := (p_{n'}^{(s)})_{n' \in \Delta_s}$, where $p_{n'}^{(s)} \in \mathbb{R}_{\geq 0}$ is the price proposed by agent s for node n' in his neighborhood.
- $\vec{q}_s := (q_{n'n''}^{(s)})_{(n',n'') \in \Phi_s}$, where $q_{n'n''}^{(s)} \in \mathbb{R}_{\geq 0}$ is the FTR price proposed by agent s for transmission line $n'n''$ in his neighborhood.

It is worth noting that, as $|\Phi_s| = 2 \times |\Delta_s| - 2 \times |\mathcal{N}_s^{(E)}| - 2 \times |\mathcal{N}_s^{(D)}|$, the dimensionality of message space of agent s is $3 \times |\Delta_s|$.

In this study, the vector of messages of all agents is shown with bold letter and it is denoted by $\vec{\mathbf{m}} := (\vec{\mathbf{w}}, \vec{\mathbf{v}}, \vec{\mathbf{p}}, \vec{\mathbf{q}})$.

4.2 Outcome Function

The outcome function consists of two components, an allocation rule and a payment rule. We present each component separately.

4.2.1 Allocation Rule

After receiving the agents' messages $\vec{\mathbf{m}} := (\vec{m}_s)_{s\in\mathcal{S}}$, the ISO forms the surrogate function $\mathrm{W}' := \sum_{s\in\mathcal{S}}\left(\sum_{n\in\mathcal{N}_s^{(D)}} v_n^{(s)}.f^{(D)}(d_n^{(s)}) - \sum_{n\in\mathcal{N}_s^{(E)}} w_n^{(s)}.f^{(E)}(e_n^{(s)})\right)$ (that replaces W in (**OPF**)) and solves an optimization problem described by

$$\max_{\Xi} \mathrm{W}' = \sum_{s\in\mathcal{S}}\left(\sum_{n\in\mathcal{N}_s^{(D)}} v_n^{(s)}.f^{(D)}(d_n^{(s)}) - \sum_{n\in\mathcal{N}_s^{(E)}} w_n^{(s)}.f^{(E)}(e_n^{(s)})\right) \tag{\textbf{Surrogate}}$$

subject to:

$$-O_n + \sum_{s\in\Psi_n^{(E)}} e_n^{(s)} - \sum_{s\in\Psi_n^{(D)}} d_n^{(s)} \geq \sum_{n'\in\mathcal{R}_n}\left(B_{nn'}.\theta_{nn'} + \frac{1}{2}.G_{nn'}.\theta_{nn'}^2\right), \qquad \forall n\in\mathcal{N}, \tag{12}$$

$$0 \leq e_n^{(s)} \leq \overline{E}_n^{(s)}, \qquad \forall s\in\mathcal{S}, \forall n\in\mathcal{N}_s^{(E)}, \tag{13}$$

$$0 \leq d_n^{(s)} \leq \overline{D}_n^{(s)}, \qquad \forall s\in\mathcal{S}, \forall n\in\mathcal{N}_s^{(D)}, \tag{14}$$

$$B_{nn'}.\theta_{nn'} + \frac{1}{2}.G_{nn'}.\theta_{nn'}^2 \leq \overline{F}_{nn'}, \qquad \forall n\in\mathcal{N}, \forall n'\in\mathcal{R}_n, \tag{15}$$

$$\theta_{nn'} + \theta_{n'n} = 0, \qquad \forall n\in\mathcal{N}, \forall n'\in\mathcal{R}_n, \tag{16}$$

$$\sum_{n\in\mathcal{N}}\left(\sum_{s\in\Psi_n^{(E)}} e_n^{(s)} - \sum_{s\in\Psi_n^{(D)}} d_n^{(s)} - O_n - \frac{1}{2}.\sum_{n'\in\mathcal{R}_n} G_{nn'}.\theta_{nn'}^2\right) \geq 0. \tag{17}$$

Because of Assumptions 8 and 9 and constraints (12)-(17), the above problem (**Surrogate**) is a strictly concave optimization problem with a convex domain, thus it has a unique solution which we denote by

$$\hat{e}_n^{(s)}(\vec{\mathbf{m}}) = \hat{e}_n^{(s)}(\vec{\mathbf{w}}, \vec{\mathbf{v}}), \qquad \forall s\in\mathcal{S}, \forall n\in\mathcal{N}_s^{(E)}, \tag{18}$$

$$\hat{d}_n^{(s)}(\vec{\mathbf{m}}) = \hat{d}_n^{(s)}(\vec{\mathbf{w}}, \vec{\mathbf{v}}), \qquad \forall s\in\mathcal{S}, \forall n\in\mathcal{N}_s^{(D)}, \tag{19}$$

$$\hat{\theta}_{nn'}(\vec{\mathbf{m}}) = \hat{\theta}_{nn'}(\vec{\mathbf{w}}, \vec{\mathbf{v}}), \qquad \forall n\in\mathcal{N}, \forall n'\in\mathcal{R}_n, \tag{20}$$

$$\hat{\lambda}_n(\vec{\mathbf{m}}) = \hat{\lambda}_n(\vec{\mathbf{w}}, \vec{\mathbf{v}}), \qquad \forall n\in\mathcal{N}, \tag{21}$$

$$\hat{\mu}_{nn'}(\vec{\mathbf{m}}) = \hat{\mu}_{nn'}(\vec{\mathbf{w}}, \vec{\mathbf{v}}), \qquad \forall n\in\mathcal{N}, \forall n'\in\mathcal{R}_n, \tag{22}$$

$$\hat{L}_{nn'}(\vec{\mathbf{m}}) = G_{nn'}.\left(\hat{\theta}_{nn'}(\vec{\mathbf{w}}, \vec{\mathbf{v}})\right)^2, \qquad \forall n\in\mathcal{N}, \forall n'\in\mathcal{R}_n, \tag{23}$$

$$\hat{\lambda}_{ref}(\vec{\mathbf{m}}) = \hat{\lambda}_{ref}(\vec{\mathbf{w}}, \vec{\mathbf{v}}), \tag{24}$$

where $\hat{\lambda}_n(\vec{\mathbf{w}}, \vec{\mathbf{v}})$, $\hat{\mu}_{nn'}(\vec{\mathbf{w}}, \vec{\mathbf{v}})$, and $\hat{\lambda}_{ref}(\vec{\mathbf{w}}, \vec{\mathbf{v}})$ denote the centroid of the set of Lagrange multipliers associated with the constraints (12), (16), and (17), respectively, and $\hat{L}_{nn'}(\vec{\mathbf{w}}, \vec{\mathbf{v}})$ is the loss in line connecting node n to node n'.

Based on this solution, the ISO allocates a production of $\hat{e}_n^{(s)}(\vec{\mathbf{m}})$ units of energy to agent s at node n, and a consumption of $\hat{d}_n^{(s)}(\vec{\mathbf{m}})$ units of energy to agent s at node n, $s\in\mathcal{S}$, $n\in\mathcal{N}$.

4.2.2 Payment Rule

The payment function of the proposed mechanism consists of two components: energy and FTR payments. It is given by (25)-(27) below,

$$t_s(\vec{\mathbf{m}}) = \sum_{n\in\mathcal{N}_s^{(D)}} P_n^{(s)}(\vec{\mathbf{p}}).\hat{d}_n^{(s)}(\vec{\mathbf{m}}) - \sum_{n\in\mathcal{N}_s^{(E)}} P_n^{(s)}(\vec{\mathbf{p}}).\hat{e}_n^{(s)}(\vec{\mathbf{m}}) - \sum_{n'\in\mathcal{N},n''\in\mathcal{R}_{n'}} Q_{n'n''}^{(s)}(\vec{\mathbf{q}}).\alpha_{n'n''}^{(s)}$$
$$+ \sum_{n'\in\Delta_s}\left(p_{n'}^{(s)} - \hat{\lambda}_{n'}(\vec{\mathbf{w}},\vec{\mathbf{v}}) - \hat{\lambda}_{ref}(\vec{\mathbf{w}},\vec{\mathbf{v}})\right)^2 + \sum_{(n',n'')\in\Phi_s}\left(q_{n'n''}^{(s)} - \hat{\mu}_{n'n''}(\vec{\mathbf{w}},\vec{\mathbf{v}}) - \frac{\hat{\lambda}_{ref}(\vec{\mathbf{w}},\vec{\mathbf{v}}).\hat{L}_{n'n''}(\vec{\mathbf{w}},\vec{\mathbf{v}})}{2.\overline{F}_{n'n''}}\right)^2, \tag{25}$$

$$P_n^{(s)}(\vec{\mathbf{p}}) = \frac{\sum_{s'\in\Omega_n-\{s\}} p_n^{(s')}}{(|\Omega_n|-1)}\ , \tag{26}$$

$$Q_{n'n''}^{(s)}(\vec{\mathbf{q}}) = \frac{\sum_{s'\in\Upsilon_{n'n''}-\{s\}} q_{n'n''}^{(s')}}{(|\Upsilon_{n'n''}|-1)}\ , \tag{27}$$

The first, second, and fourth terms on the right hand side (RHS) of (25) represent payments due to energy production and consumption. The third and fifth terms on the RHS of (25) represent FTR payments.

We now proceed to interpret the various components of the proposed mechanism.

4.3 Interpretation of the Mechanism

4.3.1 Interpretation of the Message Space

In the proposed mechanism, a typical message of all the agents consist of two parts; weight factors $(\vec{\mathbf{w}},\vec{\mathbf{v}})$ and prices $(\vec{\mathbf{p}},\vec{\mathbf{q}})$.

The weight factors $(\vec{\mathbf{w}},\vec{\mathbf{v}})$ represent the agents' bids for energy production and consumption. Note that agents announce finite-dimensional vectors instead of utilities and cost functions (that belong to an infinite dimensional message space). Thus, the mechanism's space, the space where a typical message $\vec{\mathbf{m}}$ belongs, is finite-dimensional, in contrast to direct revelation mechanisms (such as VCG), which have infinite dimensional message spaces. As shown in Section 5.3, when the proposed mechanism implements the ISO's objective function in Nash equilibrium, the following equations (28)-(29) are satisfied at all Nash equilibria $\vec{\mathbf{m}}^* := (\vec{\mathbf{w}}^*,\vec{\mathbf{v}}^*,\vec{\mathbf{p}}^*,\vec{\mathbf{q}}^*)$ of the game induced by it.

$$\left.\frac{\partial c_n^{(s)}}{\partial e_n^{(s)}}\right|_{(e_n^{(s)})^*} = (w_n^{(s)})^*.\left.\frac{\partial f^{(E)}}{\partial y}\right|_{y=(e_n^{(s)})^*}, \qquad \forall s\in\mathcal{S},\forall n\in\mathcal{N}_s^{(E)}, \tag{28}$$

$$\left.\frac{\partial u_n^{(s)}}{\partial d_n^{(s)}}\right|_{(d_n^{(s)})^*} = (v_n^{(s)})^*.\left.\frac{\partial f^{(D)}}{\partial y}\right|_{y=(d_n^{(s)})^*}, \qquad \forall s\in\mathcal{S},\forall n\in\mathcal{N}_s^{(D)}, \tag{29}$$

where $\left((e_n^{(s)})^*\right)_{s\in\mathcal{S},n\in\mathcal{N}_s^{(E)}}$ and $\left((d_n^{(s)})^*\right)_{s\in\mathcal{S},n\in\mathcal{N}_s^{(D)}}$ are the unique optimal solution of problem (**OPF**).

Thus, the only information revealed by the agents is their marginal costs and their marginal utilities at the Nash equilibria of the game induced by the mechanism.

Each agent s announces prices $(\vec{p}_s, \vec{q}_s)$ for energy and FTRs not only for his nodes and the transmission lines connected to them but also for his neighboring nodes. This happens because each agent is dealing with a local public goods problem (see discussion in Section 2.5).

4.3.2 Interpretation of the ISO's Surrogate Optimization Problem

In the ISO's optimization problem (**Surrogate**), constraint (17) is implied by (12). Nevertheless, we include (17) in the ISO's optimization problem because in this way we introduce the reference energy price (REP) $\hat{\lambda}_{ref}(\vec{\mathbf{w}}, \vec{\mathbf{v}})$, which is used to calculate FTR prices; $\hat{\lambda}_{ref}(\vec{\mathbf{w}}, \vec{\mathbf{v}})$ is also a component of the locational marginal prices (LMPs) $\hat{\lambda}_{ref}(\vec{\mathbf{w}}, \vec{\mathbf{v}}) + \hat{\lambda}_n(\vec{\mathbf{w}}, \vec{\mathbf{v}})$, $n \in \mathcal{N}$.

The concept of nodal spot pricing was introduced by Schweppe et al. [62]. Under nodal pricing, the LMP is generally composed of three components: the REP, the marginal congestion component (MCC), and the marginal loss component (MLC). It has been proven, using the envelope theorem [63], that the LMP at node $n \in \mathcal{N}$ is equal to $\hat{\lambda}_{ref}(\vec{\mathbf{w}}, \vec{\mathbf{v}}) + \hat{\lambda}_n(\vec{\mathbf{w}}, \vec{\mathbf{v}})$, $n \in \mathcal{N}$. Therefore, $\hat{\lambda}_n(\vec{\mathbf{w}}, \vec{\mathbf{v}})$ represents the sum of MCC and MLC of price at node $n \in \mathcal{N}$. The design of nodal pricing explicitly acknowledges that the location of a generator or flexible demand should reflect the marginal price the generator receives or the demand pays; all generators (respectively, flexible demands) in node $n \in \mathcal{N}$ must be paid (respectively, pay) the same marginal price. As noted in Section 2.1, the slack node, also known as reference node, is not predetermined. This does not contradict the determination of $\hat{\lambda}_{ref}(\vec{\mathbf{w}}, \vec{\mathbf{v}})$ for the following reason. The REP $\hat{\lambda}_{ref}(\vec{\mathbf{w}}, \vec{\mathbf{v}})$ is equal to the Lagrange multiplier associated with constraint (17), which describes the power balance in the network (i.e., the difference between production and consumption is equal to the losses), and can be determined without selecting any specific node as the slack node in the electricity network.

4.3.3 Interpretation of the Payment Rule

The price a producer (respectively, a consumer) at node n gets paid per unit of energy of he produces (respectively, pays per unit of energy he consumes) does not depend on his message (cf. (26)). Similarly, the price per unit of FTRs agent s receives in the line connecting node n' to node n'' does not depend on his message/proposal (cf. (27)). Such a pricing/payment scheme/rule induces price-taking behavior among the strategic agents.

The term $\sum_{n' \in \Delta_s} \left(p_{n'}^{(s)} - \hat{\lambda}_{n'}(\vec{\mathbf{w}}, \vec{\mathbf{v}}) - \hat{\lambda}_{ref}(\vec{\mathbf{w}}, \vec{\mathbf{v}}) \right)^2$ incentivizes agent, $s \in \mathcal{S}$, to bid at any node n', where his generators and flexible demands are located, a price $p_{n'}^{(s)}$ per unit of energy that is equal to the LMP at node n'. At equilibrium, this term is equal to zero (see Section 5). Thus, at equilibrium each producer (respectively, consumer) at node n' gets paid per unit of production (respectively, pays per unit of consumption) the LMP at node n'.

Consider the term $\sum_{(n',n'') \in \Phi_s} \left(q_{n'n''}^{(s)} - \hat{\mu}_{n'n''}(\vec{\mathbf{w}}, \vec{\mathbf{v}}) - \frac{\hat{\lambda}_{ref}(\vec{\mathbf{w}}, \vec{\mathbf{v}}).\hat{L}_{n'}(\vec{\mathbf{w}}, \vec{\mathbf{v}})}{2.\overline{F}_{nn'}} \right)^2$ in the payment function

(25). The term $\hat{\mu}_{n'n''}(\vec{\mathbf{w}}, \vec{\mathbf{v}})$ represents the congestion shadow prices for line $n'n''$; the term $\frac{\hat{\lambda}_{ref}(\vec{\mathbf{w}},\vec{\mathbf{v}}).\hat{L}_{n'}(\vec{\mathbf{w}},\vec{\mathbf{v}})}{2.\overline{F}_{nn'}}$ represents the price for losses in line $n'n''$. Therefore, the term $\sum_{(n',n'')\in\Phi_s}\left(q^{(s)}_{n'n''} - \hat{\mu}_{n'n''}(\vec{\mathbf{w}}, \vec{\mathbf{v}}) - \frac{\hat{\lambda}_{ref}(\vec{\mathbf{w}},\vec{\mathbf{v}}).\hat{L}_{n'}(\vec{\mathbf{w}},\vec{\mathbf{v}})}{2.\overline{F}_{nn'}}\right)^2$ incentivizes agent s, who owns FTRs in line $n'n''$, to bid a price $q^{(s)}_{n'n''}$ that is equal to congestion shadow price and losses price for line $n'n''$. At equilibrium, this term is equal to zero (see Section 5).

The fact that the terms $\sum_{(n',n'')\in\Phi_s}\left(q^{(s)}_{n'n''} - \hat{\mu}_{n'n''}(\vec{\mathbf{w}}, \vec{\mathbf{v}}) - \frac{\hat{\lambda}_{ref}(\vec{\mathbf{w}},\vec{\mathbf{v}}).\hat{L}_{n'}(\vec{\mathbf{w}},\vec{\mathbf{v}})}{2.\overline{F}_{nn'}}\right)^2$ and $\sum_{n'\in\Delta_s}\left(p^{(s)}_{n'} - \hat{\lambda}_{n'}(\vec{\mathbf{w}}, \vec{\mathbf{v}}) - \hat{\lambda}_{ref}(\vec{\mathbf{w}}, \vec{\mathbf{v}})\right)^2$ are zero at any Nash equilibrium $\vec{\mathbf{m}}^* := (\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*, \vec{\mathbf{p}}^*, \vec{\mathbf{q}}^*)$ of the game induced by the proposed mechanism along with (26) and (27), establish that at any Nash equilibrium $\vec{\mathbf{m}}^*$ the payment of agent s is given by

$$t_s(\vec{\mathbf{m}}^*) = \sum_{n\in\mathcal{N}_s^{(D)}} \left(\hat{\lambda}_n(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*) + \hat{\lambda}_{ref}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*)\right).\hat{d}^{(s)}_n(\vec{\mathbf{m}}^*) - \sum_{n\in\mathcal{N}_s^{(E)}} \left(\hat{\lambda}_n(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*) + \hat{\lambda}_{ref}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*)\right).\hat{e}^{(s)}_n(\vec{\mathbf{m}}^*)$$
$$- \sum_{n'\in\mathcal{N},n''\in\mathcal{R}_{n'}} \left(\hat{\mu}_{n'n''}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*) + \frac{\hat{\lambda}_{ref}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*).\hat{L}_{n'n''}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*)}{2.\overline{F}_{n'n''}}\right).\alpha^{(s)}_{n'n''}. \tag{30}$$

Thus, at equilibrium the price per unit of energy at node n' is equal to the LMP at node n', and the price per unit of FTRs in line $n'n''$ is equal to the congestion shadow price for line $n'n''$ plus the price for losses in line $n'n''$.

Remark 1. *We note that the message space of our mechanism has a higher dimension than the mechanism proposed in [45]. The mechanism of [45] has the minimum dimension message space among all mechanisms in equilibrium form. However, as pointed out in [64, 65, 66], there is no tâtonnement-process with a message space that has the minimum dimension of a mechanism in equilibrium form. Our choice of message space allows us to discover a tâtonnement-process which, under certain conditions, converges to one of the Nash equilibria of the game induced by our mechanism (see Section 6).*

5 Properties of Proposed Surrogate Optimization-based Mechanism

In this section, we prove that the proposed surrogate optimization-based mechanism possesses properties (P1)-(P6). By its definition, the mechanism has finite message space. Thus, property (P6) is satisfied. We establish the remaining properties.

5.1 Preliminary Results

Lemma 5.1. *Consider agent s and a fixed message $\vec{\mathbf{m}}_{-s}$ of all agents other than s; then there exists a message $\vec{m}_s$ for agent s such that any admissible production vector $(\tilde{e}^{(s)}_n)_{n\in\mathcal{N}_s^{(E)}}$ and any admissible consumption vector $(\tilde{d}^{(s)}_n)_{n\in\mathcal{N}_s^{(D)}}$ are possible to achieve by the allocation rule* (18)-(19).

The proof of this Lemma is presented in Appendix B.

Lemma 5.2 (Nash equilibrium existence). *The set of Nash equilibria of the game induced by the proposed mechanism is non-empty.*

The proof of this Lemma is presented in Appendix C.

Lemma 5.3. *Let* $\vec{\mathbf{m}}^* := (\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*, \vec{\mathbf{p}}^*, \vec{\mathbf{q}}^*)$ *be a Nash equilibrium of the game induced by the proposed mechanism. Then, for any agent* s*, the price proposal vectors* $\vec{p}_s^* := \left((p_{n'}^{(s)})^*\right)_{n' \in \Delta_s}$ *and* $\vec{q}_s^* := \left((q_{n'n''}^{(s)})^*\right)_{(n',n'') \in \Phi_s}$ *are*

$$(p_{n'}^{(s)})^* = P_{n'}^{(s)}(\vec{\mathbf{p}}^*) = \hat{\lambda}_{n'}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*) + \hat{\lambda}_{ref}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*), \qquad \forall s \in \mathcal{S}, \forall n' \in \Delta_s, \tag{31}$$

$$(q_{n'n''}^{(s)})^* = Q_{n'n''}^{(s)}(\vec{\mathbf{q}}^*) = \hat{\mu}_{n'n''}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*) + \frac{\hat{\lambda}_{ref}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*).\hat{L}_{n'n''}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*)}{2.\overline{F}_{n'n''}}, \qquad \forall s \in \mathcal{S}, \forall (n', n'') \in \Phi_s. \tag{32}$$

The proof of this Lemma is given in Appendix D.

As pointed earlier, the proposed mechanism induces price-taking behavior by strategic agents as each agent pays (or receives) the average of prices proposed by other agents. At a Nash equilibrium, the energy prices for agents are all the same as indicated in (31). At a Nash equilibrium, the same property for prices of FTRs is established by (32).

5.2 Budget Balance at Nash Equilibria

The budget balance property is established through Theorem 1.

Theorem 1 (Budget balance at Nash equilibria). *Consider any Nash equilibrium* $\vec{\mathbf{m}}^*$ *of the game induced by the proposed mechanism. Then, the proposed mechanism is budget-balanced at* $\vec{\mathbf{m}}^*$*.*

The proof of Theorem 1 is presented in Appendix E.

5.3 Implementation in Nash Equilibria

Theorem 2 (Implementation in Nash equilibria). *The allocations corresponding to all Nash equilibria of the game induced by the proposed mechanism maximize the social welfare function.*

The proof of Theorem 2 appears in Appendix F.

We note that $\left(\hat{\lambda}_n(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*)\right)_{n \in \mathcal{N}}$ captures the capacity constraints and losses of the transmission lines. Thus, production vector $\left(\hat{e}_n^{(s)}(\vec{\mathbf{m}}^*)\right)_{s \in \mathcal{S}, n \in \mathcal{N}_s^{(E)}}$ and consumption vector $\left(\hat{d}_n^{(s)}(\vec{\mathbf{m}}^*)\right)_{s \in \mathcal{S}, n \in \mathcal{N}_s^{(D)}}$ at any Nash equilibrium $\vec{\mathbf{m}}^*$ are selected so that the capacities and losses of transmission lines are taken into the account.

Remark 2. *There are many Nash equilibrium messages* $\vec{\mathbf{m}}^* = (\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*, \vec{\mathbf{p}}^*, \vec{\mathbf{q}}^*)$*. For a fixed set of Lagrange multipliers that are a solution of problem* (**Surrogate**)*, and under the condition that* $\hat{\lambda}_n(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*)$*,* $\hat{\mu}_{nn'}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*)$*, and* $\hat{\lambda}_{ref}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*)$ *are defined by the centroid of the set of Lagrange multipliers, distinct equilibrium messages differ only in the first two components* $\vec{\mathbf{w}}^*$ *and* $\vec{\mathbf{v}}^*$*. For each equilibrium, there is unique* *allocation* $\left(\hat{e}_n^{(s)}(\vec{\mathbf{m}}^*)\right)_{s\in\mathcal{S},n\in\mathcal{N}_s^{(E)}}$ *and* $\left(\hat{d}_n^{(s)}(\vec{\mathbf{m}}^*)\right)_{s\in\mathcal{S},n\in\mathcal{N}_s^{(D)}}$ *and corresponding prices* $\vec{\mathbf{p}}^*$ *and* $\vec{\mathbf{q}}^*$. *Furthermore, for any two different equilibria* $\vec{\mathbf{m}'}^*$ *and* $\vec{\mathbf{m}}^*$ *we have* $\left(\hat{e}_n^{(s)}(\vec{\mathbf{m}'}^*)\right)_{s\in\mathcal{S},n\in\mathcal{N}_s^{(E)}} = \left(\hat{e}_n^{(s)}(\vec{\mathbf{m}}^*)\right)_{s\in\mathcal{S},n\in\mathcal{N}_s^{(E)}}$, $\left(\hat{d}_n^{(s)}(\vec{\mathbf{m}'}^*)\right)_{s\in\mathcal{S},n\in\mathcal{N}_s^{(D)}} = \left(\hat{d}_n^{(s)}(\vec{\mathbf{m}}^*)\right)_{s\in\mathcal{S},n\in\mathcal{N}_s^{(D)}}$, $\vec{\mathbf{p}'}^* = \vec{\mathbf{p}}^*$ *and* $\vec{\mathbf{q}'}^* = \vec{\mathbf{q}}^*$. *Therefore, under the above assumptions, each Nash equilibrium uniquely defined a corresponding total utility for each strategic agent.*

Remark 3. *In problem* (**Surrogate**)*, the set of Lagrange multipliers is not uniquely defined. Therefore, for different sets of Lagrange multipliers that are part of the solution of problem* (**Surrogate**) *we have different equilibrium price vectors* $\vec{\mathbf{p}}^*$ *and* $\vec{\mathbf{q}}^*$ *(the equilibrium allocation remains the same). As a result, we may have different total utilities for the agents, and different distribution of the social welfare among agents.*

Remark 4. *In our problem formulation, we did not take into account the agents' initial endowments. If we consider initial endowments* $\psi_s, s \in \mathcal{S}$, *such that* $\sum_{s\in\mathcal{S}} \psi_s = \hat{\psi}$, *that is if we assume that agent s's total utility is* $U_s(.) := \sum_{n\in\mathcal{N}_s^{(D)}} u_n^{(s)}(d_n^{(s)}) - \sum_{n\in\mathcal{N}_s^{(E)}} c_n^{(s)}(e_n^{(s)}) - t_s + \psi_s$, *then for any fixed vector* $(\psi_s)_{s\in\mathcal{S}}$ *of endowments our proposed mechanism yields one Pareto optimal level of total utilities* $(U_s^*)_{s\in\mathcal{S}}$. *By varying the agents' initial endowments (without changing their total sum* $\hat{\psi}$*) we can achieve, based on the second welfare theorem [], all points in the Pareto frontier.*

5.4 Price Efficiency

Price efficiency is established through Theorem 3.

Theorem 3 (Price efficiency)**.** *At any Nash equilibrium* $\vec{\mathbf{m}}^*$ *of the game induced by the proposed mechanism, the price that agent s at node n receives per unit of energy production is equal to the price that he pays per unit of energy consumption. This price is equal to* $\hat{\lambda}_{ref}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*) + \hat{\lambda}_n(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*)$, *where* $\hat{\lambda}_{ref}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*)$ *is the REP and* $\hat{\lambda}_n(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*)$ *represents the sum of MCC and MLC of price at node n. Moreover, this price is equal to the price the participants (producers or consumers) would pay or get paid in the situation where the ISO has centralized information and solves the problem* (**OPF**)*.*

The proof of Theorem 3 appears in Appendix G.

5.5 Individual Rationality

We can have different definitions for individual rationality property. As stated in [41], a mechanism is said to be individually rational *"if no participant would lose profit at the equilibrium"*. This condition is needed because participants have an outside option to quit and not participate in the game induced by the mechanism. In this case their profit, termed *"reservation utility"*, is equal to zero. This definition of individual rationality is provided for excludable public goods.

As stated in [29] for non-excludable public goods, a mechanism is individually rational if the net reward/gain of any agent at any equilibrium of the game induced by the mechanism is greater than the net reward he receives when he unilaterally opts out of the game but still benefits from the public goods.

In what follows, we first establish individual rationality for excludable public goods (Theorem 4 below). Then, we discuss how individual rationality can be established for non-excludable public goods (Theorem 5 below).

5.5.1 Individual Rationality for Excludable Public Goods

The authors of [41, 46] claim that the electricity market problem is a public goods problem with excludable public goods. A producer or a consumer can turn off his generators or flexible demands and get zero profit. This claim is correct when we assume that this option (turning off the generator or flexible demand) is feasible. By considering such an assumption, we can adopt the first definition of individual rationality when the reservation utility is zero.

If turning off is not a feasible option, then an agent, say agent s', has to provide a minimum amount of power $\left(\underline{e}_n^{(s')}\right)_{n\in\mathcal{N}_{s'}^{(E)}}$ from his generators, as there is fixed consumption O_n in (12) and (17). [5] The production of $\left(\underline{e}_n^{(s')}\right)_{n\in\mathcal{N}_{s'}^{(E)}}$ costs $c_n^{(s')}(\underline{e}_n^{(s')}), \forall n \in \mathcal{N}_{s'}^{(E)}$. Moreover, agent s' can turn off his demand and make $\left(\underline{d}_n^{(s')}\right)_{n\in\mathcal{N}_{s'}^{(D)}}$ equal to zero. In this case, a feasible solution for the optimization problem (**Surrogate**) exists as (12) and (17) are satisfied. Furthermore, since agent s' does not participate in the mechanism he incurs a negative profit (the cost $\sum_{n\in\mathcal{N}_{s'}^{(E)}} c_n^{(s')}(\underline{e}_n^{(s')})$).

From the above discussion we conclude that individual rationality for excludable public goods can be established if we prove that at every Nash equilibrium the profit of a participating agent is non-negative. We prove the following result.

Theorem 4 (Individual rationality for excludable public goods)**.** *The proposed mechanism is individually rational for excludable public goods. That is, at each Nash equilibrium of the game induced by the mechanism, each agent's profit is non-negative, i.e., it is greater than his reservation utility.*

The proof of this theorem is presented in Appendix H.

5.5.2 Individual Rationality for Non-Excludable Public Goods

We consider the definition of individual rationality for non-excludable public goods ([29]) and proceed as follows. Generators can sell their capacity to other markets, e.g., a reserve market, with a fixed price λ'. Note that in this case they do not incur any production cost. Flexible demands can also opt out and participate in demand response markets. They sell their capacity amounts of their flexible demands with fixed price λ''. However, they do not gain any utility as they do not consume anything. Therefore, the

[5] In this case, Assumption 3 is not satisfied and is replaced by the following Assumption 3$'$: For the network under consideration, for any agent $s' \in \mathcal{S}$, any feasible profile of production $(e_n^{(s')} \in [\underline{e}_n^{(s')}, \overline{E}_n^{(s')}])_{n\in\mathcal{N}_{s'}^{(E)}}$ and consumption $(d_n^{(s')} \in \mathcal{D}_n^{(s')})_{n\in\mathcal{N}_{s'}^{(D)}}$ satisfying the network constraints (the network's topology and the lines' thermal capacity and stability constraints) and the physical laws (Kirchhoff's voltage and current laws) can be achieved.

individual rationality constraint for a producer and a consumer becomes

$$\left(\hat{\lambda}_n(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*) + \hat{\lambda}_{ref}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*)\right).\hat{e}_n^{(s)}(\vec{\mathbf{m}}^*) - c_n^{(s)}\left(\hat{e}_n^{(s)}(\vec{\mathbf{m}}^*)\right) \geq \lambda'.\hat{e}_n^{(s)}(\vec{\mathbf{m}}^*), \qquad \forall s \in \mathcal{S}, \forall n \in \mathcal{N}_s^{(E)}, \tag{33}$$

$$u_n^{(s)}\left(\hat{d}_n^{(s)}(\vec{\mathbf{m}}^*)\right) - \left(\hat{\lambda}_n(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*) + \hat{\lambda}_{ref}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*)\right).\hat{d}_n^{(s)}(\vec{\mathbf{m}}^*) \geq \lambda''.\hat{d}_n^{(s)}(\vec{\mathbf{m}}^*), \qquad \forall s \in \mathcal{S}, \forall n \in \mathcal{N}_s^{(D)}. \tag{34}$$

Based on the above discussion, we can modify the cost and utility functions of generators and flexible demands, respectively, as in (35)-(36) below,

$$(c_n^{(s)}(e_n^{(s)}))^{(ext)} := c_n^{(s)}(e_n^{(s)}) + \lambda'.e_n^{(s)}, \qquad \forall s \in \mathcal{S}, \forall n \in \mathcal{N}_s^{(E)}, \tag{35}$$

$$(u_n^{(s)}(d_n^{(s)}))^{(ext)} := u_n^{(s)}(d_n^{(s)}) - \lambda''.d_n^{(s)}, \qquad \forall s \in \mathcal{S}, \forall n \in \mathcal{N}_s^{(D)}. \tag{36}$$

The first and second terms on the RHS of (35) are the operational cost of production and the opportunity cost, respectively. The first and second terms on the RHS of (36) are the total utility and the opportunity cost of consumption, respectively. It is worth mentioning that $(c_n^{(s)}(e_n^{(s)}))^{(ext)}$ and $(u_n^{(s)}(d_n^{(s)}))^{(ext)}$ remain strictly convex and concave, respectively. However, λ'' must be properly selected so that $(u_n^{(s)}(d_n^{(s)}))^{(ext)}$ is strictly increasing, $\forall d_n^{(s)} \in \mathcal{D}_n^{(s)}$ ($(c_n^{(s)}(e_n^{(s)}))^{(ext)}$ is increasing for all $\lambda' \in \mathbb{R}_{\geq 0}$). Then, Assumptions 1 and 2 will be satisfied. To have $(u_n^{(s)}(d_n^{(s)}))^{(ext)}$ strictly increasing, λ'' must be selected less than $\left.\frac{\partial u_n^{(s)}}{\partial d_n^{(s)}}\right|_{\hat{d}_n^{(s)}}$ for any feasible $\hat{d}_n^{(s)}$, $\forall s \in \mathcal{S}$, $\forall n \in \mathcal{N}_s^{(D)}$.

By considering (35)-(36) for the cost and utility functions, respectively, individual rationality for non-excludable public goods can be established if we prove that at every Nash equilibrium, the profit of each participating agent is non-negative. We prove the following result.

Theorem 5 (Individual rationality for non-excludable public goods)**.** *The proposed mechanism for non-excludable public goods is individually rational. That is, at each Nash equilibrium of the game induced by the mechanism, each agent's profit is greater than his reservation utility, i.e., the profits he gets from other markets.*

With the choice of λ'' indicated above, the proof of Theorem 5 can be established by the same argument as Theorem 4. Furthermore, other properties such as budget balance, implementation in Nash equilibria, and price efficiency can be established, by arguments similar to those used in Theorems 1-3, when the cost and utility functions are given by (35) and (36), respectively, and λ'' is chosen as indicated above.

5.6 Off-Equilibrium Feasibility

Theorem 6. *For any message* $\vec{\mathbf{m}} := (\vec{\mathbf{w}}, \vec{\mathbf{v}}, \vec{\mathbf{p}}, \vec{\mathbf{q}})$*, the allocation resulting from the solution of the problem (**Surrogate**) is a feasible solution of the problem (**OPF**).*

The proof of this theorem is straightforward. Any allocation resulting from the solution of the problem (**Surrogate**), for any message $\vec{\mathbf{m}} := (\vec{\mathbf{w}}, \vec{\mathbf{v}}, \vec{\mathbf{p}}, \vec{\mathbf{q}})$, satisfies constraints (12)-(17). Note that constraint (17) is redundant, as it can be derived by the sum of constraints (12) for all $n \in \mathcal{N}$. Then, constraints (12)-(16)

are equivalent to the constraints (6)-(10). Therefore, for any message $\vec{\mathbf{m}} := (\vec{\mathbf{w}}, \vec{\mathbf{v}}, \vec{\mathbf{p}}, \vec{\mathbf{q}})$, the allocation of the proposed mechanism is a feasible solution of the problem (**OPF**).

This importance of an off-equilibrium feasibility property is discussed in Section 6. Note that off-equilibrium feasibility is not established in the mechanisms proposed in [19], [67].

6 A Best-Estimate Algorithm/Tâtonnement-Process

A tâtonnement-process is defined by an update rule, $\langle\delta\rangle : \prod_{s\in\mathcal{S}} \mathcal{M}_s \to \prod_{s\in\mathcal{S}} \mathcal{M}_s$, that specifies a new message $\vec{\mathbf{m}} \in \prod_{s\in\mathcal{S}} \mathcal{M}_s$ for every previously announced message $\vec{\mathbf{m}} \in \prod_{s\in\mathcal{S}} \mathcal{M}_s$. A fixed point of $\langle\delta\rangle$ (if it exists) is the equilibrium message profile. In this section, we present a best-estimate tâtonnement-process and prove that it converges to a Nash equilibrium of the game induced by the mechanism presented in Section 4. The tâtonnement-process works as follows: At iteration τ, the ISO solves the optimization problem (**Surrogate**) and communicates to each strategic agent, s, the solution of the problem along with all Lagrange multipliers associated with the nodes in Δ_s and transmission lines in Φ_s. Then, the update at iteration $\tau+1$ is performed by each agent, s, according to (37)-(40) below,

$$(w_n^{(s)})^{(\tau+1)} = \frac{\left.\frac{\partial c_n^{(s)}}{\partial e_n^{(s)}}\right|_{(\hat{e}_n^{(s)})^{(\tau)}}}{\left.\frac{\partial f^{(E)}}{\partial y}\right|_{y=(\hat{e}_n^{(s)})^{(\tau)}}}, \qquad \forall s\in\mathcal{S}, \forall n \in \mathcal{N}_s^{(E)}, \tag{37}$$

$$(v_n^{(s)})^{(\tau+1)} = \frac{\left.\frac{\partial u_n^{(s)}}{\partial d_n^{(s)}}\right|_{(\hat{d}_n^{(s)})^{(\tau)}}}{\left.\frac{\partial f^{(D)}}{\partial y}\right|_{y=(\hat{d}_n^{(s)})^{(\tau)}}}, \qquad \forall s\in\mathcal{S}, \forall n \in \mathcal{N}_s^{(D)}, \tag{38}$$

$$(p_{n'}^{(s)})^{(\tau+1)} = (\hat{\lambda}_{n'})^{(\tau)} + (\hat{\lambda}_{ref})^{(\tau)}, \qquad \forall s\in\mathcal{S}, \forall n' \in \Delta_s, \tag{39}$$

$$(q_{nn'}^{(s)})^{(\tau+1)} = (\hat{\mu}_{nn'})^{(\tau)} + \frac{(\hat{L}_{nn'})^{(\tau)}.(\hat{\lambda}_{ref})^{(\tau)}}{2.\overline{F}_{nn'}}, \qquad \forall s\in\mathcal{S}, \forall (n,n') \in \Phi_s. \tag{40}$$

Equations (37)-(40) show that the update by agent s is based on the information he receives from the ISO and his private information (i.e., utility functions of his demands and cost functions of his generators).

We study convergence of the tâtonnement-process described by (37)-(40) by choosing

$$f^{(E)}(y) = \exp\left(\frac{y}{\gamma_e}\right) - 1, \tag{41}$$

$$f^{(D)}(y) = \log\left(\frac{y}{\gamma_d} + 1\right), \tag{42}$$

as surrogate cost and utility functions, respectively. Note that $f^{(E)}(y)$ and $f^{(D)}(y)$ satisfy Assumptions 8 and 9, respectively. With the choice of $f^{(E)}(y)$ and $f^{(D)}(y)$, we obtain the following result.

Theorem 7 (Convergence of proposed tâtonnement-process)**.** *Choose* (41) *and* (42) *as surrogate cost and utility functions, respectively, define*

$$\gamma_e \coloneqq \max_{(s\in\mathcal{S},n\in\mathcal{N}_s^{(E)})} \max_{e_n^{(s)}\in\mathcal{E}_n^{(s)}} \max\left(\frac{\left.\frac{\partial c_n^{(s)}}{\partial e_n^{(s)}}\right|_{e_n^{(s)}}}{\left.\frac{\partial^2 c_n^{(s)}}{\partial\left(e_n^{(s)}\right)^2}\right|_{e_n^{(s)}}}, \frac{\left.\frac{\partial^2 c_n^{(s)}}{\partial\left(e_n^{(s)}\right)^2}\right|_{e_n^{(s)}}}{\left|\left.\frac{\partial^3 c_n^{(s)}}{\partial\left(e_n^{(s)}\right)^3}\right|_{e_n^{(s)}}\right|}\right), \tag{43}$$

$$\gamma_d \coloneqq \min_{(s\in\mathcal{S},n\in\mathcal{N}_s^{(D)})} \min_{d_n^{(s)}\in\mathcal{D}_n^{(s)}} \min\left(\frac{\left.\frac{\partial u_n^{(s)}}{\partial d_n^{(s)}}\right|_{d_n^{(s)}}}{\left|\left.\frac{\partial^2 u_n^{(s)}}{\partial\left(d_n^{(s)}\right)^2}\right|_{d_n^{(s)}}\right|} - d_n^{(s)}, \frac{2.\left|\left.\frac{\partial^2 u_n^{(s)}}{\partial\left(d_n^{(s)}\right)^2}\right|_{d_n^{(s)}}\right|}{\left|\left.\frac{\partial^3 u_n^{(s)}}{\partial\left(d_n^{(s)}\right)^3}\right|_{d_n^{(s)}}\right|} - d_n^{(s)}\right), \tag{44}$$

and assume that the conditions (45)-(46) below are satisfied.

$$\left|\gamma_e \left.\frac{\partial^2 c_n^{(s)}}{\partial(e_n^{(s)})^2}\right|_0 - \left.\frac{\partial c_n^{(s)}}{\partial e_n^{(s)}}\right|_0\right| < \frac{1}{T_1.EI_{(n,s)} + T_2.DI_{(n,s)}}, \qquad \forall s\in\mathcal{S}, \forall n\in\mathcal{N}_s^{(E)}, \tag{45}$$

$$\left|\gamma_d \left.\frac{\partial^2 u_n^{(s)}}{\partial(d_n^{(s)})^2}\right|_0 + \left.\frac{\partial u_n^{(s)}}{\partial d_n^{(s)}}\right|_0\right| < \frac{1}{T_3.EI_{(n,s)} + T_4.DI_{(n,s)}}, \qquad \forall s\in\mathcal{S}, \forall n\in\mathcal{N}_s^{(D)}, \tag{46}$$

where $EI_{(n,s)}$ and $DI_{(n,s)}$ are fixed indices for producers and consumers related to their locations in the network, and T_1, T_2, T_3, T_4 are fixed constants which depend on the network structure and parameters. Then, the tâtonnement-process described by (37)-(40) is a contraction map. The unique fixed point of this contraction map is a Nash equilibrium of the game induced by the mechanism proposed in Section 4. Furthermore, the allocations (energy production and consumption) resulting at each step of the proposed tâtonnement-process are feasible solutions of the problem (**OPF**).

The proof of this theorem and definitions of indices $EI_{(n,s)}$, $DI_{(n,s)}$, T_1, T_2, T_3, and T_4 appear in Appendix I.

Example 1 (Quadratic cost and utility functions)**.** *Generators and flexible demands usually have quadratic cost and utility functions. For instance,*

$$c_n^{(s)}(e_n^{(s)}) = \frac{\imath_n^{(s)}}{2}.(e_n^{(s)})^2 + \jmath_n^{(s)}.e_n^{(s)}, \qquad \forall s\in\mathcal{S}, \forall n\in\mathcal{N}_s^{(E)}, \tag{47}$$

$$u_n^{(s)}(d_n^{(s)}) = -\frac{\eth_n^{(s)}}{2}.(d_n^{(s)})^2 + \ell_n^{(s)}.d_n^{(s)}, \qquad \overline{D}_n^{(s)} < \frac{\ell_n^{(s)}}{\eth_n^{(s)}}, \qquad \forall s\in\mathcal{S}, \forall n\in\mathcal{N}_s^{(D)}, \tag{48}$$

where $\imath_n^{(s)}$, $\jmath_n^{(s)}$, $\eth_n^{(s)}$, and $\ell_n^{(s)}$ are parameters of quadratic cost and utility functions.

In this case, γ_e and γ_d are defined by

$$\gamma_e \coloneqq \max_{(s\in\mathcal{S},n\in\mathcal{N}_s^{(E)})} \left(\overline{E}_n^{(s)} + \frac{\jmath_n^{(s)}}{\imath_n^{(s)}}\right), \tag{49}$$

$$\gamma_d := \min_{(s\in\mathcal{S},n\in\mathcal{N}_s^{(E)})}\left(\frac{\ell_n^{(s)}}{\eth_n^{(s)}} - 2.\overline{D}_n^{(s)}\right), \tag{50}$$

and the analogues of (45) *and* (46) *are given by (51)-(52).*

$$\left|\gamma_e \imath_n^{(s)} - \jmath_n^{(s)}\right| < \frac{1}{T_1.EI_{(n,s)} + T_2.DI_{(n,s)}}, \qquad \forall s\in\mathcal{S}, \forall n\in\mathcal{N}_s^{(E)}, \tag{51}$$

$$\left|\gamma_d \eth_n^{(s)} - \ell_n^{(s)}\right| < \frac{1}{T_3.EI_{(n,s)} + T_4.DI_{(n,s)}}, \qquad \forall s\in\mathcal{S}, \forall n\in\mathcal{N}_s^{(D)}. \tag{52}$$

Assuming (51)-(52), Theorem 7 can be proved for quadratic cost and utility functions.

Remark 5. *Conditions (45)-(46) impose limitations on the first and second derivatives of the cost and utility functions, respectively, at zero production and consumption. These conditions can be justified by the results of [68] and [69] on stability of tâtonnement-processes. The conditions are not valid for all networks. In the numerical case study presented in Section 8, (45)-(46) are not satisfied. In such a case, we modify the tâtonnement-process described by (37)-(40) so that the resulting algorithm is convergent (see Section 8). Then, the unique fixed point of this modified contraction map is also a Nash equilibrium of the game induced by the mechanism proposed in Section 4.*

Remark 6. *In practice, the best-estimate algorithm stops when the difference between all the components of two successive estimates is less, in absolute value, than some predetermined number $\epsilon \in \mathbb{R}_{>0}$. In this case, even though the outcome of the tâtonnement-process is not a Nash equilibrium of the game induced by the mechanism, it is still a feasible solution of problem (**OPF**) (see Theorem 7).*

Remark 7. *The game induced by the mechanism proposed in Section 4 may have many Nash equilibria. The tâtonnement-process defined in this section is a contraction map, thus it has a unique fixed point which is one of the Nash equilibria of the game induced by the proposed mechanism.*

7 Extension to Sparse Electricity Networks

In the model of Section 2 we assume that there are no isolated or intermediary nodes. However, electricity networks are usually sparse. This means that there are network nodes with no agents located in them (i.e., *"intermediary"* nodes); furthermore, it is possible that there are nodes without any agents located at them or at any of the nodes in their immediate neighbors (i.e., *"isolated"* nodes). In this section, we discuss how the proposed mechanism can be extended to the case of sparse electricity networks.

First, we define by $\mathcal{R}_n^{(ext)}$ the extended neighbors of node n as follows: For any $n' \in \mathcal{R}_n^{(ext)}$ there exists a path from node $n \in \mathcal{N}$ to node n' that does not include any agent other than the one at node n and, possibly, the one at node n'. Then, we define $\Delta_s^{(ext)}$, the set of extended nodes of agent s by $\Delta_s^{(ext)} := \cup_{n\in(\mathcal{N}_s^{(E)}\cup\mathcal{N}_s^{(D)})}\mathcal{R}_n^{(ext)}$. We define by $\Omega_n^{(ext)}$ the set of all agents s that have node n belonging to $\Delta_s^{(ext)}$. We define $\Phi_s^{(ext)}$, the extended set of neighboring transmission lines of agent s as follows: For any $(n', n'') \in \Phi_s^{(ext)}$ there exist a path from node $n \in (\mathcal{N}_s^{(E)} \cup \mathcal{N}_s^{(D)})$ to node n' that does not include any agent other than agent s and, possibly, the ones at node n''. Finally, we define $\Upsilon_{n'n''}^{(ext)}$ to be the set of agents s that the transmission line (n', n'') included in $\Phi_s^{(ext)}$.

To extend the mechanism proposed in Section 4 to sparse networks we consider the following message space. The price vectors $\vec{p}_s$ and $\vec{q}_s$ in (11) are replaced by the vectors $(p_{n'}^{(s)})_{n' \in \Delta_s^{(ext)}}$ and $(q_{n'n''}^{(s)})_{(n',n'') \in \Phi_s^{(ext)}}$. As far as the outcome function is concerned, the fourth and fifth terms in (25) must include all neighbors in $\Delta_s^{(ext)}$ and all transmission lines in $\Phi_s^{(ext)}$. In addition, all LMPs and FTR prices must be calculated in (26) and (27) with respect to the extended neighbors in the sets $\Omega_n^{(ext)}$ and $\Upsilon_{n'n''}^{(ext)}$, respectively. With these modifications, we can prove that the resulting mechanism possesses the same properties as the mechanism in Section 4.

Example 2 (A sparse electricity network)**.** *Consider the network of Figure 2. Assume an agent has generators at nodes* 1 *and* 7*. In this case,* $\Delta_s = \{1, 2, 3, 5, 7\}$, $\Delta_s^{(ext)} = \{1, 2, 3, 4, 5, 6, 7\}$, $\Phi_s = \{(1, 3), (1, 2), (3, 1), (3, 2), (5, 7), (7, 5)\}$, *and* $\Phi_s^{(ext)} = \{$*all transmission lines*$\}$*. Then, the agent must propose the price vectors* $(p_{n'}^{(s)})_{n' \in \Delta_s^{(ext)}}$ *and* $(q_{n'n''}^{(s)})_{(n',n'') \in \Phi_s^{(ext)}}$*.*

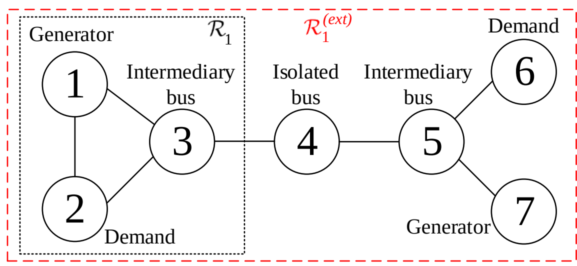

Figure 2: An example of sparse electricity networks.

8 Numerical Case Study

To validate the effectiveness of the proposed mechanism and tâtonnement-process, we consider a three nodes electricity network with three identical transmission lines. Each 138 kV transmission line has an inductance of 14.59 Ohms, a resistance of 1.82 Ohms, and a capacity of 390 MW. This simple electricity network, which has also been used in [12] for validation purposes, is illustrated in Figure 3.

As shown in Figure 3, each node has one generator and one flexible demand. The must-run demands of nodes, i.e., $(O_n)_{n \in \mathcal{N}}$, are considered equal to zero. In addition, there are three strategic agents ($\mathcal{S} = \{1, 2, 3\}$), the characteristics of which are displayed in Table 1.

Under the above data, the ISO should issue 390 MW of FTRs for each transmission line. We assume that agent 1 owns 210 MW share of FTRs of each line, and each of agents 2 and 3 own 90 MW shares in each line.

The dispatch and the agents' payoffs at an equilibrium of the proposed mechanism applied to the above network are presented in Tables 2 and 3, respectively. From Table 2 we conclude that the optimal

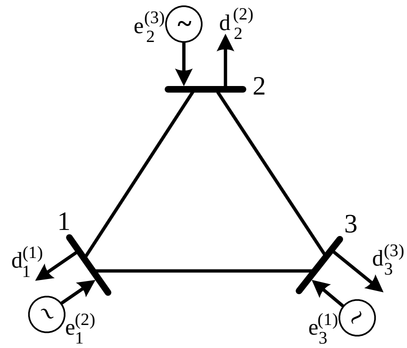

Figure 3: A simple electricity transmission network.

Table 1: Utility and cost functions of agents.

s	$\mathcal{N}_s^{(D)}$	$\mathcal{N}_s^{(E)}$	$u_n^{(s)}(d_n^{(s)})$	$c_n^{(s)}(e_n^{(s)})$	$\overline{D}_n^{(s)}$	$\overline{E}_n^{(s)}$
1	$\{1\}$	$\{3\}$	$-0.15(d_1^{(1)})^2 + 100d_1^{(1)}$	$0.15(e_3^{(1)})^2 + 75e_3^{(1)}$	100	50
2	$\{2\}$	$\{1\}$	$-0.10(d_2^{(2)})^2 + 200d_2^{(2)}$	$0.05(e_1^{(2)})^2 + 30e_1^{(2)}$	200	500
3	$\{3\}$	$\{2\}$	$-0.05(d_3^{(3)})^2 + 400d_3^{(3)}$	$0.10(e_2^{(3)})^2 + 50e_2^{(3)}$	400	150

dispatch (solution of problem (**OPF**)) is implemented in Nash equilibrium, and from Table 3 we conclude that individual rationality and budget balance are satisfied.

Table 2: Dispatch at Nash equilibrium of the proposed mechanism.

n	$\sum_{s\in\Psi_n^{(E)}} \hat{e}_n^{(s)}(\vec{\mathbf{m}}^*)$	$\sum_{n\in\Psi_n^{(D)}} \hat{d}_n^{(s)}(\vec{\mathbf{m}}^*)$	$\hat{\lambda}_n(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*)$	$\hat{\lambda}_{ref}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*)$
1	469.46	76.95	57.17	19.73
2	144.69	155.32	59.20	19.73
3	19.41	391.82	61.08	19.73

8.1 Feasibility and Efficiency of Implemented Dispatch

We investigate the following issues. First, we check if the dispatch of Table 2 is a feasible solution of the alternate-current (AC) power flow model. Second, we check the efficiency of the dispatch of Table 2 by comparing it with the optimal dispatch obtained when the AC power flow model is used instead of the convex DC approximation.

To study the feasibility of Table 2's dispatches within the context of the AC power flow model we proceed as follows: We consider the dispatch of Table 2 at all nodes except one, say node 1, and use them in the package of MATPOWER 6.0 in MATLAB, [70], to obtain the solution for the AC power flow model (MATPOWER 6.0 requires that we should consider a slack node, thus we take node 1 to be that

Table 3: Payoff at Nash equilibrium of the proposed mechanism.

s	$\sum_{n\in\mathcal{N}_s^{(D)}} u_n^{(s)}(\hat{d}_n^{(s)}(\vec{\mathbf{m}}^*)) - \sum_{n\in\mathcal{N}_s^{(E)}} c_n^{(s)}(\hat{e}_n^{(s)}(\vec{\mathbf{m}}^*))$	$t_s(\vec{\mathbf{m}}^*)$	$\mathrm{U}_s(\vec{\mathbf{m}}^*)$
1	5293.6	4322.3	$971.3 \geq 0$
2	3548.6	-24829.4	$28378 \geq 0$
3	139726.0	19922.1	$119803.9 \geq 0$
		0	

node). According to the solution of the AC power flow model in Table 4, the generator at the slack node produces 1.11 MW less than the amount corresponding to the solution of the convex DC approximation. The difference between the two dispatches is due to the fact that in the model of Section 2 we do not consider losses due to the terms neglected in the convex DC approximation. As a result of this difference, the dispatch associated with all Nash equilibria resulting from the convex DC approximation model does not satisfy the power balance equations in the AC power flow model. Thus, the dispatch of Table 2 is not a feasible solution of the AC power flow model. Nevertheless, the difference between the dispatches of Tables 2 and 4 is very small (less than 0.25%).

Table 4: AC power flow of the equilibrium dispatch.

n	$\hat{\theta}_n(\vec{\mathbf{m}}^*)$	$\sum_{s\in\Psi_n^{(E)}} \hat{e}_n^{(s)}(\vec{\mathbf{m}}^*)$	$\sum_{s\in\Psi_n^{(D)}} \hat{d}_n^{(s)}(\vec{\mathbf{m}}^*)$
1	0.00	468.35	76.95
2	-0.09	144.69	155.32
3	0.34	19.41	391.82

In Table 5, we compare the dispatch of Table 2 with the solution of the OPF problem when: (i) the exact AC power flow model is used; and (ii) the DC approximation model is used. For the OPF problem associated with the exact AC model, we assume that the voltage magnitudes at all nodes are limited between 124.2 kV and 151.8 kV. We observe that the dispatch of Table 2 is a more accurate approximation of the optimal solution of the AC power flow model than the dispatch resulting from the optimal solution of the power flow problem for the DC approximation model.

8.2 Tâtonnement-Process Convergence

Generators and flexible demands in this case study have quadratic cost and utility functions, and are given by (47) and (48), respectively. The quantities γ_e and γ_d are defined by (49) and (50), and are equal to 800 and 233.33, respectively. If (51)-(52) are satisfied for the network considering in this section, the tâtonnement-process described by (37)-(40) converges to a Nash equilibrium of the game induced by the mechanism proposed in Section 4 (see Theorem 7 and Remark 7).

We now check if (51)-(52) are satisfied. The left hand side (LHS) and RHS of (51) and (52) are presented in Table 6. The ratios of RHS to the LHS of (51) and (52) are denoted by $\sigma_n^{(s)}$ and $\rho_n^{(s)}$,

Table 5: Comparing the dispatch of Table 2 with the solutions of the OPF problem with AC and DC power flow models.

	n	$\sum_{s\in\Psi_n^{(E)}} \hat{e}_n^{(s)}(\vec{\mathbf{m}}^*)$	$\sum_{s\in\Psi_n^{(D)}} \hat{d}_n^{(s)}(\vec{\mathbf{m}}^*)$	$\hat{\lambda}_n(\vec{\mathbf{w}}^*,\vec{\mathbf{v}}^*)+\hat{\lambda}_{ref}(\vec{\mathbf{w}}^*,\vec{\mathbf{v}}^*)$
OPF with DC power flow	1	481.2	72.9	78.1
	2	140.6	159.4	78.1
	3	10.4	400	78.1
Implemented Nash equilibria	1	469.5	76.9	76.9
	2	144.7	155.3	78.9
	3	19.4	391.8	80.8
OPF with AC power flow	1	473.0	75.6	77.3
	2	144.3	155.7	78.8
	3	17.8	369.5	80.3

respectively, and appear in Table 6. Since $\sigma_n^{(s)}$ and $\rho_n^{(s)}$ are less than one for all generators and flexible demands, we conclude that conditions (51) and (52) in Theorem 7 do not hold, therefore, we cannot conclude that the tâtonnement-process (37)-(40) converges to a Nash equilibrium of the game induced by the mechanism of Section 4. In this case, we modify the tâtonnement-process (37)-(40) as follows: We replace (37)-(38) with (53)-(54) given below,

$$(w_n^{(s)})^{(\tau+1)} = (1-b).(w_n^{(s)})^{(\tau)} + b.\frac{\left.\frac{\partial c_n^{(s)}}{\partial e_n^{(s)}}\right|_{(\hat{e}_n^{(s)})^{(\tau)}}}{\left.\frac{\partial f^{(E)}}{\partial y}\right|_{y=(\hat{e}_n^{(s)})^{(\tau)}}}, \qquad \forall s\in\mathcal{S}, \forall n\in\mathcal{N}_s^{(E)}, \tag{53}$$

$$(v_n^{(s)})^{(\tau+1)} = (1-b).(v_n^{(s)})^{(\tau)} + b.\frac{\left.\frac{\partial u_n^{(s)}}{\partial d_n^{(s)}}\right|_{(\hat{d}_n^{(s)})^{(\tau)}}}{\left.\frac{\partial f^{(D)}}{\partial y}\right|_{y=(\hat{d}_n^{(s)})^{(\tau)}}}, \qquad \forall s\in\mathcal{S}, \forall n\in\mathcal{N}_s^{(D)}, \tag{54}$$

where b is a positive number less than $\frac{\sqrt{1+L^2}-1}{L^2}$ and L is defined as

$$L := \max_{s\in\mathcal{S}}\left(\max_{n\in\mathcal{N}_s^{(E)}}\left(\frac{1}{\sigma_n^{(s)}}\right), \max_{n\in\mathcal{N}_s^{(D)}}\left(\frac{1}{\rho_n^{(s)}}\right)\right). \tag{55}$$

In the modified tâtonnement-process, the weight factors $(w_n^{(s)})_{s\in\mathcal{S},n\in\mathcal{N}_s^{(E)}}$ and $(v_n^{(s)})_{s\in\mathcal{S},n\in\mathcal{N}_s^{(D)}}$ are updating smoothly through the use of coefficient b. With this modification and by choosing (41) and (42) as surrogate cost and utility functions, respectively, we can prove the following result.

Theorem 8 (Convergence of modified tâtonnement-process)**.** *Choose* (41) *and* (42) *as surrogate cost and utility functions, respectively. Then, the algorithm described by (53)-(54) and (39)-(40) converges to the fixed-point of the tâtonnement-process (37)-(40); this fixed point is a Nash equilibrium of the game induced by mechanism proposed in Section 4. Furthermore, the allocations (energy production and consumption) resulting at each step of the modified tâtonnement-process are feasible solutions of the problem* (**OPF**)*.*

The proof of Theorem 8 appears in Appendix J.

Table 6: LHS and RHS of (51) and (52).

	LHS of (51)	RHS of (51)	$\sigma_n^{(s)}$		LHS of (52)	RHS of (52)	$\rho_n^{(s)}$
$e_3^{(1)}$	165	4.25	0.026	$d_1^{(1)}$	30	5.31	0.177
$e_1^{(2)}$	50	4.25	0.085	$d_2^{(2)}$	63.3	5.31	0.084
$e_2^{(3)}$	110	4.25	0.039	$d_3^{(3)}$	96.6	5.31	0.055

Using the modified tâtonnement-process, the convergence of weight factors $(w_n^{(s)})_{s\in\mathcal{S},n\in\mathcal{N}_s^{(E)}}$ and $(v_n^{(s)})_{s\in\mathcal{S},n\in\mathcal{N}_s^{(D)}}$ is illustrated in Figure 4. Moreover, the convergence of productions, consumptions, and LMPs, is depicted in Figures 5, 6, and 7, respectively.

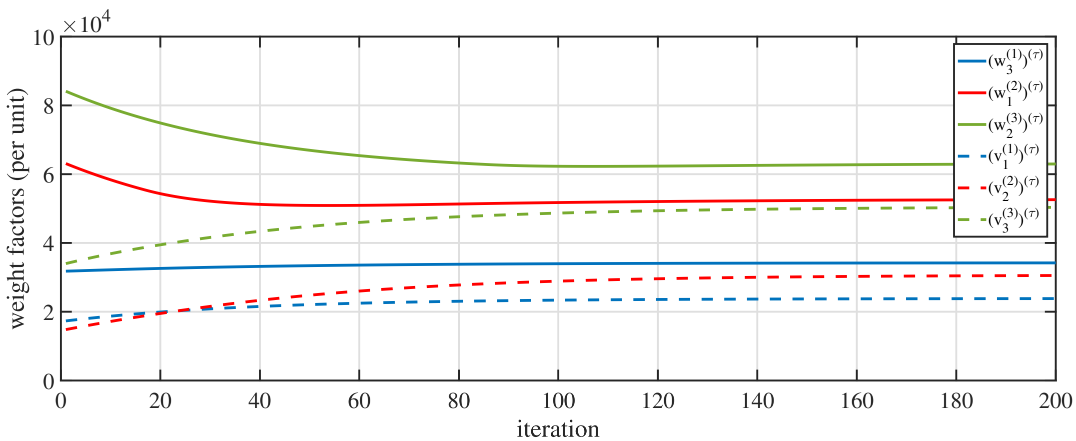

Figure 4: Convergence of weight factors.

8.3 Comparison with Other Existing Mechanisms

At equilibrium, the surrogate mechanism proposed in this paper is compared to two other conventional market mechanisms, the marginal price clearing mechanism in the power pool and the VCG mechanism [42]. The scenarios in Table 7 are used to comprehensively analyze the ISO's objective changes under different market mechanisms, including three strategic agents in three nodes electricity network.

Table 7: Comparison of market mechanisms.

Mechanism	ISO's Objective	$\sum_{s\in\mathcal{S}} t_s$
Proposed surrogate mechanism	148568.2	0
Power pool	140625.4	3049.8
VCG	148568.2	-3289.5

According to the comparison results in Table 7, the ISO's objective at equilibrium is lower under

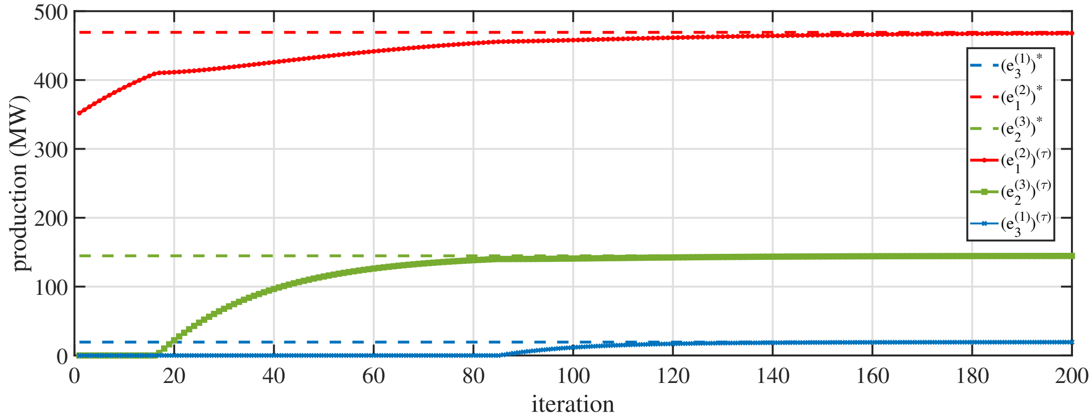

Figure 5: Convergence of production of generators.

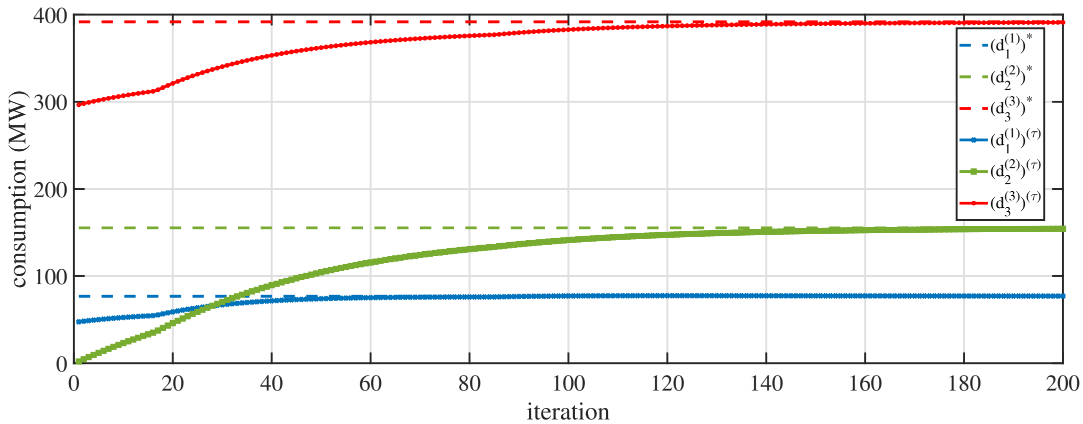

Figure 6: Convergence of consumption of demands.

the conventional power pool model. Furthermore, the ISO's objective under the proposed surrogate mechanism and the VCG mechanism are the same. Although the market can converge to the optimal ISO's objective under both the VCG and proposed surrogate mechanisms, the VCG mechanism cannot reach budget balance on its own. Furthermore, at the equilibrium of the conventional power pool market, a weak budget balance will be established. The proposed surrogate mechanism's advantage is precisely its ability to maximize the ISO's objective while balancing the budget without the need for third-party monetary subsidies.

9 Conclusion and Future Work

We have presented a market design for electricity networks with strategic agents possessing asymmetric information. The electricity network model takes into account the congestion in transmission lines, losses, and FTRs. Our approach to market design is based on mechanism design for local public goods.

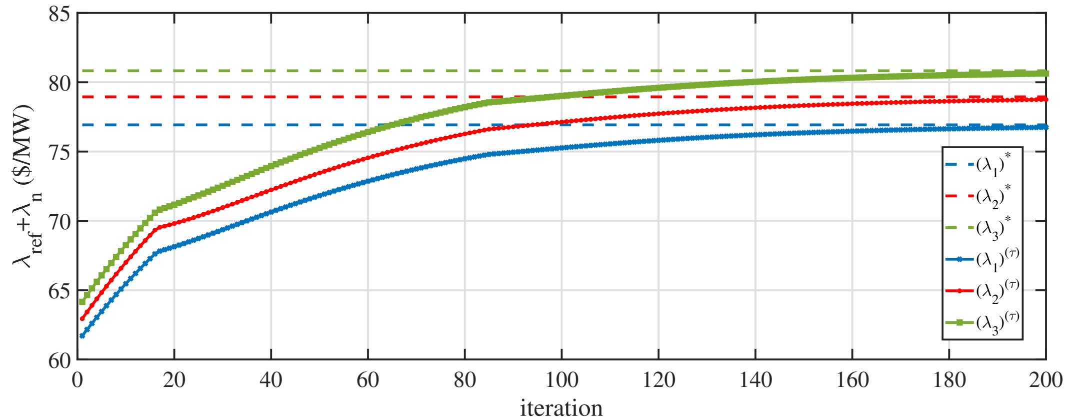

Figure 7: Convergence of LMPs.

Our design consists of a mechanism along with a tâtonnement-process which, under certain conditions, converges to a Nash equilibrium of the game induced by the mechanism. Our mechanism has a finite message space and possesses several desirable properties, namely: (i) it is budget balanced; (ii) the allocations corresponding to each Nash equilibrium of the game induced by it maximize the strategic agents' social welfare, that is, the sum of agents' utilities; (iii) it is price efficient; (iv) it is individually rational; and (v) the allocations corresponding to off-equilibrium messages are feasible.

We presented a case study where we highlighted the properties of the proposed mechanism, and indicated how the tâtonnement-process must be modified if the conditions sufficient to guarantee its convergence are not satisfied.

The proposed design approach can be potentially extended to simultaneous energy and ancillary service markets (i.e., multi-commodities market), and other networked systems/markets with externalities such as spectrum allocation in wireless networks with interferences.

Appendices

A Karush-Kuhn-Tucker Conditions:

A.1 KKT Conditions for Problem (OPF)

Because of Assumptions 1 and 2 and constraints (6)-(10), (**OPF**) is a strictly concave optimization problem with a convex domain, where Slater's constraint qualification condition is satisfied. As a result, the KKT conditions are sufficient and necessary for optimality. Define the Lagrangain $\mathcal{L}^{(OPF)}$ associated with (**OPF**),

$$\mathcal{L}^{(OPF)} = \sum_{s\in\mathcal{S}} \left(\sum_{n\in\mathcal{N}_s^{(D)}} u_n^{(s)}(d_n^{(s)}) - \sum_{n\in\mathcal{N}_s^{(E)}} c_n^{(E)}(e_n^{(s)}) \right)$$

$$
\begin{aligned}
&-\sum_{n\in\mathcal{N}}\lambda_n^{(O)}.\left(O_n-\sum_{s\in\Psi_n^{(E)}}e_n^{(s)}+\sum_{s\in\Psi_n^{(D)}}d_n^{(s)}+\sum_{n'\in\mathcal{R}_n}\left(B_{nn'}.\theta_{nn'}+\frac{1}{2}.G_{nn'}.\theta_{nn'}^2\right)\right)\\
&-\sum_{s\in\mathcal{S}}\left(\sum_{n\in\mathcal{N}_s^{(D)}}\left(\overline{\varsigma}_n^{(s,O)}.(d_n^{(s)}-\overline{D}_n^{(s)})-\underline{\varsigma}_n^{(s,O)}.d_n^{(s)}\right)-\sum_{n\in\mathcal{N}_s^{(E)}}\left(\overline{\varphi}_n^{(s,O)}.(e_n^{(s)}-\overline{E}_n^{(s)})-\underline{\varphi}_n^{(s,O)}.e_n^{(s)}\right)\right)\\
&-\sum_{n\in\mathcal{N}}\sum_{n'\in\mathcal{R}_n}\mu_{nn'}^{(O)}.\left(B_{nn'}.\theta_{nn'}+\frac{1}{2}.G_{nn'}.(\theta_{nn'})^2-\overline{F}_{nn'}\right)-\sum_{n\in\mathcal{N}}\sum_{n'\in\mathcal{R}_n}\chi_{nn'}^{(O)}.\left(\theta_{nn'}+\theta_{n'n}\right),
\end{aligned}
\tag{56}
$$

where $\lambda_n^{(O)}$, $\underline{\varphi}_n^{(s,O)}$, $\overline{\varphi}_n^{(s,O)}$, $\underline{\varsigma}_n^{(s,O)}$, $\overline{\varsigma}_n^{(s,O)}$, $\mu_{nn'}^{(O)}$, and $\chi_{nn'}^{(O)}$ denote the centroid of set of Lagrange multiplier vectors corresponding to constraints (6)-(10). The KKT conditions are

$$\frac{\partial\mathcal{L}^{(OPF)}}{\partial d_n^{(s)}}=\left.\frac{\partial u_n^{(s)}}{\partial d_n^{(s)}}\right|_{(d_n^{(s)})^*}-\lambda_n^{(O)}+\underline{\varsigma}_n^{(s,O)}-\overline{\varsigma}_n^{(s,O)}=0,\qquad\forall s\in\mathcal{S},\forall n\in\mathcal{N}_s^{(D)},\tag{57}$$

$$\frac{\partial\mathcal{L}^{(OPF)}}{\partial e_n^{(s)}}=-\left.\frac{\partial c_n^{(s)}}{\partial e_n^{(s)}}\right|_{(e_n^{(s)})^*}+\lambda_n^{(O)}+\underline{\varphi}_n^{(s,O)}-\overline{\varphi}_n^{(s,O)}=0,\qquad\forall s\in\mathcal{S},\forall n\in\mathcal{N}_s^{(E)},\tag{58}$$

$$\frac{\partial\mathcal{L}^{(OPF)}}{\partial\theta_{nn'}}=\sum_{n'\in\mathcal{R}_n}\left((B_{nn'}+G_{nn'}.\theta_{nn'}^*).(\lambda_n^{(O)}+\mu_{nn'}^{(O)})\right)-\chi_{nn'}^{(O)}-\chi_{n'n}^{(O)}=0,\qquad\forall n\in\mathcal{N},\tag{59}$$

$$
\begin{aligned}
&\Bigg(O_n-\sum_{s\in\Psi_n^{(E)}}(e_n^{(s)})^*+\sum_{s\in\Psi_n^{(D)}}(d_n^{(s)})^*\\
&\qquad+\sum_{n'\in\mathcal{R}_n}\left(B_{nn'}.\theta_{nn'}^*+\frac{1}{2}.G_{nn'}.(\theta_{nn'}^*)^2\right)\Bigg).\lambda_n^{(O)}=0,\qquad\forall n\in\mathcal{N},
\end{aligned}
\tag{60}
$$

$$(e_n^{(s)})^*.\underline{\varphi}_n^{(s,O)}=0,\qquad\forall s\in\mathcal{S},\forall n\in\mathcal{N}_s^{(E)},\tag{61}$$

$$\left((e_n^{(s)})^*-\overline{E}_n^{(s)}\right).\overline{\varphi}_n^{(s,O)}=0,\qquad\forall s\in\mathcal{S},\forall n\in\mathcal{N}_s^{(E)},\tag{62}$$

$$(d_n^{(s)})^*.\underline{\varsigma}_n^{(s,O)}=0,\qquad\forall s\in\mathcal{S},\forall n\in\mathcal{N}_s^{(D)},\tag{63}$$

$$\left((d_n^{(s)})^*-\overline{D}_n^{(s)}\right).\overline{\varsigma}_n^{(s,O)}=0,\qquad\forall s\in\mathcal{S},\forall n\in\mathcal{N}_s^{(D)},\tag{64}$$

$$\left(B_{nn'}\theta_{nn'}^*+\frac{1}{2}.G_{nn'}.(\theta_{nn'}^*)^2-\overline{F}_{nn'}\right).\mu_{nn'}^{(O)}=0,\qquad\forall n\in\mathcal{N},\forall n'\in\mathcal{R}_n,\tag{65}$$

$$(6)-(10)\big|_{(\theta_{nn'}^*)_{n\in\mathcal{N},n'\in\mathcal{R}_n},((e_n^{(s)})^*)_{s\in\mathcal{S},n\in\mathcal{N}_s^{(E)}},((d_n^{(s)})^*)_{s\in\mathcal{S},n\in\mathcal{N}_s^{(D)}}},\tag{66}$$

$$
\begin{aligned}
&(\lambda_n^{(O)})_{n\in\mathcal{N}},(\underline{\varphi}_n^{(s,O)})_{s\in\mathcal{S},n\in\mathcal{N}_s^{(E)}},(\overline{\varphi}_n^{(s,O)})_{s\in\mathcal{S},n\in\mathcal{N}_s^{(E)}},\\
&(\underline{\varsigma}_n^{(s,O)})_{s\in\mathcal{S},n\in\mathcal{N}_s^{(D)}},(\overline{\varsigma}_n^{(s,O)})_{s\in\mathcal{S},n\in\mathcal{N}_s^{(D)}},(\mu_{nn'}^{(O)})_{n\in\mathcal{N},n'\in\mathcal{R}_n}\geq 0,
\end{aligned}
\tag{67}
$$

where $(\theta_{nn'}^*)_{n\in\mathcal{N},n'\in\mathcal{R}_n}$, $((e_n^{(s)})^*)_{s\in\mathcal{S},n\in\mathcal{N}_s^{(E)}}$, and $((d_n^{(s)})^*)_{s\in\mathcal{S},n\in\mathcal{N}_s^{(D)}}$ are the unique solution of problem (**OPF**).

A.2 KKT Conditions for Problem (Surrogate)

Because of Assumptions 8 and 9 and constraints (12)-(17), and since the weight factos $(w_n^{(s)})_{s\in\mathcal{S},n\in\mathcal{N}_s^{(E)}}$ and $(v_n^{(s)})_{s\in\mathcal{S},n\in\mathcal{N}_s^{(D)}}$ are strictly positive, for any fixed message profile $(\vec{\mathbf{w}},\vec{\mathbf{v}})$, (**Surrogate**) is strictly concave optimization problem with a convex domain, where Slater's constraint qualification condition is satisfied. As a result, the KKT conditions are sufficient and necessary for optimality. Define the Lagrangain $\mathcal{L}^{(SG)}$ associated with (**Surrogate**),

$$\begin{aligned}
\mathcal{L}^{(SG)} = &\sum_{s\in\mathcal{S}}\left(\sum_{n\in\mathcal{N}_s^{(D)}} v_n^{(s)}.f^{(D)}(d_n^{(s)}) - \sum_{n\in\mathcal{N}_s^{(E)}} w_n^{(s)}.f^{(E)}(e_n^{(s)})\right)\\
&-\sum_{n\in\mathcal{N}}\lambda_n\left(O_n - \sum_{s\in\Psi_n^{(E)}} e_n^{(s)} + \sum_{s\in\Psi_n^{(D)}} d_n^{(s)} + \sum_{n'\in\mathcal{R}_n}\left(B_{nn'}.\theta_{nn'} + \frac{1}{2}.G_{nn'}.\theta_{nn'}^2\right)\right)\\
&-\sum_{s\in\mathcal{S}}\left(\sum_{n\in\mathcal{N}_s^{(D)}}\left(\overline{\varsigma}_n^{(s)}.(d_n^{(s)} - \overline{D}_n^{(s)}) - \underline{\varsigma}_n^{(s)}.d_n^{(s)}\right) - \sum_{n\in\mathcal{N}_s^{(E)}}\left(\overline{\varphi}_n^{(s)}.(e_n^{(s)} - \overline{E}_n^{(s)}) - \underline{\varphi}_n^{(s)}.e_n\right)\right)\\
&-\sum_{n\in\mathcal{N}}\sum_{n'\in\mathcal{R}_n}\mu_{nn'}.\left(B_{nn'}.\theta_{nn'} + \frac{1}{2}.G_{nn'}.(\theta_{nn'})^2 - \overline{F}_{nn'}\right) - \sum_{n\in\mathcal{N}}\sum_{n'\in\mathcal{R}_n}\chi_{nn'}.\left(\theta_{nn'} + \theta_{n'n}\right)\\
&-\sum_{n\in\mathcal{N}}\lambda_{ref}.\left(O_n - \sum_{s\in\Psi_n^{(E)}} e_n^{(s)} + \sum_{s\in\Psi_n^{(D)}} d_n^{(s)} + \frac{1}{2}.\sum_{n'\in\mathcal{R}_n} G_{nn'}.\theta_{nn'}^2\right),
\end{aligned}\tag{68}$$

where λ_n, $\underline{\varphi}_n^{(s)}$, $\overline{\varphi}_n^{(s)}$, $\underline{\varsigma}_n^{(s)}$, $\overline{\varsigma}_n^{(s)}$, $\mu_{nn'}$, $\chi_{nn'}$, and λ_{ref} denote the centroid of set of Lagrange multiplier vectors corresponding to (12)-(17). The KKT conditions are

$$\frac{\partial\mathcal{L}^{(SG)}}{\partial d_n^{(s)}} = v_n^{(s)}.\left.\frac{\partial f^{(D)}}{\partial y}\right|_{y=\hat{d}_n^{(s)}} - \lambda_n - \lambda_{ref} + \underline{\varsigma}_n^{(s)} - \overline{\varsigma}_n^{(s)} = 0, \qquad \forall s\in\mathcal{S},\forall n\in\mathcal{N}_s^{(D)}, \tag{69}$$

$$\frac{\partial\mathcal{L}^{(SG)}}{\partial e_n^{(s)}} = -w_n^{(s)}.\left.\frac{\partial f^{(E)}}{\partial y}\right|_{y=\hat{e}_n^{(s)}} + \lambda_n + \lambda_{ref} + \underline{\varphi}_n^{(s)} - \overline{\varphi}_n^{(s)} = 0, \qquad \forall s\in\mathcal{S},\forall n\in\mathcal{N}_s^{(E)}, \tag{70}$$

$$\frac{\partial\mathcal{L}^{(SG)}}{\partial\theta_{nn'}} = \sum_{n'\in\mathcal{R}_n}\left((B_{nn'} + G_{nn'}.\hat{\theta}_{nn'}).(\lambda_n + \mu_{nn'}) + G_{nn'}.\hat{\theta}_{nn'}.\lambda_{ref}\right) - \chi_{nn'} - \chi_{n'n} = 0, \forall n\in\mathcal{N}, \tag{71}$$

$$\left(O_n - \sum_{s\in\Psi_n^{(E)}}\hat{e}_n^{(s)} + \sum_{s\in\Psi_n^{(D)}}\hat{d}_n^{(s)} + \sum_{n'\in\mathcal{R}_n}\left(B_{nn'}.\hat{\theta}_{nn'} + \frac{1}{2}.G_{nn'}.\hat{\theta}_{nn'}^2\right)\right).\lambda_n = 0, \qquad \forall n\in\mathcal{N}, \tag{72}$$

$$\hat{e}_n^{(s)}.\underline{\varphi}_n^{(s)} = 0, \qquad \forall s\in\mathcal{S},\forall n\in\mathcal{N}_s^{(E)}, \tag{73}$$

$$\left(\hat{e}_n^{(s)} - \overline{E}_n^{(s)}\right).\overline{\varphi}_n^{(s)} = 0, \qquad \forall s\in\mathcal{S},\forall n\in\mathcal{N}_s^{(E)}, \tag{74}$$

$$\hat{d}_n^{(s)}.\underline{\varsigma}_n^{(s)} = 0, \qquad \forall s\in\mathcal{S},\forall n\in\mathcal{N}_s^{(D)}, \tag{75}$$

$$\left(\hat{d}_n^{(s)} - \overline{D}_n^{(s)}\right).\overline{\varsigma}_n^{(s)} = 0, \qquad \forall s\in\mathcal{S},\forall n\in\mathcal{N}_s^{(D)}, \tag{76}$$

$$\left(B_{nn'}.\hat{\theta}_{nn'} + \frac{1}{2}.G_{nn'}.\hat{\theta}^2_{nn'} - \overline{F}_{nn'}\right).\mu_{nn'} = 0, \qquad \forall n \in \mathcal{N}, \forall n' \in \mathcal{R}_n, \tag{77}$$

$$\sum_{n\in\mathcal{N}} \left(O_n - \sum_{s\in\Psi_n^{(E)}} \hat{e}_n^{(s)} + \sum_{s\in\Psi_n^{(D)}} \hat{d}_n^{(s)} + \frac{1}{2}. \sum_{n'\in\mathcal{R}_n} G_{nn'}.\hat{\theta}^2_{nn'} \right).\lambda_{ref} = 0, \tag{78}$$

$$(12) - (17)|_{(\hat{\theta}_{nn'})_{n\in\mathcal{N},n'\in\mathcal{R}_n}, (\hat{e}_n^{(s)})_{s\in\mathcal{S},n\in\mathcal{N}_s^{(E)}}, (\hat{d}_n^{(s)})_{s\in\mathcal{S},n\in\mathcal{N}_s^{(D)}}}, \tag{79}$$

$$(\lambda_n)_{n\in\mathcal{N}}, (\underline{\varphi}_n^{(s)})_{s\in\mathcal{S},n\in\mathcal{N}_s^{(E)}}, (\overline{\varphi}_n^{(s)})_{s\in\mathcal{S},n\in\mathcal{N}_s^{(E)}},$$
$$(\underline{\varsigma}_n^{(s)})_{s\in\mathcal{S},n\in\mathcal{N}_s^{(D)}}, (\overline{\varsigma}_n^{(s)})_{s\in\mathcal{S},n\in\mathcal{N}_s^{(D)}}, (\mu_{nn'})_{n\in\mathcal{N},n'\in\mathcal{R}_n}, \lambda_{ref} \geq 0, \tag{80}$$

where $(\hat{\theta}_{nn'})_{n\in\mathcal{N},n'\in\mathcal{R}_n}$, $(\hat{e}_n^{(s)})_{s\in\mathcal{S},n\in\mathcal{N}_s^{(E)}}$, and $(\hat{d}_n^{(s)})_{s\in\mathcal{S},n\in\mathcal{N}_s^{(D)}}$ are the unique solution of problem (**Surrogate**).

B Proof of Lemma 5.1

Proof. Consider a fixed message $\vec{\mathbf{m}}_{-s}$ of all agents except agent s, an admissible production vector $(\tilde{e}_n^{(s)})_{n\in\mathcal{N}_s^{(E)}}$, and an admissible consumption vector $(\tilde{d}_n^{(s)})_{n\in\mathcal{N}_s^{(D)}}$ for agent s. We want to prove that there exists a message $\vec{m}_s$ for agent s such that

$$\hat{e}_n^{(s)}(\vec{m}_s, \vec{\mathbf{m}}_{-s}) = \tilde{e}_n^{(s)}, \qquad \forall n \in \mathcal{N}_s^{(E)}, \tag{81}$$
$$\hat{d}_n^{(s)}(\vec{m}_s, \vec{\mathbf{m}}_{-s}) = \tilde{d}_n^{(s)}, \qquad \forall n \in \mathcal{N}_s^{(D)}, \tag{82}$$

For that matter, consider the following modification of problem (**Surrogate**). Set $(e_n^{(s)})_{n\in\mathcal{N}_s^{(E)}}$ and $(d_n^{(s)})_{n\in\mathcal{N}_s^{(D)}}$ equal to fixed vectors $(\tilde{e}_n^{(s)})_{n\in\mathcal{N}_s^{(E)}}$ and $(\tilde{d}_n^{(s)})_{n\in\mathcal{N}_s^{(D)}}$. Call the resulting problem (**Surrogate**$^{(-s)}$), which maximizes the sum of surrogate functions of all agents except agent s, when production vector $(\tilde{e}_n^{(s)})_{n\in\mathcal{N}_s^{(E)}}$ and consumption vector $(\tilde{d}_n^{(s)})_{n\in\mathcal{N}_s^{(D)}}$ are reserved for agent s. Because of Assumption 3, the resulting problem (**Surrogate**$^{(-s)}$) has at least one feasible solution. In addition, because of Assumptions 8 and 9, constraints (12)-(17), and the fact that the weight factors $(w_n^{(s')})_{s'\in\mathcal{S}-\{s\},n\in\mathcal{N}_{s'}^{(E)}}$ and $(v_n^{(s')})_{s'\in\mathcal{S}-\{s\},n\in\mathcal{N}_{s'}^{(D)}}$ are strictly positive by the specification of the mechanism, the resulting problem (**Surrogate**$^{(-s)}$) is a strictly concave optimization problem with a convex domain where Slater's constraint qualification condition is satisfied. Thus, the KKT conditions are sufficient and necessary for optimality.

Denote by $(\tilde{e}_n^{(s')})_{s'\in\mathcal{S}-\{s\},n\in\mathcal{N}_{s'}^{(E)}}$, $(\tilde{d}_n^{(s')})_{s'\in\mathcal{S}-\{s\},n\in\mathcal{N}_{s'}^{(E)}}$, and $(\tilde{\theta}_{nn'})_{n\in\mathcal{N},n'\in\mathcal{R}_n}$, an optimal solution of problem (**Surrogate**$^{(-s)}$), and by $(\lambda_n)_{n\in\mathcal{N}}$, $(\underline{\varphi}_n^{(s')})_{s\in\mathcal{S}-\{s\},n\in\mathcal{N}_{s'}^{(E)}}$, $(\overline{\varphi}_n^{(s')})_{s'\in\mathcal{S}-\{s\},n\in\mathcal{N}_{s'}^{(E)}}$, $(\underline{\varsigma}_n^{(s')})_{s'\in\mathcal{S}-\{s\},n\in\mathcal{N}_{s'}^{(D)}}$, $(\overline{\varsigma}_n^{(s')})_{s'\in\mathcal{S}-\{s\},n\in\mathcal{N}_{s'}^{(D)}}$, $(\mu_{nn'})_{n\in\mathcal{N},n'\in\mathcal{R}_n}$ $(\chi_{nn'})_{n\in\mathcal{N},n'\in\mathcal{R}_n}$, λ_{ref} a set of Lagrange multipliers which along with $(\tilde{e}_n^{(s')})_{s'\in\mathcal{S}-\{s\},n\in\mathcal{N}_{s'}^{(E)}}$ and $(\tilde{d}_n^{(s')})_{s'\in\mathcal{S}-\{s\},n\in\mathcal{N}_{s'}^{(E)}}$ satisfy the KKT conditions of problem (**Surrogate**$^{(-s)}$). If we write the KKT conditions of problem (**Surrogate**) (see Appendix A.2) and we select message vector

$\vec{m}_s = (\vec{w}_s, \vec{v}_s, \vec{p}_s, \vec{q}_s)$ for agent s so that

$$w_n^{(s)} = \frac{\lambda_n + \lambda_{ref}}{\frac{\partial f^{(E)}}{\partial y}\Big|_{y=\tilde{e}_n^{(s)}}}, \qquad \forall n \in \mathcal{N}_s^{(E)}, \tag{83}$$

$$v_n^{(s)} = \frac{\lambda_n + \lambda_{ref}}{\frac{\partial f^{(D)}}{\partial y}\Big|_{y=\tilde{d}_n^{(s)}}}, \qquad \forall n \in \mathcal{N}_s^{(D)}, \tag{84}$$

then, the KKT conditions of problem (**Surrogate**$^{(-s)}$) are satisfied Furthermore, by (18)-(19) and (83)-(84) we obtain (81)-(82). To complete the proof of Lemma, we need to prove that $\lambda_n + \lambda_{ref} > 0, \forall n \in \mathcal{N}$; this will show that $(w_n^{(s)})_{s\in\mathcal{S}, n\in\mathcal{N}_s^{(E)}}$ and $(v_n^{(s)})_{s\in\mathcal{S}, n\in\mathcal{N}_s^{(D)}}$, defined by (83) and (84), are strictly positive. We prove that $\lambda_n + \lambda_{ref} > 0, \forall n \in \mathcal{N}$ by contradiction. Suppose $\exists n \in \mathcal{N}$ such that $\lambda_n + \lambda_{ref} = 0$. Then, the constraint (12) for n and the constraint (17) are non-binding in problem (**Surrogate**$^{(-s)}$), and this contradicts the optimality of $(\tilde{e}_n^{(s')})_{s'\in\mathcal{S}-\{s\}, n\in\mathcal{N}_{s'}^{(E)}}$, $(\tilde{d}_n^{(s')})_{s'\in\mathcal{S}-\{s\}, n\in\mathcal{N}_{s'}^{(E)}}$, and $(\tilde{\theta}_{nn'})_{n\in\mathcal{N}, n'\in\mathcal{R}_n}$ in problem (**Surrogate**$^{(-s)}$) because the productions $(\tilde{e}_n^{(s')})_{s'\in\Omega_n-\{s\}, n'\in\mathcal{N}_{s'}^{(E)}}$ can be reduced or the consumptions $(\tilde{d}_n^{(s')})_{s'\in\Omega_n-\{s\}, n\in\mathcal{N}_{s'}^{(E)}}$ can be increased and, as a result, the objective of problem (**Surrogate**$^{(-s)}$) will be improved. □

C Proof of Lemma 5.2; Nash Equilibrium Existence

Proof. We define the message profile $\vec{\mathbf{m}}^* := (\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*, \vec{\mathbf{p}}^*, \vec{\mathbf{q}}^*)$ given by (85)-(88) below,

$$(w_n^{(s)})^* = \frac{\frac{\partial c_n^{(s)}}{\partial e_n^{(s)}}\Big|_{(e_n^{(s)})^*}}{\frac{\partial f^{(E)}}{\partial y}\Big|_{y=(e_n^{(s)})^*}}, \qquad \forall s \in \mathcal{S}, \forall n \in \mathcal{N}_s^{(E)}, \tag{85}$$

$$(v_n^{(s)})^* = \frac{\frac{\partial u_n^{(s)}}{\partial d_n^{(s)}}\Big|_{(d_n^{(s)})^*}}{\frac{\partial f^{(D)}}{\partial y}\Big|_{y=(d_n^{(s)})^*}}, \qquad \forall s \in \mathcal{S}, \forall n \in \mathcal{N}_s^{(D)}, \tag{86}$$

$$(p_{n'}^{(s)})^* = \hat{\lambda}_{n'}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*) + \hat{\lambda}_{ref}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*), \qquad \forall s \in \mathcal{S}, \forall n' \in \Delta_s, \tag{87}$$

$$(q_{nn'}^{(s)})^* = \hat{\mu}_{nn'}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*) + \frac{\hat{L}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*).\hat{\lambda}_{ref}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*)}{2.\overline{F}_{nn'}}, \qquad \forall s \in \mathcal{S}, \forall (n, n') \in \Phi_s, \tag{88}$$

where $\left((e_n^{(s)})^*\right)_{s\in\mathcal{S}, n\in\mathcal{N}_s^{(E)}}$ and $\left((d_n^{(s)})^*\right)_{s\in\mathcal{S}, n\in\mathcal{N}_s^{(D)}}$ are the unique optimal solution of problem (**OPF**), and $\hat{\lambda}_{n'}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*)$, $\hat{\mu}_{nn'}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*)$, $\hat{L}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*)$, $\hat{\lambda}_{ref}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*)$ are the solution of problem (**Surrogate**) and are defined by (21)-(24).

We want to show that $\vec{\mathbf{m}}^* := (\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*, \vec{\mathbf{p}}^*, \vec{\mathbf{q}}^*)$ is a Nash equilibrium of the game induced by the proposed mechanism. We proceed in three stages.

First stage:

Claim 1. *Consider a fixed message profile $\vec{\mathbf{m}}_{-s}$ for all agents other than s and let $\vec{m}'_s$ be the best response of agent s to $\vec{\mathbf{m}}_{-s}$. Define $\Lambda_s(\vec{m}_s, \vec{\mathbf{m}}_{-s})$ by*

$$\Lambda_s(\vec{m}_s, \vec{\mathbf{m}}_{-s}) := \sum_{n' \in \Delta_s} \left(p_{n'}^{(s)} - \hat{\lambda}_{n'}(\vec{m}_s, \vec{\mathbf{m}}_{-s}) - \hat{\lambda}_{ref}(\vec{m}_s, \vec{\mathbf{m}}_{-s}) \right)^2$$
$$+ \sum_{(n',n'') \in \Phi_s} \left(q_{n'n''}^{(s)} - \hat{\mu}_{n'n''}(\vec{m}_s, \vec{\mathbf{m}}_{-s}) - \frac{\hat{\lambda}_{ref}(\vec{m}_s, \vec{\mathbf{m}}_{-s}).\hat{L}_{n'n''}(\vec{m}_s, \vec{\mathbf{m}}_{-s})}{2.\overline{F}_{n'n''}} \right)^2. \tag{89}$$

We prove $\Lambda_s(\vec{m}'_s, \vec{\mathbf{m}}_{-s}) = 0, \forall s \in \mathcal{S}$.

Proof. We establish this claim by contradiction. Assume $\Lambda_s(\vec{m}'_s, \vec{\mathbf{m}}_{-s}) \neq 0$, then agent s's utility at $(\vec{m}'_s, \vec{\mathbf{m}}_{-s})$ is

$$\mathrm{U}_s(\vec{m}'_s, \vec{\mathbf{m}}_{-s}) = \sum_{n \in \mathcal{N}_s^{(D)}} \left(u_n^{(s)}(\hat{d}_n^{(s)}(\vec{m}'_s, \vec{\mathbf{m}}_{-s})) - P_n^{(s)}(\vec{\mathbf{m}}_{-s}).\hat{d}_n^{(s)}(\vec{m}'_s, \vec{\mathbf{m}}_{-s}) \right)$$
$$- \sum_{n \in \mathcal{N}_s^{(E)}} \left(c_n^{(s)}(\hat{e}_n^{(s)}(\vec{m}'_s, \vec{\mathbf{m}}_{-s})) - P_n^{(s)}(\vec{\mathbf{m}}_{-s}).\hat{e}_n^{(s)}(\vec{m}'_s, \vec{\mathbf{m}}_{-s}) \right)$$
$$+ \sum_{n' \in \mathcal{N}, n'' \in \mathcal{R}_{n'}} Q_{n'n''}^{(s)}(\vec{\mathbf{m}}_{-s}).\alpha_{n'n''}^{(s)} - \Lambda_s(\vec{m}'_s, \vec{\mathbf{m}}_{-s}). \tag{90}$$

Agent s can change his message to $\vec{m}''_s := (\vec{w}'_s, \vec{v}'_s, \vec{p}''_s, \vec{q}''_s)$, where $(\vec{w}'_s, \vec{v}'_s)$ are not changed while the price proposal $(\vec{p}''_s, \vec{q}''_s)$ is changed to

$$(p_{n'}^{(s)})'' = \hat{\lambda}_{n'}(\vec{m}'_s, \vec{\mathbf{m}}_{-s}) + \hat{\lambda}_{ref}(\vec{m}'_s, \vec{\mathbf{m}}_{-s}), \qquad \forall n' \in \Delta_s, \tag{91}$$
$$(q_{n'n''}^{(s)})'' = \hat{\mu}_{n'n''}(\vec{m}'_s, \vec{\mathbf{m}}_{-s}) + \frac{\hat{\lambda}_{ref}(\vec{m}'_s, \vec{\mathbf{m}}_{-s}).\hat{L}_{n'n''}(\vec{m}'_s, \vec{\mathbf{m}}_{-s})}{2.\overline{F}_{n'n''}}, \qquad \forall (n', n'') \in \Phi_s. \tag{92}$$

Then, agent s's utility at $(\vec{m}''_s, \vec{\mathbf{m}}_{-s})$ is given by (93).

$$\mathrm{U}_s(\vec{m}''_s, \vec{\mathbf{m}}_{-s}) = \sum_{n \in \mathcal{N}_s^{(D)}} \left(u_n^{(s)}(\hat{d}_n^{(s)}(\vec{m}''_s, \vec{\mathbf{m}}_{-s})) - P_n^{(s)}(\vec{\mathbf{m}}_{-s}).\hat{d}_n^{(s)}(\vec{m}''_s, \vec{\mathbf{m}}_{-s}) \right)$$
$$- \sum_{n \in \mathcal{N}_s^{(E)}} \left(c_n^{(s)}(\hat{e}_n^{(s)}(\vec{m}''_s, \vec{\mathbf{m}}_{-s})) - P_n^{(s)}(\vec{\mathbf{m}}_{-s}).\hat{e}_n^{(s)}(\vec{m}''_s, \vec{\mathbf{m}}_{-s}) \right)$$
$$+ \sum_{n' \in \mathcal{N}, n'' \in \mathcal{R}_{n'}} Q_{n'n''}^{(s)}(\vec{\mathbf{m}}_{-s}).\alpha_{n'n''}^{(s)}. \tag{93}$$

Using (18)-(19), we conclude that the first three terms of (90) are equal to the three terms of (93). Since $\Lambda_s(\vec{m}'_s, \vec{\mathbf{m}}_{-s})$ is positive, as it is the sum of positive square terms, we conclude $\mathrm{U}_s(\vec{m}''_s, \vec{\mathbf{m}}_{-s}) > \mathrm{U}_s(\vec{m}'_s, \vec{\mathbf{m}}_{-s})$. This means $\vec{m}'_s$ is not a best response to $\vec{\mathbf{m}}_{-s}$, a contradiction. □

Second stage:

Using the result of Claim 1, agent s's utility at $(\vec{m}'_s, \vec{\mathbf{m}}_{-s})$ is given by

$$\begin{aligned} \mathrm{U}_s(\vec{m}'_s, \vec{\mathbf{m}}_{-s}) = &\sum_{n\in\mathcal{N}_s^{(D)}} \Big(u_n^{(s)}(\hat{d}_n^{(s)}(\vec{m}'_s, \vec{\mathbf{m}}_{-s})) - P_n^{(s)}(\vec{\mathbf{m}}_{-s}).\hat{d}_n^{(s)}(\vec{m}'_s, \vec{\mathbf{m}}_{-s})\Big) \\ &- \sum_{n\in\mathcal{N}_s^{(E)}} \Big(c_n^{(s)}(\hat{e}_n^{(s)}(\vec{m}'_s, \vec{\mathbf{m}}_{-s})) - P_n^{(s)}(\vec{\mathbf{m}}_{-s}).\hat{e}_n^{(s)}(\vec{m}'_s, \vec{\mathbf{m}}_{-s})\Big) \\ &+ \sum_{n'\in\mathcal{N}, n''\in\mathcal{R}_{n'}} Q_{n'n''}^{(s)}(\vec{\mathbf{m}}_{-s}).\alpha_{n'n''}^{(s)}, \end{aligned} \tag{94}$$

where the message profile $\vec{m}_s$ is replaced by best response message profile $\vec{m}'_s$ of agent s and $\vec{\mathbf{m}}_{-s}$ is fixed. Note that the third term of (94) does not depend on the message vector $\vec{m}'_s$. In addition, using the argument of Lemma 5.1, there exists a message profile for agent s such that any production and consumption vector, such as $(\hat{e}_n^{(s)}(\vec{m}'_s, \vec{\mathbf{m}}_{-s}))_{n\in\mathcal{N}_s^{(E)}}$ and $(\hat{d}_n^{(s)}(\vec{m}'_s, \vec{\mathbf{m}}_{-s}))_{n\in\mathcal{N}_s^{(D)}}$, can be achieved. Therefore, $(\hat{e}_n^{(s)}(\vec{m}'_s, \vec{\mathbf{m}}_{-s}))_{n\in\mathcal{N}_s^{(E)}}$ and $(\hat{d}_n^{(s)}(\vec{m}'_s, \vec{\mathbf{m}}_{-s}))_{n\in\mathcal{N}_s^{(D)}}$ are the solutions of the following optimization problem for agent s, called problem $(\mathbf{BR}^{(s)})$.

$$\max_{e_n^{(s)}, d_n^{(s)}} \sum_{n\in\mathcal{N}_s^{(D)}} \Big(u_n^{(s)}(d_n^{(s)}) - P_n^{(s)}(\vec{\mathbf{m}}_{-s}).d_n^{(s)}\Big) - \sum_{n\in\mathcal{N}_s^{(E)}} \Big(c_n^{(s)}(e_n^{(s)}) - P_n^{(s)}(\vec{\mathbf{m}}_{-s}).e_n^{(s)}\Big) \tag{$\mathbf{BR}^{(s)}$}$$

subject to:

$$0 \le d_n^{(s)} \le \overline{D}_n^{(s)}, \qquad \forall n \in \mathcal{N}_s^{(D)}, \tag{95}$$

$$0 \le e_n^{(s)} \le \overline{E}_n^{(s)}, \qquad \forall n \in \mathcal{N}_s^{(E)}. \tag{96}$$

Because of Assumptions 1 and 2 and constraints (95)-(96), $(\mathbf{BR}^{(s)})$ is strictly concave optimization problem with a convex domain, where Slater's constraint qualification condition is satisfied. As a result, the KKT conditions are sufficient and necessary for optimality. Define the Lagrangain $\mathcal{L}^{(BR^{(s)})}$ associated with $(\mathbf{BR}^{(s)})$. Then, the KKT conditions for problem $(\mathbf{BR}^{(s)})$ are

$$\frac{\partial \mathcal{L}^{(BR^{(s)})}}{\partial d_n^{(s)}} = \frac{\partial u_n^{(s)}}{\partial d_n^{(s)}}\bigg|_{\hat{d}_n^{(s)}(\vec{m}'_s, \vec{\mathbf{m}}_{-s})} - P_n^{(s)}(\vec{\mathbf{m}}_{-s}) - \overline{\zeta}_n^{(s)} + \underline{\zeta}_n^{(s)} = 0, \qquad \forall n \in \mathcal{N}_s^{(D)}, \tag{97}$$

$$\frac{\partial \mathcal{L}^{(BR^{(s)})}}{\partial e_n^{(s)}} = -\frac{\partial c_n^{(s)}}{\partial e_n^{(s)}}\bigg|_{\hat{e}_n^{(s)}(\vec{m}'_s, \vec{\mathbf{m}}_{-s})} + P_n^{(s)}(\vec{\mathbf{m}}_{-s}) - \overline{\beta}_n^{(s)} + \underline{\beta}_n^{(s)} = 0, \qquad \forall n \in \mathcal{N}_s^{(E)}, \tag{98}$$

$$(\overline{D}_n^{(s)} - \hat{d}_n^{(s)}(\vec{m}'_s, \vec{\mathbf{m}}_{-s})).\overline{\zeta}_n^{(s)} = 0, \qquad \forall n \in \mathcal{N}_s^{(D)}, \tag{99}$$

$$\hat{d}_n^{(s)}(\vec{m}'_s, \vec{\mathbf{m}}_{-s}).\underline{\zeta}_n^{(s)} = 0, \qquad \forall n \in \mathcal{N}_s^{(D)}, \tag{100}$$

$$(\overline{E}_n^{(s)} - \hat{e}_n^{(s)}(\vec{m}'_s, \vec{\mathbf{m}}_{-s})).\overline{\beta}_n^{(s)} = 0, \qquad \forall n \in \mathcal{N}_s^{(E)}, \tag{101}$$

$$\hat{e}_n^{(s)}(\vec{m}'_s, \vec{\mathbf{m}}_{-s}).\underline{\beta}_n^{(s)} = 0, \qquad \forall n \in \mathcal{N}_s^{(E)}, \tag{102}$$

$$(95) - (96), \tag{103}$$

where $\underline{\zeta}_n^{(s)}$, $\overline{\zeta}_n^{(s)}$, $\underline{\beta}_n^{(s)}$, and $\overline{\beta}_n^{(s)}$ denote the centroid of set of Lagrange multiplier vectors corresponding to constraints (95)-(96).

Third stage:

Write the KKT conditions of problem (**Surrogate**) (see Appendix A.2) for the fixed message profile $(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*)$ described by (85)-(86). Note that because of (18) and (19), $\hat{e}_n^{(s)}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*) = \hat{e}_n^{(s)}(\vec{\mathbf{m}}^*)$ and $\hat{d}_n^{(s)}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*) = \hat{d}_n^{(s)}(\vec{\mathbf{m}}^*)$. Using the KKT conditions (13)-(14), (69)-(70), (73)-(76), and setting $\underline{\varphi}_n^{(s)} = \underline{\zeta}_n^{(s)}$, $\overline{\varphi}_n^{(s)} = \overline{\zeta}_n^{(s)}$, $\underline{\varsigma}_n^{(s)} = \underline{\beta}_n^{(s)}$, $\overline{\varsigma}_n^{(s)} = \overline{\beta}_n^{(s)}$, the KKT conditions of problem $(\mathbf{BR}^{(s)})$ described by (97)-(103) are satisfied for all agents $s \in \mathcal{S}$. This shows that for the message profile $(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*)$ defined by (85)-(86), agent s cannot achieve any better production and consumption vectors than $(\hat{e}_n^{(s)}(\vec{m}_s^*, \vec{\mathbf{m}}_{-s}^*))_{n\in\mathcal{N}_s^{(E)}}$ and $(\hat{d}_n^{(s)}(\vec{m}_s^*, \vec{\mathbf{m}}_{-s}^*))_{n\in\mathcal{N}_s^{(D)}}$ by any unilateral deviation from $\vec{m}_s^*$. The only incentive for deviation agent s could possibly have is to reduce payment by changing his price proposal. However, at message profile $\vec{\mathbf{m}}^*$, the price proposal $(\vec{\mathbf{p}}^*, \vec{\mathbf{q}}^*)$ is selected by (87)-(88) and the function $\Lambda_s(\vec{m}_s^*, \vec{\mathbf{m}}_{-s}^*)$ (defined by (89)) is equal to zero. Therefore, for each agent s, message $\vec{m}_s^*$ is the best response to $\vec{\mathbf{m}}_{-s}^*$. Consequently, the message profile $\vec{\mathbf{m}}^*$ is a Nash equilibrium of the game induced by the proposed mechanism. □

D Proof of Lemma 5.3

Proof. Let $\vec{\mathbf{m}}^* := (\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*, \vec{\mathbf{p}}^*, \vec{\mathbf{q}}^*)$ be a Nash equilibrium of the game induced by the proposed mechanism. Assume that all agents except agent s adhere to the message profile $\vec{\mathbf{m}}_{-s}^*$, and agent s broadcasts message $\vec{m}_s := (\vec{w}_s^*, \vec{v}_s^*, \vec{p}_s, \vec{q}_s)$, with arbitrary price vectors $\vec{p}_s := (p_{n'}^{(s)})_{n'\in\Delta_s} \in \mathbb{R}^{|\Delta_s|}$ and $\vec{q}_s := (q_{n'n''}^{(s)})_{(n',n'')\in\Phi_s} \in \mathbb{R}^{|\Phi_s|}$. We prove that agent s's price proposals vectors in $\vec{\mathbf{m}}^*$ are as (31) and (32).

Since $\vec{\mathbf{m}}^*$ is a Nash equilibrium of the game induced by the proposed mechanism, we must have

$$\begin{aligned}&\mathrm{U}_s\left((\hat{e}_n^{(s)}(\vec{m}_s, \vec{\mathbf{m}}_{-s}^*))_{n\in\mathcal{N}_s^{(E)}}, (\hat{d}_n^{(s)}(\vec{m}_s, \vec{\mathbf{m}}_{-s}^*))_{n\in\mathcal{N}_s^{(D)}}, t_s(\vec{m}_s, \vec{\mathbf{m}}_{-s}^*)\right)\\ &\quad\le \mathrm{U}_s\left((\hat{e}_n^{(s)}(\vec{\mathbf{m}}^*))_{n\in\mathcal{N}_s^{(E)}}, (\hat{d}_n^{(s)}(\vec{\mathbf{m}}^*))_{n\in\mathcal{N}_s^{(D)}}, t_s(\vec{\mathbf{m}}^*)\right), \qquad \forall s \in \mathcal{S}. \end{aligned} \tag{104}$$

Since by (18) and (19) productions and consumptions do not depend on the price proposals, we have

$$\begin{aligned}&\sum_{n\in\mathcal{N}_s^{(D)}} u_n^{(s)}(\hat{d}_n^{(s)}(\vec{m}_s, \vec{\mathbf{m}}_{-s}^*)) - \sum_{n\in\mathcal{N}_s^{(E)}} c_n^{(s)}(\hat{e}_n^{(s)}(\vec{m}_s, \vec{\mathbf{m}}_{-s}^*))\\ &\quad= \sum_{n\in\mathcal{N}_s^{(D)}} u_n^{(s)}(\hat{d}_n^{(s)}(\vec{\mathbf{m}}^*)) - \sum_{n\in\mathcal{N}_s^{(E)}} c_n^{(s)}(\hat{e}_n^{(s)}(\vec{\mathbf{m}}^*)), \qquad \forall s \in \mathcal{S}. \end{aligned} \tag{105}$$

Using (4), (104), and (105), we get

$$t_s\left(\vec{m}_s, \mathbf{m}_{-\mathbf{s}}^*\right) \ge t_s\left(\vec{\mathbf{m}}^*\right), \qquad \forall s \in \mathcal{S}, \tag{106}$$

where the payment function $t_s(\vec{\mathbf{m}})$ is defined by (25).

Because of (26) and (27), we have

$$P_n^{(s)}(\vec{m}_s, \vec{\mathbf{m}}_{-s}^*) = P_n^{(s)}(\vec{\mathbf{m}}^*), \qquad \forall s \in \mathcal{S}, \forall n \in (\mathcal{N}_s^{(E)} \cup \mathcal{N}_s^{(D)}), \tag{107}$$

$$Q^{(s)}_{n'n''}(\vec{m}_s, \vec{\mathbf{m}}^*_{-s}) = Q^{(s)}_{n'n''}(\vec{\mathbf{m}}^*), \qquad \forall s \in \mathcal{S}, \forall n' \in \mathcal{N}, \forall n'' \in \mathcal{R}_{n'}. \tag{108}$$

Therefore, because of (18)-(19) and (107)-(108), we obtain

$$\begin{aligned}
&\sum_{n\in\mathcal{N}_s^{(D)}} P^{(s)}_n(\vec{m}_s, \vec{\mathbf{m}}^*_{-s}).\hat{d}^{(s)}_n(\vec{m}_s, \vec{\mathbf{m}}^*_{-s}) - \sum_{n\in\mathcal{N}_s^{(E)}} P^{(s)}_n(\vec{m}_s, \vec{\mathbf{m}}^*_{-s}).\hat{e}^{(s)}_n(\vec{m}_s, \vec{\mathbf{m}}^*_{-s}) \\
&\quad - \sum_{n'\in\mathcal{N}, n''\in\mathcal{R}_{n'}} Q^{(s)}_{n'n''}(\vec{m}_s, \vec{\mathbf{m}}^*_{-s}).\alpha^{(s)}_{n'n''} = \sum_{n\in\mathcal{N}_s^{(D)}} P^{(s)}_n(\vec{\mathbf{m}}^*).\hat{d}^{(s)}_n(\vec{\mathbf{m}}^*) \\
&\quad - \sum_{n\in\mathcal{N}_s^{(E)}} P^{(s)}_n(\vec{\mathbf{m}}^*).\hat{e}^{(s)}_n(\vec{\mathbf{m}}^*) - \sum_{n'\in\mathcal{N}, n''\in\mathcal{R}_{n'}} Q^{(s)}_{n'n''}(\vec{\mathbf{m}}^*).\alpha^{(s)}_{n'n''}, \qquad \forall s \in \mathcal{S}.
\end{aligned} \tag{109}$$

Substituting (25) and (109) in (106), and using (21)-(24), we obtain

$$\begin{aligned}
&\sum_{n'\in\Delta_s} \left(p^{(s)}_{n'} - \hat{\lambda}_{n'}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*) - \hat{\lambda}_{ref}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*)\right)^2 \\
&+ \sum_{(n',n'')\in\Phi_s} \left(q^{(s)}_{n'n''} - \hat{\mu}_{n'n''}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*) - \frac{\hat{\lambda}_{ref}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*).\hat{L}_{n'n''}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*)}{2.\overline{F}_{n'n''}}\right)^2 \\
&\geq \sum_{n'\in\Delta_s} \left((p^{(s)}_{n'})^* - \hat{\lambda}_{n'}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*) - \hat{\lambda}_{ref}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*)\right)^2 \\
&+ \sum_{(n',n'')\in\Phi_s} \left((q^{(s)}_{n'n''})^* - \hat{\mu}_{n'n''}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*) - \frac{\hat{\lambda}_{ref}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*).\hat{L}_{n'n''}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*)}{2.\overline{F}_{n'n''}}\right)^2, \forall s \in \mathcal{S}.
\end{aligned} \tag{110}$$

Thus, $\vec{p}^*_s := \left((p^{(s)}_{n'})^*\right)_{n'\in\Delta_s} \in \mathbb{R}^{|\Delta_s|}_{\geq 0}$ and $\vec{q}^*_s := \left((q^{(s)}_{n'n''})^*\right)_{(n',n'')\in\Phi_s} \in \mathbb{R}^{|\Phi_s|}_{\geq 0}$ are the optimal solution of the following problem.

$$\begin{aligned}
&\min_{\vec{p}_s\in\mathbb{R}^{|\Delta_s|}_{\geq 0}, \vec{q}_s\in\mathbb{R}^{|\Phi_s|}_{\geq 0}} \sum_{n'\in\Delta_s} \left(p^{(s)}_{n'} - \hat{\lambda}_{n'}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*) - \hat{\lambda}_{ref}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*)\right)^2 \\
&\quad + \sum_{(n',n'')\in\Phi_s} \left(q^{(s)}_{n'n''} - \hat{\mu}_{n'n''}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*) - \frac{\hat{\lambda}_{ref}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*).\hat{L}_{n'n''}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*)}{2.\overline{F}_{n'n''}}\right)^2.
\end{aligned} \tag{111}$$

This solution is

$$(p^{(s)}_{n'})^* = \hat{\lambda}_{n'}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*) + \hat{\lambda}_{ref}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*), \qquad \forall s \in \mathcal{S}, \forall n' \in \Delta_s, \tag{112}$$

$$(q^{(s)}_{n'n''})^* = \hat{\mu}_{n'n''}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*) + \frac{\hat{\lambda}_{ref}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*).\hat{L}_{n'n''}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*)}{2.\overline{F}_{n'n''}}, \qquad \forall s \in \mathcal{S}, \forall (n', n'') \in \Phi_s. \tag{113}$$

Substituting (112) and (113) in (26) and (27) completes the proof. □

E Proof of Theorem 1; Budget Balance

Proof. Using the results of Lemma 5.3, and substituting (31)-(32) in (25), the payment function for agent s at any Nash equilibrium $\vec{\mathbf{m}}^*$ of the game induced by the proposed mechanism is given (30). Then, using (30) the sum of payments to all agents at any Nash equilibrium $\vec{\mathbf{m}}^*$ is

$$\sum_{s\in\mathcal{S}} t_s(\vec{\mathbf{m}}^*) = \sum_{s\in\mathcal{S}} \left(\sum_{n\in\mathcal{N}_s^{(D)}} \left(\hat{\lambda}_n(\vec{\mathbf{m}}^*) + \hat{\lambda}_{ref}(\vec{\mathbf{m}}^*)\right).\hat{d}_n^{(s)}(\vec{\mathbf{m}}^*) - \sum_{n\in\mathcal{N}_s^{(E)}} \left(\hat{\lambda}_n(\vec{\mathbf{m}}^*) + \hat{\lambda}_{ref}(\vec{\mathbf{m}}^*)\right).\hat{e}_n^{(s)}(\vec{\mathbf{m}}^*) \right.$$
$$\left. - \sum_{n'\in\mathcal{N}} \sum_{n''\in\mathcal{R}_{n'}} \left(\hat{\mu}_{n'n''}(\vec{\mathbf{m}}^*) + \frac{\hat{\lambda}_{ref}(\vec{\mathbf{m}}^*).\hat{L}_{n'n''}(\vec{\mathbf{m}}^*)}{2.\overline{F}_{n'n''}} \right).\alpha_{n'n''}^{(s)} \right). \tag{114}$$

Based on the formulation of problem (**Surrogate**) described in Section 4.2 and using the method of calculation of LMPs described in [71], we obtain

$$\hat{\lambda}_n(\vec{\mathbf{m}}^*) = \sum_{n'\in\mathcal{N}} \sum_{n''\in\mathcal{R}_{n'}} \left(\frac{1}{2}.\hat{\lambda}_{ref}(\vec{\mathbf{m}}^*). \left.\frac{\partial \hat{L}_{n'n''}(\vec{\mathbf{m}})}{\partial \hat{e}_n(\vec{\mathbf{m}})}\right|_{\hat{e}_n(\vec{\mathbf{m}}^*)} + \mu_{n'n''}(\vec{\mathbf{m}}^*).\overline{F}_{n'n''} \right), \qquad \forall n\in\mathcal{N}. \tag{115}$$

Replacing (115) into the RHS of (114), and using $\sum_{s\in\mathcal{S}} \alpha_{n'n''}^{(s)} = \overline{F}_{n'n''}$ for all transmission lines, along with Kron's formula (see [72], pages 2144-2145), we find that the RHS of (114) is equal to zero. This means that we have budget balance at any Nash equilibrium $\vec{\mathbf{m}}^*$ of the game induced by the proposed mechanism. □

F Proof of Theorem 2; Implementation in Nash Equilibria

Proof. To prove that the outcome corresponding to every Nash equilibrium $\vec{\mathbf{m}}^*$ of the game induced by the proposed mechanism implements the unique solution of problem (**OPF**), it is sufficient to show that we can recover the KKT conditions of problem (**OPF**) from the KKT conditions of problem ($\mathbf{BR}^{(s)}$) for all agents $s \in \mathcal{S}$ along with the KKT conditions of problem (**Surrogate**). Then, by Theorem 1, at any Nash equilibrium of the game induced by the proposed mechanism the ISO's objective function in problem (**OPF**) is equal to the social welfare function. Thus, we conclude that the social welfare maximizing dispatch is implemented at every Nash equilibrium.

Write the KKT conditions of problem ($\mathbf{BR}^{(s)}$) for all agents $s \in \mathcal{S}$ (see (97)-(103)) along with the KKT conditions of problem (**Surrogate**) (see Appendix A.2). Let $(\tilde{e}_n^{(s)})_{s\in\mathcal{S},n\in\mathcal{N}_s^{(E)}}$ and $(\tilde{d}_n^{(s)})_{s\in\mathcal{S},n\in\mathcal{N}_s^{(E)}}$, $(\underline{\zeta}_n^{(s)})_{s\in\mathcal{S},n\in\mathcal{N}_s^{(D)}}$, $(\overline{\zeta}_n^{(s)})_{s\in\mathcal{S},n\in\mathcal{N}_s^{(D)}}$, $(\underline{\beta}_n^{(s)})_{s\in\mathcal{S},n\in\mathcal{N}_s^{(E)}}$, $(\overline{\beta}_n^{(s)})_{s\in\mathcal{S},n\in\mathcal{N}_s^{(E)}}$ describe a Nash equilibrium of the game along with the corresponding dual variables of problems ($\mathbf{BR}^{(s)}$) for each agent $s \in \mathcal{S}$. Using the result of Lemma 5.3, substitute $P_n^{(s)}(\vec{\mathbf{m}}^*)$ with $\hat{\lambda}_n(\vec{\mathbf{m}}^*) + \hat{\lambda}_{ref}(\vec{\mathbf{m}}^*)$ in (97) and (98). Then, the KKT conditions of problem (**OPF**) are satisfied (see Appendix A.1). □

G Proof of Theorem 3; Price Efficiency

Proof. Let $\vec{\mathbf{m}}^* := (\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*, \vec{\mathbf{p}}^*, \vec{\mathbf{q}}^*)$ be a Nash equilibrium of the game induced by the proposed mechanism. As proved in Lemma 5.3, the price that agent s at node n receives per unit of energy production is equal to the price that he pays per unit of energy consumption and it is equal to

$$P_n^{(s)}(\vec{\mathbf{p}}^*) = \hat{\lambda}_n(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*) + \hat{\lambda}_{ref}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*), \qquad \forall s \in \mathcal{S}, \forall n \in (\mathcal{N}_s^{(E)} \cup \mathcal{N}_s^{(D)}), \tag{116}$$

where $\hat{\lambda}_n(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*)$ and $\hat{\lambda}_{ref}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*)$ are the Lagrange multipliers of KKT conditions of problem (**Surrogate**) (see Appendix A.2).

As proved in Theorem 2, by writing the KKT conditions of problem ($\mathbf{BR}^{(s)}$) for all agents (given by (97)-(103) for all agents $s \in \mathcal{S}$) along with the KKT conditions of problem (**Surrogate**) (given in Appendix A.2), we establish the KKT conditions of problem (**OPF**) (given in Appendix A.1). Then, we conclude that $\hat{\lambda}_n(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*) + \hat{\lambda}_{ref}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*)$ is equal to $\lambda_n^{(O)}$, which is the Lagrange multiplier of constraint (6) in problem (**OPF**). Therefore, we conclude that price $\hat{\lambda}_n(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*) + \hat{\lambda}_{ref}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*)$ is equal to the LMP of node n. Based on the formulation of problem (**Surrogate**) described in Section 4.2 and using the method of calculation of LMPs described in [71], we conclude that $\hat{\lambda}_{ref}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*)$ is the REP and $\hat{\lambda}_n(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*)$ represents the sum of MCC and MLC of price at node n. □

H Proof of Theorem 4; Individual Rationality for Excludable Public Goods

Proof. As discussed in the paragraph above the statement of Theorem 4 (page 22), to prove individual rationality for excludable public goods when turning off the generators is either a feasible option or not, it suffices to show that the utility of each agent $s \in \mathcal{S}$ at any Nash equilibrium $\vec{\mathbf{m}}^*$ is non-negative, i.e.,

$$\mathrm{U}_s(\vec{\mathbf{m}}^*) \geq 0, \qquad \forall s \in \mathcal{S}. \tag{117}$$

To establish (117), we adopt the definition of Nash equilibrium, namely,

$$\mathrm{U}_s(\vec{\mathbf{m}}_s^*, \vec{\mathbf{m}}_{-s}^*) \geq \mathrm{U}_s(\vec{m}_s, \vec{\mathbf{m}}_{-s}^*), \qquad \forall \vec{m}_s \in \mathcal{M}_s, \forall s \in \mathcal{S}. \tag{118}$$

If we find a message vector $\vec{m}_s$ for agent s such that

$$\mathrm{U}_s(\vec{m}_s, \vec{\mathbf{m}}_{-s}^*) \geq 0, \tag{119}$$

then, using (118) and (119), we establish (117).

Consider admissible production vector $(\underline{e}_n^{(s)})_{n \in \mathcal{N}_s^{(E)}}$ and consumption vector $(0)_{n \in \mathcal{N}_s^{(D)}}$, where $\underline{e}_n^{(s)}$ is the minimum production level of generator $n \in \mathcal{N}_s^{(E)}$ (if turning off is a feasible option for generator $n \in \mathcal{N}_s^{(E)}$, then, $\underline{e}_n^{(s)}$ is equal to 0). By Lemma 5.1, we can find message $\vec{m}_s := (\vec{w}_s, \vec{v}_s, \vec{p}_s, \vec{q}_s)$ for agent s along with production vector $(\underline{e}_n^{(s)})_{n \in \mathcal{N}_s^{(E)}}$ and consumption vector $(0)_{n \in \mathcal{N}_s^{(D)}}$ such that

$$w_n^{(s)}.\left.\frac{\partial f^{(E)}}{\partial y}\right|_{y=\underline{e}_n^{(s)}} = \lambda_n + \lambda_{ref}, \qquad \forall n \in \mathcal{N}_s^{(E)}, \tag{120}$$

$$v_n^{(s)}.\left.\frac{\partial f^{(D)}}{\partial y}\right|_{y=0} = \lambda_n + \lambda_{ref}, \qquad \forall n \in \mathcal{N}_s^{(E)}, \tag{121}$$

$$p_{n'}^{(s)} = (p_{n'}^{(s)})^*, \qquad \forall n' \in \Delta_s, \tag{122}$$

$$q_{n'n''}^{(s)} = (q_{n'n''}^{(s)})^*, \qquad \forall (n', n'') \in \Phi_s, \tag{123}$$

are satisfied, where $((p_{n'}^{(s)})^*)_{n' \in \Delta_s}$ and $((q_{n'n''}^{(s)})^*)_{(n',n'') \in \Phi_s}$ are the price proposals of agent s at Nash equilibrium $\vec{\mathbf{m}}^*$ (which are equal to (31) and (32), respectively, as proved in Lemma 5.3), and λ_n and λ_{ref} are the Lagrange multipliers of problem (**Surrogate**$^{(-s)}$), which maximizes the sum of surrogate functions of all agents except agent s, when production vector $(\underline{e}_n^{(s)})_{n \in \mathcal{N}_s^{(E)}}$ and consumption vector $(0)_{n \in \mathcal{N}_s^{(D)}}$ are reserved for agent s.

Using (122)-(123), the payment described by (25) is equal to

$$\begin{aligned} t_s(\vec{m}_s, \vec{\mathbf{m}}_{-s}^*) = &\sum_{n \in \mathcal{N}_s^{(E\neq 0)}} \left(\hat{\lambda}_n\left(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*\right) + \hat{\lambda}_{ref}\left(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*\right)\right).\underline{e}_n^{(s)} \\ &+ \sum_{n' \in \mathcal{N}, n'' \in \mathcal{R}_{n'}} \left(\hat{\mu}_{n'n''}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*) + \frac{\hat{\lambda}_{ref}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*).\hat{L}_{n'n''}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*)}{2.\overline{F}_{n'n''}}\right).\alpha_{n'n''}^{(s)}. \end{aligned} \tag{124}$$

Combining (4) and (124) we find that agent s's utility for the above message $\vec{m}_s$ is

$$\begin{aligned} \mathrm{U}_s(\vec{m}_s, \vec{\mathbf{m}}_{-s}^*) = &\sum_{n \in \mathcal{N}_s^{(D)}} u_n^{(s)}(0) - \sum_{n \in \{\mathcal{N}_s^{(E)} - \mathcal{N}_s^{(E\neq 0)}\}} c_n^{(s)}(0) - \sum_{n \in \mathcal{N}_s^{(E\neq 0)}} c_n^{(s)}\left(\underline{e}_n^{(s)}\right) \\ &+ \sum_{n \in \mathcal{N}_s^{(E\neq 0)}} \left(\hat{\lambda}_n\left(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*\right) + \hat{\lambda}_{ref}\left(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*\right)\right).\underline{e}_n^{(s)} \\ &+ \sum_{n' \in \mathcal{N}, n'' \in \mathcal{R}_{n'}} \left(\hat{\mu}_{n'n''}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*) + \frac{\hat{\lambda}_{ref}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*).\hat{L}_{n'n''}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*)}{2.\overline{F}_{n'n''}}\right).\alpha_{n'n''}^{(s)}, \end{aligned} \tag{125}$$

where $\mathcal{N}_s^{(E\neq 0)}$ is the set of generators for which turning off is not a feasible option or $\underline{e}_n^{(s)} \neq 0, \forall n \in \mathcal{N}_s^{(E\neq 0)}$.

Because of Assumptions 1 and 2, we have $c_n^{(s)}(0) = 0, \forall n \in \{\mathcal{N}_s^{(E)} - \mathcal{N}_s^{(E\neq 0)}\}$ and $u_n^{(s)}(0) = 0, \forall n \in \mathcal{N}_s^{(D)}$, therefore

$$\begin{aligned} \mathrm{U}_s(\vec{m}_s, \vec{\mathbf{m}}_{-s}^*) = &\sum_{n \in \mathcal{N}_s^{(E\neq 0)}} \left(\left(\hat{\lambda}_n\left(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*\right) + \hat{\lambda}_{ref}\left(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*\right)\right).\underline{e}_n^{(s)} - c_n^{(s)}\left(\underline{e}_n^{(s)}\right)\right) \\ &+ \sum_{n' \in \mathcal{N}, n'' \in \mathcal{R}_{n'}} \left(\hat{\mu}_{n'n''}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*) + \frac{\hat{\lambda}_{ref}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*).\hat{L}_{n'n''}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*)}{2.\overline{F}_{n'n''}}\right).\alpha_{n'n''}^{(s)}. \end{aligned} \tag{126}$$

In addition, by Theorem 3, we have

$$\hat{\lambda}_n(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*) + \hat{\lambda}_{ref}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*) = \lambda_n^{(O)}, \tag{127}$$

where $\lambda_n^{(O)}$ is the LMP of node n and Lagrange multiplier of constraint (6) in problem (**OPF**). Using the KKT condition (58) of problem (**OPF**), we have

$$-\left.\frac{\partial c_n^{(s)}}{\partial e_n^{(s)}}\right|_{\underline{e}_n^{(s)}} + \lambda_n^{(O)} + \underline{\varphi}_n^{(s,O)} - \overline{\varphi}_n^{(s,O)} = 0, \qquad \forall n \in \mathcal{N}_s^{(E\neq 0)}. \tag{128}$$

Using the KKT conditions (61) and (67) of problem (**OPF**), we have $\underline{\varphi}_n^{(s,O)} = 0, \forall n \in \mathcal{N}_s^{(E\neq 0)}$ and $\overline{\varphi}_n^{(s,O)} \geq 0, \forall n \in \mathcal{N}_s^{(E\neq 0)}$. Therefore, from (61) and (67) we conclude that

$$\lambda_n^{(O)} \geq \left.\frac{\partial c_n^{(s)}}{\partial e_n^{(s)}}\right|_{\underline{e}_n^{(s)}}, \qquad \forall n \in \mathcal{N}_s^{(E\neq 0)}. \tag{129}$$

Using (125), (127), and (129), we obtain

$$\begin{aligned}\mathrm{U}_s(\vec{m}_s, \vec{\mathbf{m}}^*_{-s}) \geq & \sum_{n \in \mathcal{N}_s^{(E\neq 0)}} \left(\left.\frac{\partial c_n^{(s)}}{\partial e_n^{(s)}}\right|_{\underline{e}_n^{(s)}} .\underline{e}_n^{(s)} - c_n^{(s)}\left(\underline{e}_n^{(s)}\right)\right) \\ & + \sum_{n' \in \mathcal{N}, n'' \in \mathcal{R}_{n'}} \left(\hat{\mu}_{n'n''}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*) + \frac{\hat{\lambda}_{ref}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*).\hat{L}_{n'n''}(\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*)}{2.\overline{F}_{n'n''}} \right) .\alpha_{n'n''}^{(s)}.\end{aligned} \tag{130}$$

Since $c_n^{(s)}\left(e_n^{(s)}\right)$ is strictly convex function and $c_n^{(s)}(0) = 0$ (by Assumption 1), the first term of RHS of (130) is non-negative. Moreover, the second term of the RHS of (130), which is agent s's benefit from the FTR market at Nash equilibrium $\vec{\mathbf{m}}^*$, is non-negative. Thus, we conclude (119). This establishes (117) and proves the individual rationality of the proposed mechanism. □

I Proof of Theorem 7; Convergence of Proposed Tâtonnement-Process

Proof. Using the result of Theorem 6, we conclude that the allocations (energy production and consumption) resulting at each step of the proposed tâtonnement-process are feasible solutions of the problem (**OPF**). In the following, we show that the tâtonnement-process described by (37)-(40) is a contraction map; moreover, the unique fixed point of this contraction map is a Nash equilibrium of the game induced by the mechanism proposed in Section 4.

We proceed in five stages.

First stage:

In the tâtonnement-process described by (37)-(40), $(p_{n'}^{(s)})^{(\tau+1)}$ and $(q_{nn'}^{(s)})^{(\tau+1)}$ are solely functions of the previously announced message profile $(w_n^{(s)})^{(\tau)}$ and $(v_n^{(s)})^{(\tau)}$. They do not have any impact on the announced messages $(w_n^{(s)})^{(\tau')}$ and $(v_n^{(s)})^{(\tau')}$, $\forall \tau' > \tau$. Therefore, they do not affect the convergence and can be neglected from the convergence study of the proposed tâtonnement-process.

As a result of the above observation, proving the convergence of the tâtonnement-process described by

$$(w_n^{(s)})^{(\tau+1)} = \frac{\left.\frac{\partial c_n^{(s)}}{\partial e_n^{(s)}}\right|_{(\hat{e}_n^{(s)})^{(\tau)}}}{\left.\frac{\partial f^{(E)}}{\partial y}\right|_{y=(\hat{e}_n^{(s)})^{(\tau)}}}, \qquad \forall s \in \mathcal{S}, \forall n \in \mathcal{N}_s^{(E)}, \tag{131}$$

$$(v_n^{(s)})^{(\tau+1)} = \frac{\left.\frac{\partial u_n^{(s)}}{\partial d_n^{(s)}}\right|_{(\hat{d}_n^{(s)})^{(\tau)}}}{\left.\frac{\partial f^{(D)}}{\partial y}\right|_{y=(\hat{d}_n^{(s)})^{(\tau)}}}, \qquad \forall s \in \mathcal{S}, \forall n \in \mathcal{N}_s^{(D)}, \tag{132}$$

will establish the convergence of the tâtonnement-process described by (37)-(40). Therefore, we study the convergence of the tâtonnement-process described by (131)-(132) by choosing (41) and (42) as surrogate cost and utility functions, respectively. Then, the tâtonnement-process described by (131)-(132) is rewritten as

$$(w_n^{(s)})^{(\tau+1)} = \zeta_n^{(s)}\left(\vec{\mathbf{w}}^{(\tau)}, \vec{\mathbf{v}}^{(\tau)}\right) = \frac{\gamma_e \cdot \left.\frac{\partial c_n^{(s)}}{\partial e_n^{(s)}}\right|_{\hat{e}_n^{(s)}(\vec{\mathbf{w}}^{(\tau)}, \vec{\mathbf{v}}^{(\tau)})}}{\exp\left(\hat{e}_n^{(s)}(\vec{\mathbf{w}}^{(\tau)}, \vec{\mathbf{v}}^{(\tau)}) \big/ \gamma_e\right)}, \qquad \forall s \in \mathcal{S}, \forall n \in \mathcal{N}_s^{(E)}, \tag{133}$$

$$\begin{aligned}(v_n^{(s)})^{(\tau+1)} &= \xi_n^{(s)}\left(\vec{\mathbf{w}}^{(\tau)}, \vec{\mathbf{v}}^{(\tau)}\right) \qquad\qquad (134)\\ &= \left(\hat{d}_n^{(s)}(\vec{\mathbf{w}}^{(\tau)}, \vec{\mathbf{v}}^{(\tau)}) + \gamma_d\right) \cdot \left.\frac{\partial u_n^{(s)}}{\partial d_n^{(s)}}\right|_{\hat{d}_n^{(s)}(\vec{\mathbf{w}}^{(\tau)}, \vec{\mathbf{v}}^{(\tau)})}, \qquad \forall s \in \mathcal{S}, \forall n \in \mathcal{N}_s^{(D)},\end{aligned}$$

where $\vec{\mathbf{w}}^{(\tau)} = (w_n^{(s)})^{(\tau)}_{s\in\mathcal{S}, n\in\mathcal{N}_s^{(E)}}$ and $\vec{\mathbf{v}}^{(\tau)} = (v_n^{(s)})^{(\tau)}_{s\in\mathcal{S}, n\in\mathcal{N}_s^{(D)}}$.

Thus, we study the convergence of the tâtonnement-process $\langle A \rangle$ given by

$$A(\vec{\mathbf{w}}, \vec{\mathbf{v}}) \coloneqq \left\{ \begin{matrix} \left(\zeta_n^{(s)}(\vec{\mathbf{w}}, \vec{\mathbf{v}})\right)_{s\in\mathcal{S}, n\in\mathcal{N}_s^{(E)}} \\ \left(\xi_n^{(s)}(\vec{\mathbf{w}}, \vec{\mathbf{v}})\right)_{s\in\mathcal{S}, n\in\mathcal{N}_s^{(E)}} \end{matrix} \right\}. \tag{135}$$

Second stage:

We use the following result that is proved in [73].

Lemma I.1. *The tâtonnement-process $A(\vec{\mathbf{w}}, \vec{\mathbf{v}})$ is a contraction map if,*

$$\sup_{\vec{\mathbf{w}}, \vec{\mathbf{v}}} \|DA(\vec{\mathbf{w}}, \vec{\mathbf{v}})\| < 1, \tag{136}$$

where $DA(\vec{\mathbf{w}}, \vec{\mathbf{v}})$ is the Jacobian matrix of $A(\vec{\mathbf{w}}, \vec{\mathbf{v}})$ and $\|.\|$ is any matrix norm operator.

Therefore, using the absolute row-sum norm, the tâtonnement-process $A(\vec{\mathbf{w}}, \vec{\mathbf{v}})$ is a contraction map, if (137) and (138) are true.

$$\sum_{s'\in\mathcal{S}} \left(\sum_{n'\in\mathcal{N}_s^{(E)}} \left| \frac{\partial \zeta_n^{(s)}}{\partial w_{n'}^{(s')}}(\vec{\mathbf{w}}, \vec{\mathbf{v}}) \right| + \sum_{n'\in\mathcal{N}_s^{(D)}} \left| \frac{\partial \zeta_n^{(s)}}{\partial v_{n'}^{(s')}}(\vec{\mathbf{w}}, \vec{\mathbf{v}}) \right| \right) < 1, \qquad \forall \vec{\mathbf{w}}, \forall \vec{\mathbf{v}}, \forall s \in \mathcal{S}, \forall n \in \mathcal{N}_s^{(E)}, \tag{137}$$

$$\sum_{s'\in\mathcal{S}}\left(\sum_{n'\in\mathcal{N}_s^{(E)}}\left|\frac{\partial\xi_n^{(s)}}{\partial w_{n'}^{(s')}}(\vec{\mathbf{w}},\vec{\mathbf{v}})\right|+\sum_{n'\in\mathcal{N}_s^{(D)}}\left|\frac{\partial\xi_n^{(s)}}{\partial v_{n'}^{(s')}}(\vec{\mathbf{w}},\vec{\mathbf{v}})\right|\right)<1, \qquad \forall\vec{\mathbf{w}},\forall\vec{\mathbf{v}},\forall s\in\mathcal{S},\forall n\in\mathcal{N}_s^{(D)}. \tag{138}$$

In the subsequent stages, we determine conditions sufficient to guarantee that (137) and (138) are satisfied. For that matter, we determine upper bounds for the terms appearing in the LHS of (137) and (138), and specify the above mentioned sufficient conditions in terms of these upper bounds.

Third stage:

For the terms appearing in the LHS of (137) and (138), we write (139)-(142).

$$\left|\frac{\partial\zeta_n^{(s)}}{\partial w_{n'}^{(s')}}(\vec{\mathbf{w}},\vec{\mathbf{v}})\right|=\left|\frac{\partial\zeta_n^{(s)}}{\partial e_n^{(s)}}\left(\hat{e}_n^{(s)}(\vec{\mathbf{w}},\vec{\mathbf{v}})\right)\right|\cdot\left|\frac{\partial e_n^{(s)}}{\partial w_{n'}^{(s')}}(\vec{\mathbf{w}})\right|, \tag{139}$$

$$\left|\frac{\partial\zeta_n^{(s)}}{\partial v_{n'}^{(s')}}(\vec{\mathbf{w}},\vec{\mathbf{v}})\right|=\left|\frac{\partial\zeta_n^{(s)}}{\partial e_n^{(s)}}\left(\hat{e}_n^{(s)}(\vec{\mathbf{w}},\vec{\mathbf{v}})\right)\right|\cdot\left|\frac{\partial e_n^{(s)}}{\partial v_{n'}^{(s')}}(\vec{\mathbf{v}})\right|, \tag{140}$$

$$\left|\frac{\partial\xi_n^{(s)}}{\partial w_{n'}^{(s')}}(\vec{\mathbf{w}},\vec{\mathbf{v}})\right|=\left|\frac{\partial\xi_n^{(s)}}{\partial d_n^{(s)}}\left(\hat{d}_n^{(s)}(\vec{\mathbf{w}},\vec{\mathbf{v}})\right)\right|\cdot\left|\frac{\partial\hat{d}_n^{(s)}}{\partial w_{n'}^{(s')}}(\vec{\mathbf{w}})\right|, \tag{141}$$

$$\left|\frac{\partial\xi_n^{(s)}}{\partial v_{n'}^{(s')}}(\vec{\mathbf{w}},\vec{\mathbf{v}})\right|=\left|\frac{\partial\xi_n^{(s)}}{\partial d_n^{(s)}}\left(\hat{d}_n^{(s)}(\vec{\mathbf{w}},\vec{\mathbf{v}})\right)\right|\cdot\left|\frac{\partial d_n^{(s)}}{\partial v_{n'}^{(s')}}(\vec{\mathbf{v}})\right|. \tag{142}$$

We determines the upper bounds for different terms in the following three consecutive claims.

Claim 2 (Upper bound of $\left|\partial\zeta_n^{(s)}/\partial e_n^{(s)}\right|$)**.** *Define* γ_e *as in* (43)*. Then,*

$$\left|\frac{\partial\zeta_n^{(s)}}{\partial e_n^{(s)}}\right|\leq\left|\gamma_e.\frac{\partial^2 c_n^{(s)}}{\partial(e_n^{(s)})^2}(0)-\frac{\partial c_n^{(s)}}{\partial e_n^{(s)}}(0)\right|. \tag{143}$$

Proof. By some algebra we can show

$$\left.\frac{\partial\zeta_n^{(s)}}{\partial e_n^{(s)}}\right|_{\hat{e}_n^{(s)}(\vec{\mathbf{w}},\vec{\mathbf{v}})}=\frac{\left.\frac{\partial^2 c_n^{(s)}}{\partial(e_n^{(s)})^2}\right|_{\hat{e}_n^{(s)}(\vec{\mathbf{w}},\vec{\mathbf{v}})}-\left(\frac{1}{\gamma_e}\right)\cdot\left.\frac{\partial c_n^{(s)}}{\partial e_n^{(s)}}\right|_{\hat{e}_n^{(s)}(\vec{\mathbf{w}},\vec{\mathbf{v}})}}{\frac{1}{\gamma_e}.\exp\left(\frac{\hat{e}_n^{(s)}(\vec{\mathbf{w}},\vec{\mathbf{v}})}{\gamma_e}\right)}. \tag{144}$$

Because of Assumption 1 and the definition of γ_e in (43), the RHS of (144) is decreasing in $\hat{e}_n^{(s)}(\vec{\mathbf{w}},\vec{\mathbf{v}})$. Therefore, (143) is true. □

Claim 3 (Upper bound of $\left|\partial\xi_n^{(s)}/\partial d_n^{(s)}\right|$)**.** *Define* γ_d *as in* (44)*. Then,*

$$\left|\frac{\partial\xi_n^{(s)}}{\partial d_n^{(s)}}\right|\leq\left|\gamma_d.\frac{\partial^2 u_n^{(s)}}{\partial(d_n^{(s)})^2}(0)+\frac{\partial u_n^{(s)}}{\partial d_n^{(s)}}(0)\right|. \tag{145}$$

Proof. By some algebra we can show

$$\left.\frac{\partial \xi_n^{(s)}}{\partial d_n^{(s)}}\right|_{\hat{d}_n^{(s)}(\vec{\mathbf{w}},\vec{\mathbf{v}})} = \left.\frac{\partial^2 u_n^{(s)}}{\partial (d_n^{(s)})^2}\right|_{\hat{d}_n^{(s)}(\vec{\mathbf{w}},\vec{\mathbf{v}})} \cdot \left(\hat{d}_n^{(s)}(\vec{\mathbf{w}},\vec{\mathbf{v}}) + \gamma_d\right) + \left.\frac{\partial u_n^{(s)}}{\partial d_n^{(s)}}\right|_{\hat{d}_n^{(s)}(\vec{\mathbf{w}},\vec{\mathbf{v}})}. \tag{146}$$

Because of Assumption 2 and the definition of γ_d in (44), the RHS of (146) is decreasing in $\hat{d}_n^{(s)}(\vec{\mathbf{w}},\vec{\mathbf{v}})$. Therefore, (145) is true. □

Claim 4 (Upper bounds of $\left|\partial e_n^{(s)}/\partial w_{n'}^{(s')}\right|$, $\left|\partial e_n^{(s)}/\partial v_{n'}^{(s')}\right|$, $\left|\partial d_n^{(s)}/\partial w_{n'}^{(s')}\right|$, and $\left|\partial d_n^{(s)}/\partial v_{n'}^{(s')}\right|$)**.** *We have,*

$$\left|\frac{\partial e_n^{(s)}}{\partial w_{n'}^{(s')}}\right| \leq T_1.\frac{\left|\det\left(\begin{bmatrix}\mathbf{J_3}^\top & \mathbf{J_4}^\top\end{bmatrix}\cdot\begin{bmatrix}\mathbf{J_3} & \mathbf{J_4}\end{bmatrix}^\top\right)_{-(n,s),(n',s')}\right|}{\left|\det\left(\begin{bmatrix}\mathbf{J_3}^\top & \mathbf{J_4}^\top\end{bmatrix}\cdot\begin{bmatrix}\mathbf{J_3} & \mathbf{J_4}\end{bmatrix}^\top\right)\right|}, \tag{147}$$

$$\left|\frac{\partial e_n^{(s)}}{\partial v_{n'}^{(s')}}\right| \leq T_2.\frac{\left|\det\left(\begin{bmatrix}\mathbf{J_3}^\top & \mathbf{J_4}^\top\end{bmatrix}\cdot\begin{bmatrix}\mathbf{J_3} & \mathbf{J_4}\end{bmatrix}^\top\right)_{-(n,s),(n',s')}\right|}{\left|\det\left(\begin{bmatrix}\mathbf{J_3}^\top & \mathbf{J_4}^\top\end{bmatrix}\cdot\begin{bmatrix}\mathbf{J_3} & \mathbf{J_4}\end{bmatrix}^\top\right)\right|}, \tag{148}$$

$$\left|\frac{\partial d_n^{(s)}}{\partial w_{n'}^{(s')}}\right| \leq T_3.\frac{\left|\det\left(\begin{bmatrix}\mathbf{J_3}^\top & \mathbf{J_4}^\top\end{bmatrix}\cdot\begin{bmatrix}\mathbf{J_3} & \mathbf{J_4}\end{bmatrix}^\top\right)_{-(n,s),(n',s')}\right|}{\left|\det\left(\begin{bmatrix}\mathbf{J_3}^\top & \mathbf{J_4}^\top\end{bmatrix}\cdot\begin{bmatrix}\mathbf{J_3} & \mathbf{J_4}\end{bmatrix}^\top\right)\right|}, \tag{149}$$

$$\left|\frac{\partial d_n^{(s)}}{\partial v_{n'}^{(s')}}\right| \leq T_4.\frac{\left|\det\left(\begin{bmatrix}\mathbf{J_3}^\top & \mathbf{J_4}^\top\end{bmatrix}\cdot\begin{bmatrix}\mathbf{J_3} & \mathbf{J_4}\end{bmatrix}^\top\right)_{-(n,s),(n',s')}\right|}{\left|\det\left(\begin{bmatrix}\mathbf{J_3}^\top & \mathbf{J_4}^\top\end{bmatrix}\cdot\begin{bmatrix}\mathbf{J_3} & \mathbf{J_4}\end{bmatrix}^\top\right)\right|}. \tag{150}$$

The terms $\mathbf{J_3}$ and $\mathbf{J_4}$ are fixed matrices related to the electricity network topology. They are defined by

$$\begin{aligned}&\mathbf{J_3} = \left[(J_3)_{\kappa,n}\right]_{(\sum_{s\in\mathcal{S}}|\mathcal{N}_s^{(E)}|)\times|\mathcal{N}|},\\ &(J_3)_{\kappa,n} = \begin{cases}-1, & \textit{if agent } \max\left(s, \textit{subject to: } \sum_{s'<s}|\mathcal{N}_{s'}^{(E)}| \leq \kappa\right) \textit{ has a generator on node } n,\\ 0, & \textit{otherwise,}\end{cases}\end{aligned} \tag{151}$$

$$\begin{aligned}&\mathbf{J_4} = \left[(J_4)_{\kappa,n}\right]_{(\sum_{s\in\mathcal{S}}|\mathcal{N}_s^{(D)}|)\times|\mathcal{N}|},\\ &(J_4)_{\kappa,n} = \begin{cases}1, & \textit{if agent } \max\left(s, \sum_{s'\in\mathcal{S}}|\mathcal{N}_{s'}^{(E)}| + \sum_{s'<s}|\mathcal{N}_{s'}^{(D)}| \leq \kappa\right) \textit{ has a demand on node } n,\\ 0, & \textit{otherwise.}\end{cases}\end{aligned} \tag{152}$$

The terms T_1, T_2, T_3, and T_4 are parameters that are determined based on the utility or cost functions of generators and flexible demands, and they are defined by

$$T_1 = \frac{\frac{1}{\gamma_e}.\exp(\frac{\overline{e}}{\gamma_e}).\left(\frac{H_e}{\gamma_e}.\exp(\frac{\overline{e}}{\gamma_e})\right)^{g_{11}}.\left(\frac{H_d}{\gamma_d}\right)^{g_{12}}}{\left(\frac{h_e}{\gamma_e}\right)^{p_1}.\left(\frac{h_d}{\gamma_d+\overline{d}}\right)^{p_2}}, \tag{153}$$

$$T_2 = \frac{\frac{1}{\gamma_d}.\left(\frac{H_e}{\gamma_e}.\exp(\frac{\overline{e}}{\gamma_e})\right)^{g_{21}}.\left(\frac{H_d}{\gamma_d}\right)^{g_{22}}}{\left(\frac{h_e}{\gamma_e}\right)^{p_1}.\left(\frac{h_d}{\gamma_d+\overline{d}}\right)^{p_2}}, \tag{154}$$

$$T_3 = \frac{\frac{1}{\gamma_e}.\exp(\frac{\overline{e}}{\gamma_e}).\left(\frac{H_e}{\gamma_e}.\exp(\frac{\overline{e}}{\gamma_e})\right)^{g_{21}}.\left(\frac{H_d}{\gamma_d}\right)^{g_{22}}}{\left(\frac{h_e}{\gamma_e}\right)^{p_1}.\left(\frac{h_d}{\gamma_d+\overline{d}}\right)^{p_2}}, \tag{155}$$

$$T_4 = \frac{\frac{1}{\gamma_d}.\left(\frac{H_e}{\gamma_e}.\exp(\frac{\overline{e}}{\gamma_e})\right)^{g_{31}}.\left(\frac{H_d}{\gamma_d}\right)^{g_{32}}}{\left(\frac{h_e}{\gamma_e}\right)^{p_1}.\left(\frac{h_d}{\gamma_d+\overline{d}}\right)^{p_2}}, \tag{156}$$

where

$$h_e = \min_{(s\in\mathcal{S}, n\in\mathcal{N}_s^{(E)})}\{\frac{\partial c_n^{(s)}}{\partial e_n^{(s)}}(0)\}, \tag{157}$$

$$h_d = \min_{(s\in\mathcal{S}, n\in\mathcal{N}_s^{(D)})}\{\frac{\partial u_n^{(s)}}{\partial d_n^{(s)}}\left(\overline{D}_n^{(s)}\right)\}, \tag{158}$$

$$H_e = \max_{(s\in\mathcal{S}, n\in\mathcal{N}_s^{(E)})}\{\frac{\partial c_n^{(s)}}{\partial e_n^{(s)}}\left(\overline{E}_n^{(s)}\right)\}, \tag{159}$$

$$H_d = \max_{(s\in\mathcal{S}, n\in\mathcal{N}_s^{(D)})}\{\frac{\partial u_n^{(s)}}{\partial d_n^{(s)}}(0)\}, \tag{160}$$

$$\overline{e} = \max_{(s\in\mathcal{S}, n\in\mathcal{N}_s^{(D)})}\{\overline{E}_n^{(s)}\}, \tag{161}$$

$$\overline{d} = \max_{(s\in\mathcal{S}, n\in\mathcal{N}_s^{(D)})}\{\overline{D}_n^{(s)}\}, \tag{162}$$

$$p_1 = \sum_{s\in\mathcal{S}}|\mathcal{N}_s^{(E)}| - \frac{\sum_{s\in\mathcal{S}}|\mathcal{N}_s^{(E)}|}{\sum_{s\in\mathcal{S}}|\mathcal{N}_s^{(E)}| + \sum_{s\in\mathcal{S}}|\mathcal{N}_s^{(D)}|}.(|\mathcal{N}|+1), \tag{163}$$

$$p_2 = \sum_{s\in\mathcal{S}}|\mathcal{N}_s^{(D)}| - \frac{\sum_{s\in\mathcal{S}}|\mathcal{N}_s^{(D)}|}{\sum_{s\in\mathcal{S}}|\mathcal{N}_s^{(E)}| + \sum_{s\in\mathcal{S}}|\mathcal{N}_s^{(D)}|}.(|\mathcal{N}|+1), \tag{164}$$

$$g_{11} = \sum_{s\in\mathcal{S}}|\mathcal{N}_s^{(E)}| - 2 - \frac{\sum_{s\in\mathcal{S}}|\mathcal{N}_s^{(E)}| - 2}{\sum_{s\in\mathcal{S}}|\mathcal{N}_s^{(E)}| + \sum_{s\in\mathcal{S}}|\mathcal{N}_s^{(D)}| - 2}.|\mathcal{N}|, \tag{165}$$

$$g_{12} = \sum_{s\in\mathcal{S}}|\mathcal{N}_s^{(D)}| - \frac{\sum_{s\in\mathcal{S}}|\mathcal{N}_s^{(D)}|}{\sum_{s\in\mathcal{S}}|\mathcal{N}_s^{(E)}| + \sum_{s\in\mathcal{S}}|\mathcal{N}_s^{(D)}| - 2}.|\mathcal{N}|, \tag{166}$$

$$g_{21} = \sum_{s\in\mathcal{S}} |\mathcal{N}_s^{(E)}| - 1 - \frac{\sum_{s\in\mathcal{S}} |\mathcal{N}_s^{(E)}| - 1}{\sum_{s\in\mathcal{S}} |\mathcal{N}_s^{(E)}| + \sum_{s\in\mathcal{S}} |\mathcal{N}_s^{(D)}| - 2}.|\mathcal{N}|, \tag{167}$$

$$g_{22} = \sum_{s\in\mathcal{S}} |\mathcal{N}_s^{(D)}| - 1 - \frac{\sum_{s\in\mathcal{S}} |\mathcal{N}_s^{(D)}| - 1}{\sum_{s\in\mathcal{S}} |\mathcal{N}_s^{(E)}| + \sum_{s\in\mathcal{S}} |\mathcal{N}_s^{(D)}| - 2}.|\mathcal{N}|, \tag{168}$$

$$g_{31} = \sum_{s\in\mathcal{S}} |\mathcal{N}_s^{(E)}| - \frac{\sum_{s\in\mathcal{S}} |\mathcal{N}_s^{(E)}|}{\sum_{s\in\mathcal{S}} |\mathcal{N}_s^{(E)}| + \sum_{s\in\mathcal{S}} |\mathcal{N}_s^{(D)}| - 2}.|\mathcal{N}|, \tag{169}$$

$$g_{32} = \sum_{s\in\mathcal{S}} |\mathcal{N}_s^{(D)}| - 2 - \frac{\sum_{s\in\mathcal{S}} |\mathcal{N}_s^{(D)}| - 2}{\sum_{s\in\mathcal{S}} |\mathcal{N}_s^{(E)}| + \sum_{s\in\mathcal{S}} |\mathcal{N}_s^{(D)}| - 2}.|\mathcal{N}|. \tag{170}$$

Proof. Using the KKT conditions for problem (**Surrogate**) (see Appendix A.2), we obtain (171) that is used to calculate upper bounds on $\left|\partial e_n^{(s)}/\partial w_{n'}^{(s')}\right|$ and $\left|\partial d_n^{(s)}/\partial w_{n'}^{(s')}\right|$ as indicated below.

$$\mathbf{Q_1}.\mathbf{Y_1} = \mathbf{B_1}, \tag{171}$$

where

$$\mathbf{Q_1} = \begin{bmatrix} \mathbf{J_1} & \mathbf{0} & \mathbf{J_3} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{J_2} & \mathbf{J_4} & \mathbf{0} & \mathbf{0} \\ \mathbf{J_3}^\top & \mathbf{J_4}^\top & \mathbf{0} & \mathbf{J_5} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{J_5}^\top & \mathbf{J_6} & \mathbf{J_7} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{J_7}^\top & \mathbf{J_8} \end{bmatrix}, \tag{172}$$

$$\mathbf{Y_1}^\top = \begin{bmatrix} \frac{\partial e}{\partial w} & \frac{\partial d}{\partial w} & \frac{\partial(\lambda+\lambda_{ref})}{\partial w} & \frac{\partial \lambda_{ref}}{\partial w} & \frac{\partial \theta}{\partial w} & \frac{\partial \mu}{\partial w} \end{bmatrix}, \tag{173}$$

$$\mathbf{B_1}^\top = \begin{bmatrix} -\,\partial f^{(E)}/\partial y\big|_{y=e} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{174}$$

Using the same KKT conditions for problem (**Surrogate**) (see Appendix A.2), we obtain (175) that is used to calculate upper bounds on $\left|\partial e_n^{(s)}/\partial v_{n'}^{(s')}\right|$ and $\left|\partial d_n^{(s)}/\partial v_{n'}^{(s')}\right|$ as indicated below.

$$\mathbf{Q_2}.\mathbf{Y_2} = \mathbf{B_2}, \tag{175}$$

where

$$\mathbf{Q_2} = \begin{bmatrix} \mathbf{J_1} & \mathbf{0} & \mathbf{J_3} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{J_2} & \mathbf{J_4} & \mathbf{0} & \mathbf{0} \\ \mathbf{J_3}^\top & \mathbf{J_4}^\top & \mathbf{0} & \mathbf{J_5} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{J_5}^\top & \mathbf{J_6} & \mathbf{J_7} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{J_7}^\top & \mathbf{J_8} \end{bmatrix}, \tag{176}$$

$$\mathbf{Y_2}^\top = \begin{bmatrix} \frac{\partial e}{\partial v} & \frac{\partial d}{\partial v} & \frac{\partial(\lambda+\lambda_{ref})}{\partial v} & \frac{\partial \lambda_{ref}}{\partial v} & \frac{\partial \theta}{\partial v} & \frac{\partial \mu}{\partial v} \end{bmatrix}, \tag{177}$$

$$\mathbf{B_2}^\top = \begin{bmatrix} 0 & \partial f^{(D)}/\partial y\big|_{y=d} & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{178}$$

In the matrices $\mathbf{Q_1}$ and $\mathbf{Q_2}$ defined by (172) and (176), $\mathbf{J_1}$ and $\mathbf{J_2}$ are matrices with the following fixed components,

$$\mathbf{J_1} = \left[diag\{ w_n^{(s)}. \frac{\partial^2 f^{(E)}}{\partial y^2}\Big|_{y=\hat{e}_n^{(s)}} \} \right]_{(\sum_{s\in\mathcal{S}} |\mathcal{N}_s^{(E)}|)\times(\sum_{s\in\mathcal{S}} |\mathcal{N}_s^{(E)}|)}, \tag{179}$$

$$\mathbf{J_2} = \left[diag\{ -v_n^{(s)}. \frac{\partial^2 f^{(D)}}{\partial y^2}\Big|_{\hat{d}_n^{(s)}} \} \right]_{(\sum_{s\in\mathcal{S}} |\mathcal{N}_s^{(D)}|)\times(\sum_{s\in\mathcal{S}} |\mathcal{N}_s^{(D)}|)}, \tag{180}$$

$\mathbf{J_3}$ and $\mathbf{J_4}$ are defined by (151) and (152), respectively, and $\mathbf{J_5}$, $\mathbf{J_6}$, $\mathbf{J_7}$, and $\mathbf{J_8}$ are matrices with the following components,

$$\mathbf{J_5} = \left[(J_5)_{nn'} \right]_{|\mathcal{N}|\times|\mathcal{N}|},$$
$$(J_5)_{nn'} = \begin{cases} \sum_{n''\in\mathcal{R}_n} (B_{nn''} + G_{nn''}.\theta_{nn''}), & \text{if } n' = n, \\ -B_{nn'} - G_{nn'}.\theta_{nn'}, & \text{if } n' \neq n, n' \in \mathcal{R}_n, \\ 0, & \text{otherwise}, \end{cases} \tag{181}$$

$$\mathbf{J_6} = \left[(J_6)_{nn'} \right]_{|\mathcal{N}|\times|\mathcal{N}|},$$
$$(J_6)_{nn'} = \begin{cases} \sum_{n''\in\mathcal{R}_n} G_{nn''}.(\lambda_n + \lambda_{n''} + \mu_{nn''} + \mu_{n''n}), & \text{if } n' = n, \\ -G_{nn'}.(\lambda_n + \lambda_{n'} + \mu_{nn'} + \mu_{n'n}), & \text{if } n' \neq n, n' \in \mathcal{R}_n, \\ 0, & \text{otherwise}, \end{cases} \tag{182}$$

$$\mathbf{J_7} = \left[(J_7)_{nn'} \right]_{|\mathcal{N}|\times\sum_{n\in\mathcal{N}} |\mathcal{R}_n|},$$
$$(J_7)_{nn'} = \begin{cases} \sum_{n''\in\mathcal{R}_n} (B_{nn''} + G_{nn''}.\theta_{nn''}), & \text{if } n' = \sum_{n''<n} (|\mathcal{R}_{n''}| - 1) + 1, \\ -B_{n'n} - G_{n'n}.\theta_{n'n}, & \text{if line } nn' \text{ is } \left(n' - \sum_{n''<n} (|\mathcal{R}_{n''}| - 1)\right)_{th} \text{ line}, \\ 0, & \text{otherwise}, \end{cases} \tag{183}$$

$$\mathbf{J_8} = \left[(J_8)_{\kappa1,\kappa2} \right]_{\sum_{n\in\mathcal{N}} |\mathcal{R}_n|\times\sum_{n\in\mathcal{N}} |\mathcal{R}_n|},$$
$$(J_8)_{\kappa1,\kappa2} = \begin{cases} B_{nn'}.\theta_{nn'} + \frac{1}{2}.G_{nn'}.\theta_{nn'}^2 - \overline{F}_{nn'}, & \text{if line } n \text{ to } n' \text{ is } \left(\kappa2 - \sum_{n''<n} (|\mathcal{R}_{n''}| - 1)\right)_{th} \\ & \quad \text{line from node } \left(\kappa1 - \sum_{n''<n'} (|\mathcal{R}_{n''}| - 1)\right), \\ 0, & \text{otherwise}. \end{cases} \tag{184}$$

We can write the matrices $\mathbf{Q_1}$ and $\mathbf{Q_2}$, defined by (172) and (176), as

$$\mathbf{Q_1} = \tilde{\mathbf{Q}} + \mathbf{\Delta Q}, \tag{185}$$

$$\mathbf{Q_2} = \tilde{\mathbf{Q}} + \mathbf{\Delta Q}, \tag{186}$$

where

$$\tilde{\mathbf{Q}} = \begin{bmatrix} \mathbf{J_1} & \mathbf{0} & \mathbf{J_3} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{J_2} & \mathbf{J_4} & \mathbf{0} & \mathbf{0} \\ \mathbf{J_3^\top} & \mathbf{J_4^\top} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix}, \tag{187}$$

$$\mathbf{\Delta Q} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{J_5} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{J_5^\top} & \mathbf{J_6} - \mathbf{I} & \mathbf{J_7} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{J_7^\top} & \mathbf{J_8} - \mathbf{I} \end{bmatrix}. \tag{188}$$

Using (171), (175), and the Sherman-Morrison-Woodbury formula [74], we obtain

$$\mathbf{Y_1} = \tilde{\mathbf{Q}}^{\mathbf{-1}}.\mathbf{B_1} - \tilde{\mathbf{Q}}^{\mathbf{-1}}.\left(\mathbf{I} - \mathbf{\Delta Q}.\tilde{\mathbf{Q}}^{\mathbf{-1}}\right)^{-1}.\mathbf{\Delta Q}.\tilde{\mathbf{Q}}^{\mathbf{-1}}.\mathbf{B_1}, \tag{189}$$

$$\mathbf{Y_2} = \tilde{\mathbf{Q}}^{\mathbf{-1}}.\mathbf{B_2} - \tilde{\mathbf{Q}}^{\mathbf{-1}}.\left(\mathbf{I} - \mathbf{\Delta Q}.\tilde{\mathbf{Q}}^{\mathbf{-1}}\right)^{-1}.\mathbf{\Delta Q}.\tilde{\mathbf{Q}}^{\mathbf{-1}}.\mathbf{B_2}. \tag{190}$$

Consider the second terms in the RHSs of (189) and (190); by some algebra one can show that the second term of the vector $\mathbf{Y_1}$ in (173) (corresponding to $[\partial e/\partial w \ \ \partial d/\partial w]^\top$) and the second term of the vector $\mathbf{Y_2}$ in (177) (corresponding to $[\partial e/\partial v \ \ \partial d/\partial v]^\top$) are equal to zero. Thus, we can write

$$\left|\frac{\partial e_n^{(s)}}{\partial w_{n'}^{(s')}}\right| = \frac{\left.\frac{\partial f^{(E)}}{\partial y}\right|_{y=\hat{e}_{n'}^{(s')}} \cdot \left|\det \tilde{\mathbf{Q}}_{-(\mathbf{n},\mathbf{s}),(\mathbf{n'},\mathbf{s'})}\right|}{\left|\det \tilde{\mathbf{Q}}\right|}, \tag{191}$$

$$\left|\frac{\partial e_n^{(s)}}{\partial v_{n'}^{(s')}}\right| = \frac{\left.\frac{\partial f^{(D)}}{\partial y}\right|_{y=\hat{d}_{n'}^{(s')}} \cdot \left|\det \tilde{\mathbf{Q}}_{-(\mathbf{n},\mathbf{s}),(\mathbf{n'},\mathbf{s'})}\right|}{\left|\det \tilde{\mathbf{Q}}\right|}, \tag{192}$$

$$\left|\frac{\partial d_n^{(s)}}{\partial w_{n'}^{(s')}}\right| = \frac{\left.\frac{\partial f^{(E)}}{\partial y}\right|_{y=\hat{e}_{n'}^{(s')}} \cdot \left|\det \tilde{\mathbf{Q}}_{-(\mathbf{n},\mathbf{s}),(\mathbf{n'},\mathbf{s'})}\right|}{\left|\det \tilde{\mathbf{Q}}\right|}, \tag{193}$$

$$\left|\frac{\partial d_n^{(s)}}{\partial v_{n'}^{(s')}}\right| = \frac{\left.\frac{\partial f^{(D)}}{\partial y}\right|_{y=\hat{d}_{n'}^{(s')}} \cdot \left|\det \tilde{\mathbf{Q}}_{-(\mathbf{n},\mathbf{s}),(\mathbf{n'},\mathbf{s'})}\right|}{\left|\det \tilde{\mathbf{Q}}\right|}, \tag{194}$$

where $\tilde{\mathbf{Q}}_{-(\mathbf{n},\mathbf{s}),(\mathbf{n}',\mathbf{s}')}$ results from $\tilde{\mathbf{Q}}$ when the corresponding row of generator/demand n of agent s and column of generator/demand n' of agent s' is eliminated.

By the Schur complement formula, we have

$$\begin{aligned}\left|\det \tilde{\mathbf{Q}}\right| &= \det\left|\begin{bmatrix}\mathbf{J_1} & \mathbf{0} & \mathbf{J_3}\\ \mathbf{0} & \mathbf{J_2} & \mathbf{J_4}\\ \mathbf{J_3}^\top & \mathbf{J_4}^\top & \mathbf{0}\end{bmatrix}\right| . \det\left|\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}\right| \\ &= \left|\det\begin{bmatrix}\mathbf{J_1} & \mathbf{0}\\ \mathbf{0} & \mathbf{J_2}\end{bmatrix}\right| . \left|\det\left(\begin{bmatrix}\mathbf{J_3}^\top & \mathbf{J_4}^\top\end{bmatrix}\begin{bmatrix}\mathbf{J_1^{-1}} & \mathbf{0}\\ \mathbf{0} & \mathbf{J_2^{-1}}\end{bmatrix}\begin{bmatrix}\mathbf{J_3} & \mathbf{J_4}\end{bmatrix}^\top\right)\right|.\end{aligned} \tag{195}$$

Using the definitions of h_e, h_d, H_e, H_d, $\overline{e}$, and $\overline{\overline{d}}$ in (157), (158), (159), (160), (161), and (162), respectively, and combining them with (37) and (38), namely $w_n^{(s)}.\frac{\partial f^{(E)}}{\partial y}\Big|_{y=\hat{e}_n^{(s)}} = \frac{\partial c_n^{(s)}}{\partial e_n^{(s)}}\Big|_{\hat{e}_n^{(s)}}$ and $v_n^{(s)}.\frac{\partial f^{(D)}}{\partial y}\Big|_{y=\hat{d}_n^{(s)}} = \frac{\partial u_n^{(s)}}{\partial d_n^{(s)}}\Big|_{\hat{d}_n^{(s)}}$, in the proposed tâtonnement-process, gives the following bounds on the diagonal elements of matrices $\mathbf{J_1}$ and $\mathbf{J_2}$.

$$\frac{h_e}{\gamma_e} \le w_n^{(s)}.\left.\frac{\partial^2 f^{(E)}}{\partial y^2}\right|_{y=\hat{e}_n^{(s)}} \le \frac{H_e}{\gamma_e}\exp(\frac{\overline{e}}{\gamma_e}), \qquad \forall s\in\mathcal{S}, \forall n \in \mathcal{N}_s^{(E)}, \tag{196}$$

$$\frac{h_d}{\gamma_d+\overline{\overline{d}}} \le -v_n^{(s)}.\left.\frac{\partial^2 f^{(D)}}{\partial y^2}\right|_{\hat{d}_n^{(s)}} \le \frac{H_d}{\gamma_d}, \qquad \forall s\in\mathcal{S}, \forall n \in \mathcal{N}_s^{(D)}. \tag{197}$$

Using (195) and the bounds of (196)-(197), and after some algebra, we can show that

$$\left|\det \tilde{\mathbf{Q}}\right| \ge \left(\frac{h_e}{\gamma_e}\right)^{p_1} . \left(\frac{h_d}{\gamma_d+\overline{\overline{d}}}\right)^{p_2} . \left|\det\left(\begin{bmatrix}\mathbf{J_3}^\top & \mathbf{J_4}^\top\end{bmatrix}\begin{bmatrix}\mathbf{J_3} & \mathbf{J_4}\end{bmatrix}^\top\right)\right|, \tag{198}$$

$$\left|\det \tilde{\mathbf{Q}}_{-(\mathbf{n},\mathbf{s}),(\mathbf{n}',\mathbf{s}')}\right| \le \left(\frac{H_e}{\gamma_e}\exp(\frac{\overline{e}}{\gamma_e})\right)^{g_1} . \left(\frac{H_d}{\gamma_d}\right)^{g_2} . \left|\det\left(\begin{bmatrix}\mathbf{J_3}^\top & \mathbf{J_4}^\top\end{bmatrix}\begin{bmatrix}\mathbf{J_3} & \mathbf{J_4}\end{bmatrix}^\top\right)_{-(n,s),(n',s')}\right|, \tag{199}$$

where

$$(g_1,g_2) = \begin{cases}(g_{11},g_{12}), & \text{if } (n,s) \text{ and } (n',s') \text{ both generators,}\\ (g_{21},g_{22}), & \text{if } (n,s) \text{ is a demand and } (n',s') \text{ is a generator, or vice-versa,}\\ (g_{31},g_{32}), & \text{if } (n,s) \text{ and } (n',s') \text{ both demands,}\end{cases} \tag{200}$$

and p_1, p_2, g_{11}, g_{12}, g_{21}, g_{22}, g_{31}, and g_{32} are defined by (163), (164), (165), (166), (167), (168), (169), and (170), respectively.

Substituting (198) and (199) in (191)-(194), we obtain the upper bounds (147)-(150). □

Fourth stage:

Now assume that the conditions (45) and (46) are satisfied, where $EI_{(n,s)}$ and $DI_{(n,s)}$ are fixed indices for producers and consumers defined by

$$EI_{(n,s)} = \sum_{s' \in \mathcal{S}} \left(\sum_{n' \in \mathcal{N}_s^{(E)}} \frac{\left| \det \left(\begin{bmatrix} \mathbf{J_3}^\top & \mathbf{J_4}^\top \end{bmatrix} \begin{bmatrix} \mathbf{J_3} & \mathbf{J_4} \end{bmatrix}^\top \right)_{-(n,s),(n',s')} \right|}{\left| \det \left(\begin{bmatrix} \mathbf{J_3}^\top & \mathbf{J_4}^\top \end{bmatrix} \begin{bmatrix} \mathbf{J_3} & \mathbf{J_4} \end{bmatrix}^\top \right) \right|} \right), \tag{201}$$

$$DI_{(n,s)} = \sum_{s' \in \mathcal{S}} \left(\sum_{n' \in \mathcal{N}_s^{(D)}} \frac{\left| \det \left(\begin{bmatrix} \mathbf{J_3}^\top & \mathbf{J_4}^\top \end{bmatrix} \begin{bmatrix} \mathbf{J_3} & \mathbf{J_4} \end{bmatrix}^\top \right)_{-(n,s),(n',s')} \right|}{\left| \det \left(\begin{bmatrix} \mathbf{J_3}^\top & \mathbf{J_4}^\top \end{bmatrix} \begin{bmatrix} \mathbf{J_3} & \mathbf{J_4} \end{bmatrix}^\top \right) \right|} \right). \tag{202}$$

By some algebra we can show that constraints (137) and (138) are satisfied. These constraints are the conditions sufficient to guarantee that the tâtonnement-process described by (37)-(40) is a contraction map. Therefore, by Banach's fixed point theorem, this tâtonnement-process converges and has a unique fixed point.

Fifth stage:

Claim 5. *The unique fixed point of the tâtonnement-process described by (37)-(40) is a Nash equilibrium of the game induced by the mechanism proposed in Section 4.*

Proof. Let $\mathbf{m} = (\vec{\mathbf{w}}, \vec{\mathbf{v}}, \vec{\mathbf{p}}, \vec{\mathbf{q}})$ be the fixed point of the tâtonnement-process described by (37)-(40). Then, according to (37) and (38), we have

$$w_n^{(s)} = \left. \frac{\partial c_n^{(s)}}{\partial e_n^{(s)}} \right|_{\hat{e}_n^{(s)}(\vec{\mathbf{m}})} \Bigg/ \left. \frac{\partial f^{(E)}}{\partial y} \right|_{y=\hat{e}_n^{(s)}(\vec{\mathbf{m}})}, \qquad \forall s \in \mathcal{S}, \forall n \in \mathcal{N}_s^{(E)}, \tag{203}$$

$$v_n^{(s)} = \left. \frac{\partial u_n^{(s)}}{\partial d_n^{(s)}} \right|_{\hat{d}_n^{(s)}(\vec{\mathbf{m}})} \Bigg/ \left. \frac{\partial f^{(D)}}{\partial y} \right|_{y=\hat{d}_n^{(s)}(\vec{\mathbf{m}})}, \qquad \forall s \in \mathcal{S}, \forall n \in \mathcal{N}_s^{(D)}. \tag{204}$$

Since the production vector $(\hat{e}_n^{(s)}(\vec{\mathbf{m}}))_{s \in \mathcal{S}, n \in \mathcal{N}_s^{(E)}}$ and the consumption vector $(\hat{d}_n^{(s)}(\vec{\mathbf{m}}))_{s \in \mathcal{S}, n \in \mathcal{N}_s^{(D)}}$ satisfy the KKT conditions of problem (**Surrogate**) (see Appendix A.2), using (203)-(204) and some algebra, we can show that the production vector $(\hat{e}_n^{(s)}(\vec{\mathbf{m}}))_{s \in \mathcal{S}, n \in \mathcal{N}_s^{(E)}}$ and the consumption vector $(\hat{d}_n^{(s)}(\vec{\mathbf{m}}))_{s \in \mathcal{S}, n \in \mathcal{N}_s^{(D)}}$ also satisfy the KKT conditions of problem (**OPF**) (see Appendix A.1). Therefore, the production vector $(\hat{e}_n^{(s)}(\vec{\mathbf{m}}))_{s \in \mathcal{S}, n \in \mathcal{N}_s^{(E)}}$ and the consumption vector $(\hat{d}_n^{(s)}(\vec{\mathbf{m}}))_{s \in \mathcal{S}, n \in \mathcal{N}_s^{(D)}}$ are equal to the unique optimal solution of problem (**OPF**), i.e.,

$$\hat{e}_n^{(s)}(\vec{\mathbf{m}}) = (e_n^{(s)})^*, \qquad \forall s \in \mathcal{S}, \forall n \in \mathcal{N}_s^{(E)}, \tag{205}$$

$$\hat{d}_n^{(s)}(\vec{\mathbf{m}}) = (d_n^{(s)})^*, \qquad \forall s \in \mathcal{S}, \forall n \in \mathcal{N}_s^{(D)}. \tag{206}$$

Substituting (205)-(206) in (203)-(204) and comparing the results with (85)-(86) proves that $(\vec{\mathbf{w}}, \vec{\mathbf{v}}, \vec{\mathbf{p}}, \vec{\mathbf{q}}) = (\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*, \vec{\mathbf{x}}^*, \vec{\mathbf{q}}^*)$. Hence, the Nash equilibrium $\vec{\mathbf{m}}^* = (\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*, \vec{\mathbf{p}}^*, \vec{\mathbf{q}}^*)$ described by (85)-(88) is the unique fixed point of the tâtonnement-process described by (37)-(40) (Note that in the proof of Lemma (5.2), it is shown that the message profile $\vec{\mathbf{m}}^* = (\vec{\mathbf{w}}^*, \vec{\mathbf{v}}^*, \vec{\mathbf{p}}^*, \vec{\mathbf{q}}^*)$ described by (85)-(88) is a Nash equilibrium of the game induced by the mechanism proposed in Section 4). □

□

J Proof of Theorem 8; Convergence of Modified Tâtonnement-Process

Proof. Using the result of Theorem 6, we conclude that the allocations (energy production and consumption) resulting at each step of the modified tâtonnement-process are feasible solutions of the problem (**OPF**). In the following, we prove the convergence of modified tâtonnement-process in three stages.

First stage:

In the modified tâtonnement-process described by (53)-(54) and (39)-(40), $(p_{n'}^{(s)})^{(\tau+1)}$ and $(q_{nn'}^{(s)})^{(\tau+1)}$ are solely functions of previously announced message profile $(w_n^{(s)})^{(\tau)}$ and $(v_n^{(s)})^{(\tau)}$. Thus, they do not have any impact on the announced messages of subsequent rounds, $(w_n^{(s)})^{(\tau')}$ and $(v_n^{(s)})^{(\tau')}$ $\forall \tau' > \tau$. Therefore, they do not affect the convergence of the modified tâtonnement-process and can be ommited from the convergence study.

As a result of the above observation, proving the convergence of the tâtonnement-process described by (53)-(54) will establish the convergence of the tâtonnement-process described by (53)-(54) and (39)-(40). Therefore, we study the convergence of the tâtonnement-process described by (53)-(54) by choosing (41) and (42) as surrogate cost and utility functions, respectively. Then, the tâtonnement-process in (53)-(54) is rewritten as (207)-(208).

$$(w_n^{(s)})^{(\tau+1)} = (1-b).(w_n^{(s)})^{(\tau)} + b.\zeta_n^{(s)}\left(\vec{\mathbf{w}}^{(\tau)}, \vec{\mathbf{v}}^{(\tau)}\right), \qquad \forall s \in \mathcal{S}, \forall n \in \mathcal{N}_s^{(E)}, \tag{207}$$

$$(v_n^{(s)})^{(\tau+1)} = (1-b).(v_n^{(s)})^{(\tau)} + b.\xi_n^{(s)}\left(\vec{\mathbf{w}}^{(\tau)}, \vec{\mathbf{v}}^{(\tau)}\right), \qquad \forall s \in \mathcal{S}, \forall n \in \mathcal{N}_s^{(D)}, \tag{208}$$

where $\zeta_n^{(s)}\left(\vec{\mathbf{w}}^{(\tau)}, \vec{\mathbf{v}}^{(\tau)}\right)$ and $\xi_n^{(s)}\left(\vec{\mathbf{w}}^{(\tau)}, \vec{\mathbf{v}}^{(\tau)}\right)$ are defined by (133) and (134), respectively.

Second stage:

We prove the following inequalities.

$$\sum_{s'\in\mathcal{S}}\left(\sum_{n'\in\mathcal{N}_s^{(E)}}\left|\frac{\partial \zeta_n^{(s)}}{\partial w_{n'}^{(s')}}(\vec{\mathbf{w}}, \vec{\mathbf{v}})\right| + \sum_{n'\in\mathcal{N}_s^{(D)}}\left|\frac{\partial \zeta_n^{(s)}}{\partial v_{n'}^{(s')}}(\vec{\mathbf{w}}, \vec{\mathbf{v}})\right|\right) < L, \qquad \forall \vec{\mathbf{w}}, \forall \vec{\mathbf{v}}, \forall s \in \mathcal{S}, \forall n \in \mathcal{N}_s^{(E)}, \tag{209}$$

$$\sum_{s'\in\mathcal{S}}\left(\sum_{n'\in\mathcal{N}_s^{(E)}}\left|\frac{\partial \xi_n^{(s)}}{\partial w_{n'}^{(s')}}(\vec{\mathbf{w}}, \vec{\mathbf{v}})\right| + \sum_{n'\in\mathcal{N}_s^{(D)}}\left|\frac{\partial \xi_n^{(s)}}{\partial v_{n'}^{(s')}}(\vec{\mathbf{w}}, \vec{\mathbf{v}})\right|\right) < L, \qquad \forall \vec{\mathbf{w}}, \forall \vec{\mathbf{v}}, \forall s \in \mathcal{S}, \forall n \in \mathcal{N}_s^{(D)}, \tag{210}$$

where L is defined in (55). Using the results of Claims 2, 3, and 4 of Theorem 7 (appearing in Appendix I), the maximum of LHSs of (209) and (210) are equal to $1/\sigma_n^{(s)}$ and $1/\rho_n^{(s)}$, respectively, where $\sigma_n^{(s)}$ and $\rho_n^{(s)}$ are defined as the ratios of RHS to the LHS of (45) and (46) (or equivalently the ratios of RHS to the LHS of (51) and (52) for the case of quadratic cost and utility functions). Thus, the inequalities (209) and (210) are satisfied.

Third stage:

Because of inequalities (209) and (210), the tâtonnement-process described by (133)-(134) is a L-Lipschitzian map [75]. As a result of Theorem 2.3 in [75], the modified tâtonnement-process described by (207)-(208) converges to its fixed point, which is the same as the fixed point of the tâtonnement-process desceibed by (37)-(40). As established before, this fixed point is a Nash equilibrium of the game induced by the mechanism proposed in Section 4.

□

References

[1] C. Harris, *Electricity markets: pricing, structures and economics*, vol. 565. John Wiley & Sons, 2011.

[2] J. Lin and F. H. Magnago, *Electricity Markets: Theories and Applications.* John Wiley & Sons, 2017.

[3] W. W. Hogan, "Electricity market restructuring: reforms of reforms," *Journal of Regulatory Economics*, vol. 21, no. 1, pp. 103–132, 2002.

[4] S. M. Harvey and W. W. Hogan, "On the exercise of market power through strategic withholding in California," *World Wide Web: http://www. aeibrookings. org/publications/related/market.pdf*, 2001.

[5] P. L. Joskow and E. Kohn, "A quantitative analysis of pricing behavior in California's wholesale electricity market during summer 2000," *The Energy Journal*, vol. 23, no. 4, 2002.

[6] S. Borenstein, "The trouble with electricity markets: understanding California's restructuring disaster," *Journal of Economic Perspectives*, vol. 16, no. 1, pp. 191–211, 2002.

[7] P. L. Joskow, "California's electricity crisis," *Oxford review of Economic Policy*, vol. 17, no. 3, pp. 365–388, 2001.

[8] P. D. Klemperer and M. A. Meyer, "Supply function equilibria in oligopoly under uncertainty," *Econometrica: Journal of the Econometric Society*, pp. 1243–1277, 1989.

[9] R. Baldick, "Electricity market equilibrium models: The effect of parametrization," *IEEE Transactions on Power Systems*, vol. 17, no. 4, pp. 1170–1176, 2002.

[10] R. Egging-Bratseth, T. Baltensperger, and A. Tomasgard, "Solving oligopolistic equilibrium problems with convex optimization," *European Journal of Operational Research*, vol. 284, no. 1, pp. 44–52, 2020.

[11] A. Sheffrin, "Empirical evidence of strategic bidding in the California ISO real-time market," in *Electricity pricing in transition*, pp. 267–281, Springer, 2002.

[12] H.-P. Chao and S. Peck, "A market mechanism for electric power transmission," *Journal of Regulatory Economics*, vol. 10, no. 1, pp. 25–59, 1996.

[13] P. L. Joskow and J. Tirole, "Transmission rights and market power on electric power networks," *The Rand Journal of Economics*, pp. 450–487, 2000.

[14] T. Kristiansen, "Markets for financial transmission rights," *Energy Studies Review*, vol. 13, no. 1, pp. 25–74, 2004.

[15] P. Joskow and J. Tirole, "Merchant transmission investment," *The Journal of Industrial Economics*, vol. 53, no. 2, pp. 233–264, 2005.

[16] E. Miehling and D. Teneketzis, "Multilateral trades in interconnected power systems: a local public goods approach," in *Proceedings of the 3rd international conference on High confidence networked systems*, pp. 127–134, 2014.

[17] F. F. Wu and P. Varaiya, "Coordinated multilateral trades for electric power networks: theory and implementation," *International Journal of Electrical Power & Energy Systems*, vol. 21, no. 2, pp. 75–102, 1999.

[18] T. Groves and J. Ledyard, "Optimal allocation of public goods: A solution to the free-rider problem," *Econometrica: Journal of the Econometric Society*, pp. 783–809, 1977.

[19] L. Hurwicz, "Outcome functions yielding Walrasian and Lindahl allocations at Nash equilibrium points," *The Review of Economic Studies*, vol. 46, no. 2, pp. 217–225, 1979.

[20] Y. Chen, "A family of supermodular Nash mechanisms implementing Lindahl allocations," *Economic Theory*, vol. 19, no. 4, pp. 773–790, 2002.

[21] M. Walker, "A simple incentive compatible scheme for attaining Lindahl allocations," *Econometrica: Journal of the Econometric Society*, pp. 65–71, 1981.

[22] T. Kim, "A stable Nash mechanism implementing Lindahl allocations for quasi-linear environments," *Journal of Mathematical Economics*, vol. 22, no. 4, pp. 359–371, 1993.

[23] T. M. Stoenescu, M. Liu, and D. Teneketzis, "Multirate multicast service provisioning I: an algorithm for optimal price splitting along multicast trees," *Mathematical Methods of Operations Research*, vol. 65, no. 2, pp. 199–228, 2007.

[24] T. M. Stoenescu, M. Liu, and D. Teneketzis, "Multirate multicast service provisioning II: a tâtonnement process for rate allocation," *Mathematical Methods of Operations Research*, vol. 65, no. 3, pp. 389–415, 2007.

[25] A. Kakhbod and D. Teneketzis, "An efficient game form for multi-rate multicast service provisioning," *IEEE Journal on Selected Areas in Communications*, vol. 30, no. 11, pp. 2093–2104, 2012.

[26] S. Sharma and D. Teneketzis, "A game-theoretic approach to decentralized optimal power allocation for cellular networks," *Telecommunication Systems*, vol. 47, no. 1-2, pp. 65–80, 2011.

[27] S. Sharma and D. Teneketzis, "Local public good provisioning in networks: A Nash implementation mechanism," *IEEE Journal on Selected Areas in Communications*, vol. 30, no. 11, pp. 2105–2116, 2012.

[28] A. Kakhbod and D. Teneketzis, "Power allocation and spectrum sharing in multi-user, multi-channel systems with strategic users," in *49th IEEE Conference on Decision and Control (CDC)*, pp. 1088–1095, IEEE, 2010.

[29] P. Naghizadeh and M. Liu, "Opting out of incentive mechanisms: A study of security as a non-excludable public good," *IEEE Transactions on Information Forensics and Security*, vol. 11, no. 12, pp. 2790–2803, 2016.

[30] M. G. Kallitsis, G. Michailidis, and M. Devetsikiotis, "Optimal power allocation under communication network externalities," *IEEE Transactions on Smart Grid*, vol. 3, no. 1, pp. 162–173, 2011.

[31] A. L. Motto, F. D. Galiana, A. J. Conejo, and M. Huneault, "On Walrasian equilibrium for pool-based electricity markets," *IEEE Transactions on Power Systems*, vol. 17, no. 3, pp. 774–781, 2002.

[32] E. Miehling and D. Teneketzis, "A decentralized mechanism for computing competitive equilibria in deregulated electricity markets," in *2016 American Control Conference (ACC)*, pp. 4107–4113, IEEE, 2016.

[33] A. J. Conejo and J. A. Aguado, "Multi-area coordinated decentralized DC optimal power flow," *IEEE Transactions on Power Systems*, vol. 13, no. 4, pp. 1272–1278, 1998.

[34] J. H. Chow, W. De Mello, and K. W. Cheung, "Electricity market design: An integrated approach to reliability assurance," *Proceedings of the IEEE*, vol. 93, no. 11, pp. 1956–1969, 2005.

[35] F. D. Galiana, A. L. Motto, A. J. Conejo, and M. Huneault, "Decentralized nodal-price self-dispatch and unit commitment," in *The next generation of electric power unit commitment models*, pp. 271–292, Springer, 2002.

[36] D. Gan and E. Litvinov, "Energy and reserve market designs with explicit consideration to lost opportunity costs," *IEEE Transactions on Power Systems*, vol. 18, no. 1, pp. 53–59, 2003.

[37] F. Abbaspourtorbati, A. J. Conejo, J. Wang, and R. Cherkaoui, "Pricing electricity through a stochastic non-convex market-clearing model," *IEEE Transactions on Power Systems*, vol. 32, no. 2, pp. 1248–1259, 2017.

[38] C. Ruiz, A. J. Conejo, and S. A. Gabriel, "Pricing non-convexities in an electricity pool," *IEEE Transactions on Power Systems*, vol. 27, no. 3, pp. 1334–1342, 2012.

[39] P. R. Gribik, W. W. Hogan, S. L. Pope, *et al.*, "Market-clearing electricity prices and energy uplift," *Cambridge, MA*, 2007.

[40] S. Frank, I. Steponavice, and S. Rebennack, "Optimal power flow: a bibliographic survey I," *Energy Systems*, vol. 3, no. 3, pp. 221–258, 2012.

[41] C. Silva, B. F. Wollenberg, and C. Z. Zheng, "Application of mechanism design to electric power markets (republished)," *IEEE Transactions on Power Systems*, vol. 16, no. 4, pp. 862–869, 2001.

[42] Y. Xu and S. H. Low, "An efficient and incentive compatible mechanism for wholesale electricity markets," *IEEE Transactions on Smart Grid*, vol. 8, no. 1, pp. 128–138, 2015.

[43] P. G. Sessa, N. Walton, and M. Kamgarpour, "Exploring the Vickrey-Clarke-Groves mechanism for electricity markets," *IFAC-PapersOnLine*, vol. 50, no. 1, pp. 189–194, 2017.

[44] O. Karaca and M. Kamgarpour, "Core-selecting mechanisms in electricity markets," *IEEE Transactions on Smart Grid*, 2019.

[45] M. Rasouli and D. Teneketzis, "An efficient market design for electricity networks with strategic users possessing local information," *IEEE Transactions on Control of Network Systems*, vol. 6, no. 3, pp. 1038–1049, 2019.

[46] R. Ferrero, S. Shahidehpour, and V. Ramesh, "Transaction analysis in deregulated power systems using game theory," *IEEE Transactions on Power Systems*, vol. 12, no. 3, pp. 1340–1347, 1997.

[47] J. Lavaei and S. Sojoudi, "Competitive equilibria in electricity markets with nonlinearities," in *2012 American Control Conference (ACC)*, pp. 3081–3088, IEEE, 2012.

[48] M. Rayati, M. Toulabi, and A. M. Ranjbar, "Optimal generalized bayesian Nash equilibrium of frequency-constrained electricity market in the presence of renewable energy sources," *IEEE Transactions on Sustainable Energy*, vol. 11, no. 1, pp. 136–144, 2018.

[49] A. Sheikhi, M. Rayati, S. Bahrami, and A. M. Ranjbar, "Integrated demand side management game in smart energy hubs," *IEEE Transactions on Smart Grid*, vol. 6, no. 2, pp. 675–683, 2015.

[50] S. J. Kazempour and H. Zareipour, "Equilibria in an oligopolistic market with wind power production," *IEEE Transactions on Power Systems*, vol. 29, no. 2, pp. 686–697, 2013.

[51] S. De La Torre, J. Contreras, and A. J. Conejo, "Finding multiperiod Nash equilibria in pool-based electricity markets," *IEEE Transactions on Power Systems*, vol. 19, no. 1, pp. 643–651, 2004.

[52] L. A. Barroso, R. D. Carneiro, S. Granville, M. V. Pereira, and M. H. Fampa, "Nash equilibrium in strategic bidding: A binary expansion approach," *IEEE Transactions on Power Systems*, vol. 21, no. 2, pp. 629–638, 2006.

[53] F. Farhadi, S. J. Golestani, and D. Teneketzis, "A surrogate optimization-based mechanism for resource allocation and routing in networks with strategic agents," *IEEE Transactions on Automatic Control*, vol. 64, no. 2, pp. 464–479, 2018.

[54] A. Ehrenmann and K. Neuhoff, "A comparison of electricity market designs in networks," *Operations Research*, vol. 57, no. 2, pp. 274–286, 2009.

[55] B. Allaz and J.-L. Vila, "Cournot competition, forward markets and efficiency," *Journal of Economic Theory*, vol. 59, no. 1, pp. 1–16, 1993.

[56] J. J. Wang and J. F. Zender, "Auctioning divisible goods," *Economic Theory*, vol. 19, no. 4, pp. 673–705, 2002.

[57] J. Nash, "Non-cooperative games," *Annals of Mathematics*, pp. 286–295, 1951.

[58] S. Reichelstein and S. Reiter, "Game forms with minimal message spaces," *Econometrica: Journal of the Econometric Society*, pp. 661–692, 1988.

[59] W. Hogan, "Financial transmission right formulations," tech. rep., Mimeo, JFK School of Government, Harvard Electricity Policy Group, Harvard, 2002.

[60] X. Ma, D. Sun, and A. Ott, "Implementation of the PJM financial transmission rights auction market system," in *IEEE Power Engineering Society Summer Meeting,*, vol. 3, pp. 1360–1365, IEEE, 2002.

[61] K. Lyons, H. Fraser, and H. Parmesano, "An introduction to financial transmission rights," *The Electricity Journal*, vol. 13, no. 10, pp. 31–37, 2000.

[62] F. C. Schweppe, M. C. Caramanis, R. D. Tabors, and R. E. Bohn, *Spot pricing of electricity*. Springer Science & Business Media, 2013.

[63] R. D. Pope, "The generalized envelope theorem and price uncertainty," *International Economic Review*, pp. 75–86, 1980.

[64] S. Reiter, "There is no adjustment process with 2-dimensional message spaces for "counter-examples,"," *Northwestern University, MimeoI*, 1979.

[65] J. Jordan, "The informational requirements of local stability in decentralized allocation mechanisms," *Information, incentives and economic mechanisms: essays in honor of Leonid Hurwicz*, pp. 183–212, 1987.

[66] K. Mount and S. Reiter, "On the existence of a locally stable dynamic process with a statically minimal message space," *Information, incentives and economic mechanisms: essays in honor of Leonid Hurwicz*, 1987.

[67] S. Sanghavi and B. Hajek, "A new mechanism for the free-rider problem," *IEEE Transactions on Automatic Control*, vol. 53, no. 5, pp. 1176–1183, 2008.

[68] J. S. Jordan, "Instability in the implementation of Walrasian allocations," *Journal of Economic Theory*, vol. 39, no. 2, pp. 301–328, 1986.

[69] T. Kim, *Stability problems in the implementation of Lindahl allocations.* University of Minnesota, 1987.

[70] R. D. Zimmerman, C. E. Murillo-Sánchez, and R. J. Thomas, "MATPOWER: Steady-state operations, planning, and analysis tools for power systems research and education," *IEEE Transactions on Power Systems*, vol. 26, no. 1, pp. 12–19, 2010.

[71] E. Litvinov, T. Zheng, G. Rosenwald, and P. Shamsollahi, "Marginal loss modeling in LMP calculation," *IEEE transactions on Power Systems*, vol. 19, no. 2, pp. 880–888, 2004.

[72] R. R. Shoults, W. M. Grady, and S. Helmick, "An efficient method for computing loss formula coefficients based upon the method of least squares," *IEEE Transactions on Power Apparatus and Systems*, no. 6, pp. 2144–2152, 1979.

[73] J. Conlisk, "Quick stability checks and matrix norms," *Economica*, vol. 40, no. 160, pp. 402–409, 1973.

[74] M. A. Woodbury and M. Woodbury, "Inverting modified matrices," 1950.

[75] J. Schu, "Iterative construction of fixed points of asymptotically nonexpansive mappings," *Journal of Mathematical Analysis and Applications*, vol. 158, no. 2, pp. 407–413, 1991.